Chapter 1 Quadratic Equations in One Unknown

Follow-up

pp.10 – 35

1.1 (a) p.10

$$(x+3)(3x+1)=0$$
$$x+3=0 \quad \text{or} \quad 3x+1=0$$
$$x=\underline{\underline{-3 \text{ or } -\frac{1}{3}}}$$

(b)

$$x^2+12=7x$$
$$x^2-7x+12=0$$
$$(x-3)(x-4)=0$$
$$x-3=0 \text{ or } x-4=0$$
$$x=\underline{\underline{3 \text{ or } 4}}$$

(c)

$$4x^2+3x=1$$
$$4x^2+3x-1=0$$
$$(4x-1)(x+1)=0$$
$$x+1=0 \quad \text{or} \quad 4x-1=0$$
$$x=\underline{\underline{-1 \text{ or } \frac{1}{4}}}$$

(d)

$$x^2=\frac{3-5x}{2}$$
$$2x^2=3-5x$$
$$2x^2+5x-3=0$$
$$(2x-1)(x+3)=0$$
$$x+3=0 \quad \text{or} \quad 2x-1=0$$
$$x=\underline{\underline{-3 \text{ or } \frac{1}{2}}}$$

1.2 (a) p.11

$$3x^2-4x=0$$
$$x(3x-4)=0$$
$$x=0 \text{ or } 3x-4=0$$
$$x=\underline{\underline{0 \text{ or } \frac{4}{3}}}$$

(b)

$$(x+1)^2+5(x+1)=0$$
$$(x+1)(x+1+5)=0$$
$$(x+1)(x+6)=0$$
$$x+6=0 \quad \text{or } x+1=0$$
$$x=\underline{\underline{-6 \text{ or } -1}}$$

(c)

$$(2x+1)(x-5)=2x+1$$
$$(2x+1)(x-5)-(2x+1)=0$$
$$(2x+1)(x-5-1)=0$$
$$(2x+1)(x-6)=0$$
$$2x+1=0 \quad \text{or } x-6=0$$
$$x=\underline{\underline{-\frac{1}{2} \text{ or } 6}}$$

(d)

$$(x+4)(x-3)=(2x-7)(x-3)$$
$$(x+4)(x-3)-(2x-7)(x-3)=0$$
$$(x-3)(x+4-2x+7)=0$$
$$(x-3)(-x+11)=0$$
$$x-3=0 \text{ or } -x+11=0$$
$$x=\underline{\underline{3 \text{ or } 11}}$$

1.3 (a) p.11

$$(x+2)(x-1)=2x$$
$$x^2-x+2x-2=2x$$
$$x^2-x-2=0$$
$$(x+1)(x-2)=0$$
$$x+1=0 \quad \text{or } x-2=0$$
$$x=\underline{\underline{-1 \text{ or } 2}}$$

(b)

$$3x(x-1)+3=(2x+1)(2x-3)$$
$$3x^2-3x+3=4x^2-6x+2x-3$$
$$3x^2-3x+3=4x^2-4x-3$$
$$x^2-x-6=0$$
$$(x+2)(x-3)=0$$
$$x+2=0 \quad \text{or } x-3=0$$
$$x=\underline{\underline{-2 \text{ or } 3}}$$

1.4 (a) p.12

$$x^2+14x+49=0$$
$$(x+7)^2=0$$
$$x+7=0$$
$$x=\underline{\underline{-7}}$$

(b)

$$25x^2-20x+4=0$$
$$(5x-2)^2=0$$
$$5x-2=0$$
$$x=\underline{\underline{\frac{2}{5}}}$$

1.5 (a) p.13

$$25x^2-9=0$$
$$25x^2=9$$
$$x^2=\frac{9}{25}$$
$$x=\pm\sqrt{\frac{9}{25}}$$
$$=\underline{\underline{\pm\frac{3}{5}}}$$

(b) $(x+3)^2 - 49 = 0$

$$(x+3)^2 = 49$$
$$x + 3 = \pm 7$$
$$x = -3 - 7 \text{ or } -3 + 7$$
$$= \underline{\underline{-10 \text{ or } 4}}$$

(c) $(x-5)^2 = 3$

$$x - 5 = \pm\sqrt{3}$$
$$x = \underline{\underline{5 \pm \sqrt{3}}}$$

(d) $(2x+3)^2 = 25$

$$2x + 3 = \pm 5$$
$$2x = -3 - 5 \text{ or } -3 + 5$$
$$= -8 \text{ or } 2$$
$$\therefore \quad x = \underline{\underline{-4 \text{ or } 1}}$$

1.6 **(a)** $x^2 + 40x + 391 = 0$ p.17

$$x = \frac{-40 \pm \sqrt{(40)^2 - 4(1)(391)}}{2(1)}$$
$$= \frac{-40 \pm \sqrt{36}}{2}$$
$$= \frac{-40 \pm 6}{2}$$
$$= \frac{-40-6}{2} \text{ or } \frac{-40+6}{2}$$
$$= \underline{\underline{-23 \text{ or } -17}}$$

(b) $x^2 - x - 7 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-7)}}{2(1)}$$
$$= \underline{\underline{\frac{1 \pm \sqrt{29}}{2}}}$$

(c) $49x^2 + 42x + 9 = 0$

$$x = \frac{-42 \pm \sqrt{(42)^2 - 4(49)(9)}}{2(49)}$$
$$= \frac{-42 \pm \sqrt{0}}{98}$$
$$= -\frac{42}{98}$$
$$= \underline{\underline{-\frac{3}{7}}}$$

(d) $4x^2 - x + 5 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(5)}}{2(4)}$$
$$= \frac{1 \pm \sqrt{-79}}{8}$$

Since $\sqrt{-79}$ is not a real number, the equation has no real solutions.

1.7 **(a)** $(x+1)(x-3) = 4$ p.18

$$x^2 + x - 3x - 3 = 4$$
$$x^2 - 2x - 7 = 0$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{32}}{2}$$
$$= \frac{2 \pm 4\sqrt{2}}{2}$$
$$= \frac{2(1 \pm 2\sqrt{2})}{2}$$
$$= \underline{\underline{1 \pm 2\sqrt{2}}}$$

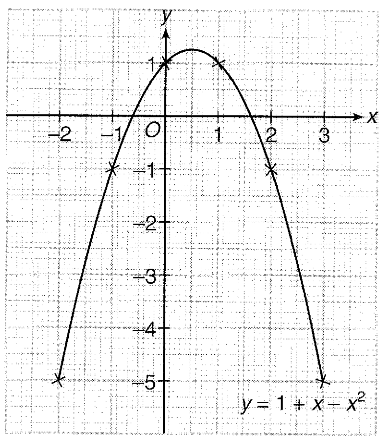
(b)

$$x^2 = 16(12 - x)$$
$$x^2 = 192 - 16x$$
$$x^2 + 16x - 192 = 0$$
$$x = \frac{-16 \pm \sqrt{16^2 - 4(1)(-192)}}{2(1)}$$
$$= \frac{-16 \pm \sqrt{1024}}{2}$$
$$= \frac{-16 \pm 32}{2}$$
$$= \frac{-16-32}{2} \text{ or } \frac{-16+32}{2}$$
$$= \underline{\underline{-24 \text{ or } 8}}$$

1.8 **(a)**

x	-2	-1	0	1	2	3
y	-5	-1	1	1	-1	-5

(b) p.21

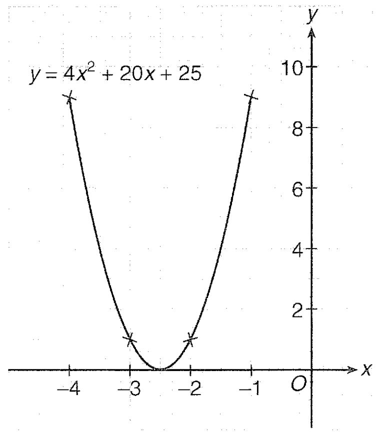


The graph cuts the x-axis at the points $(-0.6, 0)$ and $(1.6, 0)$.

$\therefore$ The solution is -0.6 or 1.6.

1.9 **(a)** $y = 4x^2 + 20x + 25$ p.22

x	-4	-3	-2	-1
y	9	1	1	9

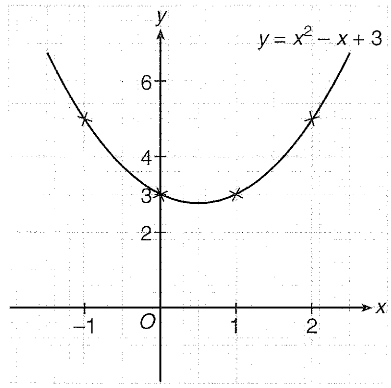


The graph touches the x-axis at the point $(-2.5, 0)$.

$\therefore$ The solution is -2.5.

1.10 **(a)** $y = x^2 - x + 3$ p.23

x	-1	0	1	2
y	5	3	3	5



(b) The graph does not intersect with the x-axis, hence the equation has no real solutions.

1.11 p.26

$y = -x^2 + px + q$ ....(1)

Putting $(-3, 0)$ into (1),

$0 = -(-3)^2 + p(-3) + q$

$0 = -9 - 3p + q$

$q = 9 + 3p$ ....(2)

Putting $(1, 0)$ into (1),

$0 = -(1)^2 + p(1) + q$

$0 = -1 + p + q$

$q = 1 - p$ ....(3)

(2) – (3):

$0 = 8 + 4p$

$p = -2$

$\therefore\ q = 1 - (-2) = 3$

$\therefore\ y = -x^2 - 2x + 3$ ....(4)

Putting $(0, c)$ into (4),

$c = -(0)^2 - 2(0) + 3$

$= \underline{\underline{3}}$

1.12 The equation has two equal roots if $\Delta = 0$, p.34

i.e. $q^2 - 4(2)(50) = 0$

$q^2 = 400$

$q = \pm\sqrt{400}$

$= \underline{\underline{\pm 20}}$

1.13 Since the graph touches the x-axis, $\Delta = 0$. p.34

$\therefore\ 4^2 - 4(3)(k) = 0$

$16 - 12k = 0$

$12k = 16$

$k = \underline{\underline{\frac{4}{3}}}$

1.14 $\Delta = (-12)^2 - 4(m + 1)(3)$ p.35

$= 144 - 12m - 12$

$= 132 - 12m$

(a) Two unequal real roots if $\Delta > 0$,

$132 - 12m > 0$, $\underline{\underline{m < 11}}$

(b) One double real root if $\Delta = 0$,

$132 - 12m = 0$, $\underline{\underline{m = 11}}$

(c) No real roots if $\Delta < 0$,

$132 - 12m < 0$, $\underline{\underline{m > 11}}$

Teacher's Example pp.9 – 35

Example 1.1T p.9

(a) $x^2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$

$x + 2 = 0$ or $x - 4 = 0$

$x = \underline{\underline{-2 \text{ or } 4}}$

(b) $x^2 = 5x + 14$

$x^2 - 5x - 14 = 0$

$(x + 2)(x - 7) = 0$

$x + 2 = 0$ or $x - 7 = 0$

$x = \underline{\underline{-2 \text{ or } 7}}$

(c) $x^2 = \dfrac{x+4}{3}$

$3x^2 - x - 4 = 0$

$(3x-4)(x+1) = 0$

$x + 1 = 0$ or $3x - 4 = 0$

$x = \underline{\underline{-1 \text{ or } \dfrac{4}{3}}}$

Example 1.2T

p.10

(a) $5x^2 = 2x$

$5x^2 - 2x = 0$

$x(5x-2) = 0$

$x = 0$ or $5x - 2 = 0$

$x = \underline{\underline{0 \text{ or } \dfrac{2}{5}}}$

(b) $x(2x+1) = 3x$

$2x^2 + x = 3x$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$x = 0$ or $x - 1 = 0$

$x = \underline{\underline{0 \text{ or } 1}}$

(c) $(2x-1)(3x+2) = 2x - 1$

$(2x-1)(3x+2) - (2x-1) = 0$

$(2x-1)(3x+2-1) = 0$

$(2x-1)(3x+1) = 0$

$3x + 1 = 0$ or $2x - 1 = 0$

$x = \underline{\underline{-\dfrac{1}{3} \text{ or } \dfrac{1}{2}}}$

Example 1.3T

p.11

(a) $(x+2)(7x-2) = 24x$

$7x^2 + 14x - 2x - 4 = 24x$

$7x^2 - 12x - 4 = 0$

$(7x+2)(x-2) = 0$

$7x + 2 = 0$ or $x - 2 = 0$

$x = \underline{\underline{-\dfrac{2}{7} \text{ or } 2}}$

(b) $(x-1)(x+2) = (x-2)(2x+1)$

$x^2 - x + 2x - 2 = 2x^2 - 4x + x - 2$

$x^2 + x - 2 = 2x^2 - 3x - 2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x - 4 = 0$

$x = \underline{\underline{0 \text{ or } 4}}$

Example 1.4T

p.12

(a) $x^2 + 8x + 16 = 0$

$(x+4)^2 = 0$

$x + 4 = 0$

$\therefore \quad x = \underline{\underline{-4}}$

(b) $9x^2 - 30x + 25 = 0$

$(3x-5)^2 = 0$

$3x - 5 = 0$

$\therefore \quad x = \underline{\underline{\dfrac{5}{3}}}$

Example 1.5T

p.16

(a) $16x^2 - 9 = 0$

$16x^2 = 9$

$x^2 = \dfrac{9}{16}$

$x = \pm\sqrt{\dfrac{9}{16}}$

$= \underline{\underline{\pm\dfrac{3}{4}}}$

(b) $16x^2 + 9 = 0$

$16x^2 = -9$

$x^2 = -\dfrac{9}{16}$

$x = \pm\sqrt{-\dfrac{9}{16}}$

$\therefore$ No real roots.

(c) $9(x-2)^2 - 16 = 0$

$9(x-2)^2 = 16$

$(x-2)^2 = \dfrac{16}{9}$

$x - 2 = \pm\sqrt{\dfrac{16}{9}}$

$x - 2 = \pm\dfrac{4}{3}$

$x = 2 - \dfrac{4}{3}$ or $2 + \dfrac{4}{3}$

$= \underline{\underline{\dfrac{2}{3} \text{ or } \dfrac{10}{3}}}$

Example 1.6T

p.17

(a) $x^2 + 30x + 161 = 0$

$$\begin{aligned} x &= \frac{-30 \pm \sqrt{(30)^2 - 4(1)(161)}}{2(1)} \\ &= \frac{-30 \pm \sqrt{256}}{2} \\ &= \frac{-30 \pm 16}{2} \\ &= \frac{-30-16}{2} \text{ or } \frac{-30+16}{2} \\ &= \underline{\underline{-23 \text{ or } -7}} \end{aligned}$$

(b) $2x^2 + 7x + 1 = 0$

$$\begin{aligned} x &= \frac{-7 \pm \sqrt{(7)^2 - 4(2)(1)}}{2(2)} \\ &= \underline{\underline{\frac{-7 \pm \sqrt{41}}{4}}} \end{aligned}$$

(c) $4x^2 + 28x + 49 = 0$

$$\begin{aligned} x &= \frac{-28 \pm \sqrt{(28)^2 - 4(4)(49)}}{2(4)} \\ &= \frac{-28 \pm \sqrt{0}}{8} \\ &= \underline{\underline{-\frac{7}{2}}} \end{aligned}$$

(d) $7x^2 + x + 1 = 0$

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{(1)^2 - 4(7)(1)}}{2(7)} \\ &= \frac{-1 \pm \sqrt{-27}}{14} \end{aligned}$$

Since $\sqrt{-27}$ is not a real number, the equation has no real solutions.

Example 1.7T

p.18

$$\begin{aligned} 12 &= x(3 + x) \\ 12 &= 3x + x^2 \\ x^2 + 3x - 12 &= 0 \end{aligned}$$

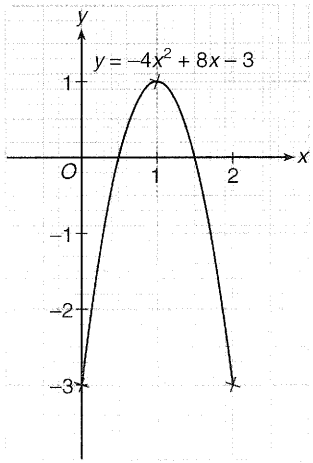
$$\begin{aligned} x &= \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-12)}}{2(1)} \\ &= \underline{\underline{\frac{-3 \pm \sqrt{57}}{2}}} \end{aligned}$$

Example 1.8T

p.21

(a) $y = -4x^2 + 8x - 3$

x	0	1	2
y	−3	1	−3



(b) The graph cuts the x-axis at the points (0.5, 0) and (1.5, 0).

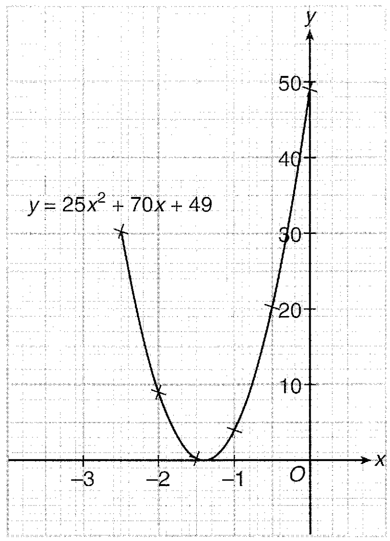
∴ The solution is 0.5 or 1.5.

Example 1.9T

p.22

(a) $y = 25x^2 + 70x + 49$

x	−2.5	−2	−1.5	−1	−0.5	0
y	30.25	9	0.25	4	20.25	49



(b) The graph touches the x-axis at the point (−1.4, 0).

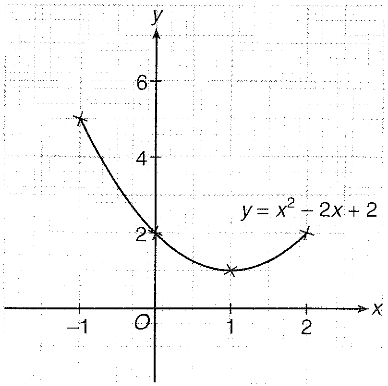
∴ The solution is −1.4.

Example 1.10T

p.23

(a) $y = x^2 - 2x + 2$

x	−1	0	1	2
y	5	2	1	2



(b) The graph does not intersect with the x-axis, hence the equation has no real solutions.

Example 1.11T

p.25

(a) Area of $\Delta ABC = \dfrac{[1+\sqrt{5}-(1-\sqrt{5})](5)}{2}$

$= \underline{\underline{5\sqrt{5} \text{ sq. units}}}$

(b) Let the point D be (a, b).

$\because$ Point D lies on the graph,

$\therefore$ $b = a^2 - 2a - 4$ ....(1)

$AB = (1+\sqrt{5}-(1-\sqrt{5})) = 2\sqrt{5}$ units

$\because$ Area of $\Delta ABC = 5\sqrt{5}$ sq. units,

$\therefore$ Area of ΔABD = Area of $ABCD$ − Area of ΔABC

$= 9\sqrt{5} - 5\sqrt{5}$

$= 4\sqrt{5}$ sq. units.

$\therefore$ $4\sqrt{5} = \dfrac{(2\sqrt{5})(b)}{2}$

$b = 4$

Substituting $b = 4$ into (1),

$4 = a^2 - 2a - 4$

$a^2 - 2a - 8 = 0$

$(a-4)(a+2) = 0$

$a = -2$ or 4

$\therefore$ The point D is $(-2, 4)$ or $(4, 4)$.

Example 1.12T

p.34

The equation has two equal roots if $\Delta = 0$,

i.e., $(-7)^2 - 4 \times 1 \times m = 0$

$49 = 4m$

$m = \underline{\underline{\dfrac{49}{4}}}$

Example 1.13T

p.34

Since the graph touches the x-axis, $\Delta = 0$.

$\therefore$ $6^2 - 4(1)(n-1) = 0$

$36 - 4(n-1) = 0$

$4(n-1) = 36$

$n - 1 = 9$

$n = \underline{\underline{10}}$

Example 1.14T

p.35

(a) $\Delta = 6^2 - 4(3)(p+2)$

$= 36 - 12(p+2)$

$= 36 - 12p - 24$

$= \underline{\underline{12 - 12p}}$

(b) (i) The equation has two unequal real roots if $\Delta > 0$,

i.e., $12 - 12p > 0$

$\underline{\underline{p < 1}}$

(ii) The equation has one double real root if $\Delta = 0$,

i.e., $12 - 12p = 0$

$\underline{\underline{p = 1}}$

(iii) The equation has no real roots if $\Delta < 0$,

i.e., $12 - 12p < 0$

$\underline{\underline{p > 1}}$

Exercise 1.1

p.7

Level 1

p.7

1. **(a)** −20, −10, 0, 20, 171

   (b) $-\dfrac{13}{6}, 0.\dot{2}\dot{1}, 1.2\dot{1}, \dfrac{4}{3}$

   (c) $\dfrac{1}{2}$, 11.789

   (d) $-\sqrt{3}, \dfrac{3\pi}{2}$

2. **(a)** −8, −4

   (b) 100, 300

   (c) $-0.\dot{1}4\dot{3}$, 16.39

   (d) $-8, -4, -\dfrac{1}{4}, -0.\dot{1}4\dot{3}, 0, \dfrac{11}{7}$, 16.39, 100, 300

3. **(a)** $0.17 = \dfrac{17}{100}$

(b) $0.017 = \dfrac{17}{1000}$

(c)
$$\begin{aligned} 0.1\dot{7} &= 0.1 + 0.0\dot{7} \\ &= \frac{1}{10} + \frac{7}{90} \\ &= \frac{16}{90} \\ &= \frac{8}{45} \end{aligned}$$

(d) Let $s = 0.\dot{1}\dot{7}$.
$$\begin{aligned} 100s &= 17.\dot{1}\dot{7} \\ 100s - s &= 17.\dot{1}\dot{7} - 0.\dot{1}\dot{7} \\ 99s &= 17 \\ s &= \frac{17}{99} \end{aligned}$$
$\therefore \quad 0.\dot{1}\dot{7} = \dfrac{17}{99}$

4. **(a)** $\because \quad \dfrac{2}{5} = 0.4$

$\dfrac{3}{16} = 0.1875$

$\therefore \quad \dfrac{3}{16}, \dfrac{2}{5}$ can be expressed as terminating decimals.

(b) $\because \quad \dfrac{5}{12} = 0.41\dot{6}$

$\dfrac{6}{7} = 0.\dot{8}5714\dot{2}$

$\dfrac{13}{24} = 0.541\dot{6}$

$\therefore \quad \dfrac{5}{12}, \dfrac{6}{7}, \dfrac{13}{24}$ can be expressed as recurring decimals.

5. **(a)** Rational

(b) Rational

(c) Rational

(d) Rational

(e) Irrational

(f) Rational

Exercise 1.2

p.14

Level 1

p.14

1.
$$\begin{aligned} x(x+3) &= 0 \\ x+3 = 0 \quad &\text{or} \quad x = 0 \\ x &= -3 \text{ or } 0 \end{aligned}$$

2.
$$\begin{aligned} (y+2)(y-5) &= 0 \\ y+2 = 0 \quad &\text{or} \quad y-5 = 0 \\ y &= -2 \text{ or } 5 \end{aligned}$$

3.
$$\begin{aligned} (x-6)^2 &= 0 \\ (x-6)(x-6) &= 0 \\ x-6 = 0 \quad &\text{or} \quad x-6 = 0 \\ x &= 6 \end{aligned}$$

4.
$$\begin{aligned} 3(y-11)^2 &= 0 \\ 3(y-11)(y-11) &= 0 \\ y-11 = 0 \quad &\text{or} \quad y-11 = 0 \\ y &= 11 \end{aligned}$$

5.
$$\begin{aligned} (2x+1)(2x+5) &= 0 \\ 2x+5 = 0 \quad &\text{or} \quad 2x+1 = 0 \\ x &= -\frac{5}{2} \text{ or } -\frac{1}{2} \end{aligned}$$

6.
$$\begin{aligned} (2y+3)(4-y) &= 0 \\ 2y+3 = 0 \quad &\text{or} \quad 4-y = 0 \\ y &= -\frac{3}{2} \text{ or } 4 \end{aligned}$$

7.
$$\begin{aligned} x^2 + x - 12 &= 0 \\ (x+4)(x-3) &= 0 \\ x+4 = 0 \quad &\text{or} \quad x-3 = 0 \\ x &= -4 \text{ or } 3 \end{aligned}$$

8.
$$\begin{aligned} y^2 + 8y + 15 &= 0 \\ (y+5)(y+3) &= 0 \\ y+5 = 0 \quad &\text{or} \quad y+3 = 0 \\ y &= -5 \text{ or } -3 \end{aligned}$$

9.
$$\begin{aligned} 4x^2 + 4x + 1 &= 0 \\ (2x+1)(2x+1) &= 0 \\ 2x+1 = 0 \quad &\text{or} \quad 2x+1 = 0 \\ x &= -\frac{1}{2} \end{aligned}$$

10. $4x^2 - 44x + 121 = 0$

$(2x - 11)(2x - 11) = 0$

$2x - 11 = 0 \quad \text{or} \quad 2x - 11 = 0$

$x = \dfrac{11}{2}$

11. $18 - 3n - n^2 = 0$

$(6 + n)(3 - n) = 0$

$6 + n = 0 \quad \text{or} \quad 3 - n = 0$

$n = -6 \text{ or } 3$

12. $12 + 4a - a^2 = 0$

$(2 + a)(6 - a) = 0$

$2 + a = 0 \quad \text{or} \quad 6 - a = 0$

$a = -2 \text{ or } 6$

13. $1 + 9x^2 - 6x = 0$

$9x^2 - 6x + 1 = 0$

$(3x - 1)(3x - 1) = 0$

$3x - 1 = 0 \quad \text{or} \quad 3x - 1 = 0$

$x = \dfrac{1}{3}$

14. $3b^2 - 7b = 0$

$b(3b - 7) = 0$

$b = 0 \text{ or } 3b - 7 = 0$

$b = 0 \text{ or } \dfrac{7}{3}$

15. $(t + 3)^2 - 36 = 0$

$(t + 3)^2 = 36$

$t + 3 = \pm 6$

$t = -3 - 6 \text{ or } -3 + 6$

$t = -9 \quad \text{or } 3$

16. $\dfrac{1}{3}(4 - 3x)^2 = 3$

$(4 - 3x)^2 = 9$

$4 - 3x = \pm 3$

$3x = 4 - 3 \text{ or } 4 + 3$

$x = \dfrac{1}{3} \quad \text{or} \quad \dfrac{7}{3}$

17. $x(2 - 3x) = 7x$

$7x - x(2 - 3x) = 0$

$x(7 - 2 + 3x) = 0$

$x(3x + 5) = 0$

$3x + 5 = 0 \quad \text{or } x = 0$

$x = -\dfrac{5}{3} \text{ or } 0$

18. $(1 - 4x)(2x - 3) = 2x - 3$

$(1 - 4x)(2x - 3) - (2x - 3) = 0$

$(2x - 3)(1 - 4x - 1) = 0$

$(2x - 3)(-4x) = 0$

$-4x = 0 \text{ or } 2x - 3 = 0$

$x = 0 \text{ or } \dfrac{3}{2}$

Level 2

p.14

19. $4x^2 + 23x + 15 = 0$

$(x + 5)(4x + 3) = 0$

$x + 5 = 0 \quad \text{or } 4x + 3 = 0$

$x = -5 \text{ or } -\dfrac{3}{4}$

20. $7x(7x + 1) + \dfrac{1}{4} = 0$

$28x(7x + 1) + 1 = 0$

$196x^2 + 28x + 1 = 0$

$(14x + 1)(14x + 1) = 0$

$14x + 1 = 0 \quad \text{or } 14x + 1 = 0$

$x = -\dfrac{1}{14}$

21. $6x^2 + 11x - 35 = 0$

$(2x + 7)(3x - 5) = 0$

$2x + 7 = 0 \quad \text{or } 3x - 5 = 0$

$x = -\dfrac{7}{2} \text{ or } \dfrac{5}{3}$

22. $6x^2 - 7x - 10 = 0$

$(6x + 5)(x - 2) = 0$

$6x + 5 = 0 \quad \text{or } x - 2 = 0$

$x = -\dfrac{5}{6} \text{ or } 2$

23. $9x^2 = \dfrac{1}{7}\left(6x - \dfrac{1}{7}\right)$

$63x^2 = 6x - \dfrac{1}{7}$

$441x^2 - 42x + 1 = 0$

$(21x - 1)(21x - 1) = 0$

$21x - 1 = 0 \quad \text{or } 21x - 1 = 0$

$x = \dfrac{1}{21}$

24. $16x^2 - 16x - 21 = 0$

$(4x+3)(4x-7) = 0$

$4x + 3 = 0$ or $4x - 7 = 0$

$x = \underline{\underline{-\frac{3}{4} \text{ or } \frac{7}{4}}}$

25. $18x^2 + 85x + 18 = 0$

$(2x+9)(9x+2) = 0$

$2x + 9 = 0$ or $9x + 2 = 0$

$x = \underline{\underline{-\frac{9}{2} \text{ or } -\frac{2}{9}}}$

26. $21x^2 - 62x + 45 = 0$

$(7x-9)(3x-5) = 0$

$7x - 9 = 0$ or $3x - 5 = 0$

$x = \underline{\underline{\frac{9}{7} \text{ or } \frac{5}{3}}}$

27. $(3x+2)(3x+1) = 20$

$9x^2 + 6x + 3x + 2 - 20 = 0$

$9x^2 + 9x - 18 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x + 2 = 0$ or $x - 1 = 0$

$x = \underline{\underline{-2 \text{ or } 1}}$

28. $3x(4x-1) = (2x+1)^2$

$12x^2 - 3x = 4x^2 + 4x + 1$

$8x^2 - 7x - 1 = 0$

$(8x+1)(x-1) = 0$

$8x + 1 = 0$ or $x - 1 = 0$

$x = \underline{\underline{-\frac{1}{8} \text{ or } 1}}$

29. $(x+2)(3x-2) = (x+6)(2x+1)$

$3x^2 + 6x - 2x - 4 = 2x^2 + 12x + x + 6$

$3x^2 + 4x - 4 = 2x^2 + 13x + 6$

$x^2 - 9x - 10 = 0$

$(x+1)(x-10) = 0$

$x + 1 = 0$ or $x - 10 = 0$

$x = \underline{\underline{-1 \text{ or } 10}}$

30. $(x-3)^2 = (1-4x)^2$

$(x-3)^2 - (1-4x)^2 = 0$

$(x - 3 + 1 - 4x)(x - 3 - 1 + 4x) = 0$

$(-3x-2)(5x-4) = 0$

$-3x - 2 = 0$ or $5x - 4 = 0$

$x = \underline{\underline{-\frac{2}{3} \text{ or } \frac{4}{5}}}$

Exercise 1.3

p.18-19

Level 1

p.18

1. $x^2 - 3x - 10 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{49}}{2}$$

$$= \frac{3-7}{2} \text{ or } \frac{3+7}{2}$$

$$= \underline{\underline{-2 \text{ or } 5}}$$

2. $2x^2 - 7x + 3 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{25}}{4}$$

$$= \frac{7-5}{4} \text{ or } \frac{7+5}{4}$$

$$= \underline{\underline{\frac{1}{2} \text{ or } 3}}$$

3. $12 - 4x - x^2 = 0$

$x^2 + 4x - 12 = 0$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{64}}{2}$$

$$= \frac{-4-8}{2} \text{ or } \frac{-4+8}{2}$$

$$= \underline{\underline{-6 \text{ or } 2}}$$

4. $20x^2 = 2 - 3x$

$20x^2 + 3x - 2 = 0$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(20)(-2)}}{2(20)}$$

$$= \frac{-3 \pm \sqrt{169}}{40}$$

$$= \frac{-3-13}{40} \text{ or } \frac{-3+13}{40}$$

$$= \underline{\underline{-\frac{2}{5} \text{ or } \frac{1}{4}}}$$

5. $x^2 - 5x + 1 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$$

$$= \underline{\underline{\frac{5 \pm \sqrt{21}}{2}}}$$

6. $2x^2 = 3x + 4$

$2x^2 - 3x - 4 = 0$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$

$= \underline{\underline{\dfrac{3 \pm \sqrt{41}}{4}}}$

7. $7 - 5x - x^2 = 0$

$x^2 + 5x - 7 = 0$

$x = \dfrac{-5 \pm \sqrt{(5)^2 - 4(1)(-7)}}{2(2)}$

$= \underline{\underline{\dfrac{-5 \pm \sqrt{53}}{2}}}$

8. $3x^2 = 7x + 3$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-3)}}{2(3)}$

$= \underline{\underline{\dfrac{7 \pm \sqrt{85}}{6}}}$

9. $x^2 - 56x + 121 = 0$

$x = \dfrac{-(-56) \pm \sqrt{(-56)^2 - 4(1)(121)}}{2(1)}$

$= \dfrac{56 \pm \sqrt{2652}}{2}$

$= \dfrac{56 - \sqrt{2652}}{2}$ or $\dfrac{56 + \sqrt{2652}}{2}$

$= \underline{\underline{2.25 \text{ or } 53.75}}$

10. $12x^2 + x - 34 = 0$

$x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(12)(-34)}}{2(12)}$

$= \dfrac{-1 \pm \sqrt{1633}}{24}$

$= \dfrac{-1 - \sqrt{1633}}{24}$ or $\dfrac{-1 + \sqrt{1633}}{24}$

$= \underline{\underline{-1.73 \text{ or } 1.64}}$

Level 2

p.19

11. $m^2 + \dfrac{11}{2}m - 210 = 0$

$2m^2 + 11m - 420 = 0$

$m = \dfrac{-11 \pm \sqrt{(11)^2 - 4(2)(-420)}}{2(2)}$

$= \dfrac{-11 \pm \sqrt{3481}}{4}$

$= \dfrac{-11 \pm 59}{4}$

$= \dfrac{-11 - 59}{4}$ or $\dfrac{-11 + 59}{4}$

$= \underline{\underline{-\dfrac{35}{2} \text{ or } 12}}$

12. $\dfrac{20}{3}p = 224 - p^2$

$p^2 + \dfrac{20}{3}p - 224 = 0$

$3p^2 + 20p - 672 = 0$

$p = \dfrac{-20 \pm \sqrt{(20)^2 - 4(3)(672)}}{2(3)}$

$= \dfrac{-20 \pm \sqrt{8464}}{6}$

$= \dfrac{-20 \pm 92}{6}$

$= \dfrac{-20 - 92}{6}$ or $\dfrac{-20 + 92}{6}$

$= \underline{\underline{-\dfrac{56}{3} \text{ or } 12}}$

13. $0.4t^2 - 4.7t - 12.6 = 0$

$t = \dfrac{-(-4.7) \pm \sqrt{(-4.7)^2 - 4(0.4)(-12.6)}}{2(0.4)}$

$= \dfrac{4.7 \pm \sqrt{42.25}}{0.8}$

$= \dfrac{4.7 \pm 6.5}{0.8}$

$= \dfrac{4.7 - 6.5}{0.8}$ or $\dfrac{4.7 + 6.5}{0.8}$

$= \underline{\underline{-\dfrac{9}{4} \text{ or } 14}}$

14. $\frac{7}{16}q^2+\frac{3}{8}q-\frac{5}{2}=0$

$7q^2+6q-40=0$ *(multiple boths sides by 16)*

$q=\frac{-6\pm\sqrt{(6)^2-4(7)(-40)}}{2(7)}$

$=\frac{-6\pm\sqrt{1156}}{14}$

$=\frac{-6-34}{14}$ or $\frac{-6+34}{14}$

$=-\frac{20}{7}$ or 2

15. $(r+3)(5r-3)=9-r^2$

$(r+3)(5r-3)=(3-r)(3+r)$

$(r+3)(5r-3)-(3-r)(3+r)=0$

$(r+3)(5r-3-3+r)=0$

$(r+3)(6r-6)=0$

$r+3=0$ or $6r-6=0$

$r=-3$ or 1

16. $(6-a)(2a+3)=4a^2+12a+9$

$(6-a)(2a+3)=(2a+3)(2a+3)$

$(6-a)(2a+3)-(2a+3)(2a+3)=0$

$(2a+3)(6-a-2a-3)=0$

$(2a+3)(3-3a)=0$

$2a+3=0$ or $3-3a=0$

$a=-\frac{3}{2}$ or 1

17. $9(y-2)^2=(3-2y)^2$

$9(y-2)^2-(3-2y)^2=0$

$[3(y-2)+(3-2y)][3(y-2)-(3-2y)]=0$

$(3y-6+3-2y)(3y-6-3+2y)=0$

$(5y-9)(y-3)=0$

$5y-9=0$ or $y-3=0$

$y=\frac{9}{5}$ or 3

18. $(4-x)(4+x)=(2x+5)(5x-2)$

$16-x^2=10x^2+25x-4x-10$

$11x^2+21x-26=0$

$x=\frac{-21\pm\sqrt{(21)^2-4(11)(-26)}}{2(11)}$

$=\frac{-21\pm\sqrt{1585}}{22}$

$=\frac{-21-\sqrt{1585}}{22}$ or $\frac{-21+\sqrt{1585}}{22}$

$=-2.76$ or 0.86

19. $3n(n+2)-4(n+1)=150$

$3n^2+6n-4n-4=150$

$3n^2+2n-154=0$

$n=\frac{-2\pm\sqrt{(2)^2-4(3)(-154)}}{2(3)}$

$=\frac{-2\pm\sqrt{1852}}{6}$

$=\frac{-2-\sqrt{1852}}{6}$ or $\frac{-2+\sqrt{1852}}{6}$

$=-7.51$ or 6.84

20. $2p(p+4)=3(p-12)$

$2p^2+8p=3p-36$

$2p^2+5p+36=0$

$p=\frac{-5\pm\sqrt{(5)^2-4(2)(36)}}{2(2)}$

$=\frac{-5\pm\sqrt{-263}}{4}$

$\therefore$ Since $\sqrt{-263}$ is not a real number, the equation has no real solutions.

Exercise 1.4

pp.26 – 29

Level 1

p.26

1. The graph cuts the x-axis at $(-3.0, 0)$ and $(0.0, 0)$.
   $\therefore$ The solution is -3.0 or 0.0.

2. The graph touches the x-axis at $(2.5, 0)$.
   $\therefore$ The solution is 2.5.

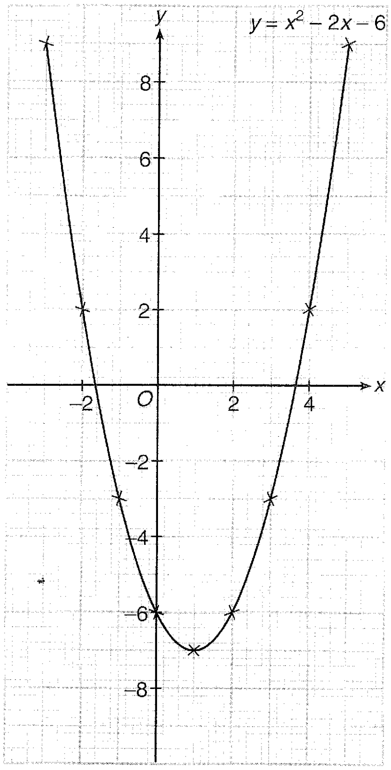
3. The graph does not intersect with the x-axis.
   $\therefore$ The equation has no real solutions.

4. The graph cuts the x-axis at $(0.3, 0)$ and $(1.7, 0)$.
   $\therefore$ The solution is 0.3 or 1.7.

5. **(a)** $y=x^2-2x-6$

x	−3	−2	−1	0	1	2	3	4
y	9	2	−3	−6	−7	−6	−3	2

(b)



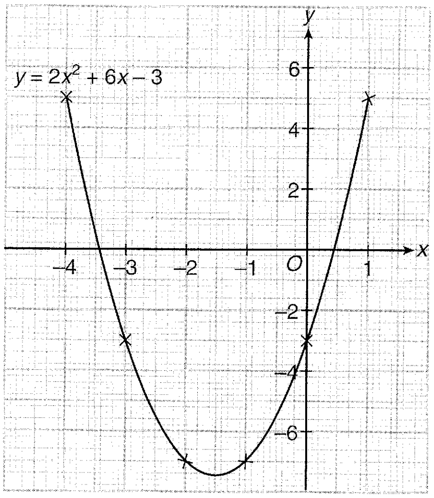
The graph cuts the x-axis at (−1.6, 0) and (3.6, 0).

∴ The solution is −1.6 or 3.6.

6. **(a)** $y = 2x^2 + 6x - 3$

x	−4	−3	−2	−1	0	1
y	5	−3	−7	−7	−3	5

(b)



The graph cuts the x-axis at (−3.4, 0) and (0.4, 0).

∴ The solution is −3.4 or 0.4.

Level 2

p.29

7. **(a)** The graph cuts the x-axis at (−1, 0) and (4, 0).

∴ The solution is −1 or 4.

(b) $\begin{cases} 0 = p(-1)^2 + q(-1) - 2 \\ 0 = p(4)^2 + q(4) - 2 \end{cases}$

$\begin{cases} p - q - 2 = 0 \text{ .... (1)} \\ 16p + 4q - 2 = 0 \text{ .... (2)} \end{cases}$

From (1):

$p = q + 2$ .... (3)

(2) can be reduced as $8p + 2q - 1 = 0$ .... (4)

Putting (3) into (4),

$$8(q + 2) + 2q - 1 = 0$$
$$8q + 16 + 2q - 1 = 0$$
$$10q = -15$$
$$q = \underline{\underline{-\frac{3}{2}}}$$

Putting $q = -\frac{3}{2}$ into (3),

$$p = -\frac{3}{2} + 2 = \underline{\underline{\frac{1}{2}}}$$

8. Putting $x = 0$, $y = 12$,

$12 = 0^2 - 0 + n$

∴ $n = \underline{\underline{12}}$

$y = -x^2 - x + 12$

When $y = 0$,

$$-x^2 - x + 12 = 0$$
$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -4 \quad \text{or} \quad 3$$

∴ The graph cuts the x-axis at (−4, 0) and (3, 0).

∴ $q = \underline{\underline{3}}$

9. When $x = 0$,

$y = 4 - 3(0) - 0^2 = 4$

∴ $F = (0, 4)$

When $y = 0$,

$$0 = 4 - 3x - x^2$$
$$x^2 + 3x - 4 = 0$$
$$(x + 4)(x - 1) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -4 \quad \text{or} \quad 1$$

∴ $D = (-4, 0)$, $E = (1, 0)$

Base of triangle $DE = 1 - (-4) = 5$

Height of triangle $OF = 4 - 0 = 4$

∴ Area of $\Delta DEF = \frac{5 \times 4}{2}$

$= \underline{\underline{10 \text{ sq. units}}}$

10. (a) When $x = 0$, $y = -4$,

$$-4 = p(0)^2 + q(0) + r$$
$$r = \underline{\underline{-4}}$$

The equation of graph can be written as $y = px^2 + qx - 4$.

When $x = 4$, $y = 0$,

$$0 = p(4)^2 + q(4) - 4$$
$$16p + 4q - 4 = 0$$
$$4p + q - 1 = 0 \quad \text{.................. (1)}$$

When $x = 5$, $y = 11$,

$$11 = p(5)^2 + q(5) - 4$$
$$25p + 5q - 15 = 0$$
$$5p + q - 3 = 0 \quad \text{................ (2)}$$

(2) − (1):

$$p - 2 = 0$$
$$p = \underline{\underline{2}}$$

Putting $p = 2$ into (1),

$$4(2) + q - 1 = 0$$
$$q = \underline{\underline{-7}}$$

(b) $y = 2x^2 - 7x - 4$

When $y = 0$,

$$2x^2 - 7x - 4 = 0$$
$$(2x + 1)(x - 4) = 0$$
$$2x + 1 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad 4$$

$$\therefore \quad \underline{\underline{A = (-\frac{1}{2}, 0)}}$$

Exercise 1.5

pp.35 – 36

Level 1

p.35

1. $\Delta = (-5)^2 - 4(1)(-4) = 41 > 0$

The equation has two unequal real roots.

2. $\Delta = (7)^2 - 4(3)(1) = 37 > 0$

The equation has two unequal real roots.

3. $\Delta = (-1)^2 - 4(7)(2) = -55 < 0$

The equation has no real roots.

4. $\Delta = \left(\frac{1}{3}\right)^2 - 4(1)\left(\frac{1}{36}\right) = 0$

The equation has one double real roots.

5. $\Delta = (-4)^2 - 4(1)(2) = 8 > 0$.

$\therefore$ There are two x-intercepts.

6. $\Delta = (-4)^2 - 4(2)(2) = 0$.

$\therefore$ There is one x-intercept.

7. $\Delta = (-4)^2 - 4(3)(2) = -8 < 0$.

$\therefore$ There are no x-intercepts.

8. $\Delta = (-4)^2 - 4(-2)(3) = 40 > 0$.

$\therefore$ There are two x-intercepts.

9. $\Delta = (-6)^2 - 4(r + 2)(1)$

$$= 36 - 4r - 8$$
$$= 28 - 4r$$

The equation has one double real root if $\Delta = 0$,

i.e., $28 - 4r = 0$

$$4r = 28$$
$$r = \underline{\underline{7}}$$

10. $\Delta = (-12)^2 - 4(1)(2p + 1)$

$$= 144 - 8p - 4$$
$$= 140 - 8p$$

The graph has only one x-intercept if $\Delta = 0$,

i.e., $140 - 8p = 0$

$$8p = 140$$
$$p = \underline{\underline{\frac{35}{2}}}$$

11. $\Delta = (5)^2 - 4(3)(m)$

$$= 25 - 12m$$

The equation has two unequal real roots if $\Delta > 0$,

i.e., $25 - 12m > 0$

$$\underline{\underline{m < \frac{25}{12}}}$$

Level 2

p.36

12. $(x - 1)(x - 2) + p = 0$

$$x^2 - 3x + (2 + p) = 0$$
$$\Delta = (-3)^2 - 4(1)(2 + p)$$
$$= 9 - 8 - 4p$$
$$= 1 - 4p$$

The equation has two equal roots if $\Delta = 0$,

i.e., $1 - 4p = 0$

$$p = \underline{\underline{\frac{1}{4}}}$$

13. $\Delta = (m + 1)^2 - 4(2)(m - 1)$

$$= m^2 + 2m + 1 - 8m + 8$$
$$= m^2 - 6m + 9$$

The equation has one double real root if $\Delta = 0$,

i.e., $m^2 - 6m + 9 = 0$

$$(m - 3)(m - 3) = 0$$
$$m - 3 = 0$$
$$m = \underline{\underline{3}}$$

14. $\Delta = (-8)^2 - 4(k)(k+6)$

$= 64 - 4k^2 - 24k$

$= -4(k^2 + 6k - 16)$

The equation has one double real root if $\Delta = 0$,

i.e., $-4(k^2 + 6k - 16) = 0$

$k^2 + 6k - 16 = 0$

$(k+8)(k-2) = 0$

$k + 8 = 0$ or $k - 2 = 0$

$k = \underline{\underline{-8 \text{ or } 2}}$

15. (a) $\Delta = (-3)^2 - 4(4)(k)$

$= 9 - 16k$

The graph touches the x-axis if $\Delta = 0$,

$9 - 16k = 0$

$k = \underline{\underline{\dfrac{9}{16}}}$

(b) $y = 4x^2 - 3x + \dfrac{9}{16}$

When $y = 0$,

$4x^2 - 3x + \dfrac{9}{16} = 0$

$\left(2x - \dfrac{3}{4}\right)\left(2x - \dfrac{3}{4}\right) = 0$

$2x - \dfrac{3}{4} = 0$

$x = \dfrac{3}{8}$

$\therefore\ p = \underline{\underline{(\tfrac{3}{8}, 0)}}$

16. (a) $y = (3 - x)(1 + x) + p$

$= 3 + 3x - x - x^2 + p$

$= -x^2 + 2x + (3 + p)$

$\Delta = 2^2 - 4(-1)(3 + p)$

$= 4 + 12 + 4p$

$= 16 + 4p$

The graph touches the x-axis if $\Delta = 0$,

i.e., $16 + 4p = 0$

$4p = -16$

$p = \underline{\underline{-4}}$

(b) $y = -x^2 + 2x + (3 - 4)$

$= -x^2 + 2x - 1$

When $y = 0$,

$-x^2 + 2x - 1 = 0$

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$x - 1 = 0$

$x = 1$

$\therefore\ Q = \underline{\underline{(1, 0)}}$

17. (a) $\Delta = 0,\ (-m)^2 - 4(-1)(m - 3) = 0$

$m^2 + 4m - 12 = 0$

$(m + 6)(m - 2) = 0$

$m + 6 = 0$ or $m - 2 = 0$

$m = \underline{\underline{-6 \text{ or } 2}}$

(b) For $m = -6$, $y = (-6 - 3) - (-6)x - x^2$

$= -x^2 + 6x - 9$

$= -(x - 3)^2$

$\therefore$ x-intercept $= 3$

For $m = 2$, $y = (2 - 3) - 2x - x^2$

$= -x^2 - 2x - 1$

$= -(x + 1)^2$

$\therefore$ x-intercept $= -1$

Revision Exercise 1

pp.40 – 45

Level 1

p.40

1. **(a)** $\sqrt{9}$, 18

(b) $-\sqrt{25}$, 0, $\sqrt{9}$, 18

(c) $-\sqrt{25}$, -0.5, 0, $\dfrac{3}{2}$, $\sqrt{9}$, $4.\dot{3}6\dot{7}$, 4.58, 18

(d) $-\sqrt{13}$, $\dfrac{\pi}{2}$, $\sqrt{7}$

2. **(a)** Rational ($\because \dfrac{3}{\sqrt{16}} = \dfrac{3}{4}$)

(b) Rational ($\because \sqrt{169} + \sqrt{196} = 13 + 14 = 27$)

(c) Irrational

(d) Irrational ($\because (\sqrt{2} - 1)^2 = 2 - 2\sqrt{2} + 1 = 3 - 2\sqrt{2}$)

(e) Rational ($\because (\sqrt{2} - 1)(\sqrt{2} + 1) = 2 - 1 = 1$)

(f) Rational

(g) Rational

(h) Rational

3. $3x(1 - x) = (4x + 7)(x - 1)$

$(4x + 7)(x - 1) + 3x(x - 1) = 0$

$(x - 1)(4x + 7 + 3x) = 0$

$(7x + 7)(x - 1) = 0$

$7x + 7 = 0$ or $x - 1 = 0$

$\therefore\ x = \underline{\underline{-1 \text{ or } 1}}$

4. $4(q+5)^2 - 121 = 0$

$$(q+5)^2 = \frac{121}{4}$$
$$q+5 = \pm\sqrt{\frac{121}{4}}$$
$$q+5 = \pm\frac{11}{2}$$
$$q = -5-\frac{11}{2} \text{ or } -5+\frac{11}{2}$$
$$= \underline{\underline{-\frac{21}{2} \text{ or } \frac{1}{2}}}$$

5. $(2t-1)^2 + (2t-1)(3t-4) = 0$

$$(2t-1)(2t-1+3t-4) = 0$$
$$(2t-1)(5t-5) = 0$$
$$2t-1 = 0 \text{ or } 5t-5 = 0$$
$$t = \underline{\underline{\frac{1}{2} \text{ or } 1}}$$

6. $(4y-3)^2 - (3y-2)^2 = (7y-5)(2y-1)$

$$(4y-3+3y-2)(4y-3-3y+2) = (7y-5)(2y-1)$$
$$(7y-5)(y-1) - (7y-5)(2y-1) = 0$$
$$(7y-5)(y-1-2y+1) = 0$$
$$(7y-5)(-y) = 0$$
$$y = 0 \text{ or } 7y-5 = 0$$
$$y = \underline{\underline{0 \text{ or } \frac{5}{7}}}$$

7. $(2m-5)(2m+5) = 3m^2 + 2m - 1$

$$4m^2 - 25 = 3m^2 + 2m - 1$$
$$m^2 - 2m - 24 = 0$$
$$(m+4)(m-6) = 0$$
$$m+4 = 0 \text{ or } m-6 = 0$$
$$m = \underline{\underline{-4 \text{ or } 6}}$$

8. $p(p+1) + (p+2)(p+3) = 14$

$$p^2 + p + p^2 + 2p + 3p + 6 = 14$$
$$2p^2 + 6p - 8 = 0$$
$$p^2 + 3p - 4 = 0$$
$$(p+4)(p-1) = 0$$
$$p+4 = 0 \text{ or } p-1 = 0$$
$$p = \underline{\underline{-4 \text{ or } 1}}$$

9. $1872 + 16p - p^2 = 0$

$$p^2 - 16p - 1872 = 0$$
$$p = \frac{-(-16)\pm\sqrt{(-16)^2 - 4(1)(-1872)}}{2(1)}$$
$$= \frac{16\pm\sqrt{7744}}{2}$$
$$= \frac{16\pm 88}{2}$$
$$= \frac{16-88}{2} \text{ or } \frac{16+88}{2}$$
$$= \underline{-36 \text{ or } 52}$$

10. $18m^2 - 3\sqrt{2}m - 2 = 0$

$$m = \frac{-(-3\sqrt{2})\pm\sqrt{(-3\sqrt{2})^2 - 4(18)(-2)}}{2(18)}$$
$$= \frac{3\sqrt{2}\pm\sqrt{162}}{36}$$
$$= \frac{3\sqrt{2}-9\sqrt{2}}{36} \text{ or } \frac{3\sqrt{2}+9\sqrt{2}}{36}$$
$$= \underline{\underline{-\frac{\sqrt{2}}{6} \text{ or } \frac{\sqrt{2}}{3}}}$$

11. $(y+5)(3y-5) = 7(y-3)$

$$3y^2 + 15y - 5y - 25 = 7y - 21$$
$$3y^2 + 3y - 4 = 0$$
$$y = \frac{-3\pm\sqrt{(3)^2 - 4(3)(-4)}}{2(3)}$$
$$= \underline{\underline{\frac{-3\pm\sqrt{57}}{6}}}$$

12. $(x-1)^2 + (x-2)^2 = 0$

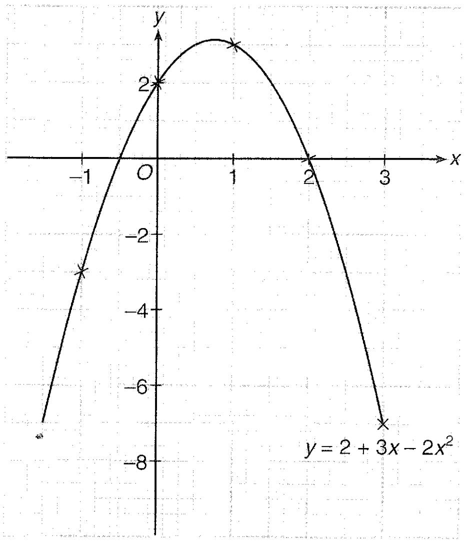
$$x^2 - 2x + 1 + x^2 - 4x + 4 = 0$$
$$2x^2 - 6x + 5 = 0$$
$$x = \frac{-(-6)\pm\sqrt{(-6)^2 - 4(2)(5)}}{2(2)}$$
$$= \frac{6\pm\sqrt{-4}}{4}$$

Since $\sqrt{-4}$ is not a real number, the equation has no real solutions.

13. (a) $y = 2 + 3x - 2x^2$

x	-1	0	1	2	3
y	-3	2	3	0	-7

(b)



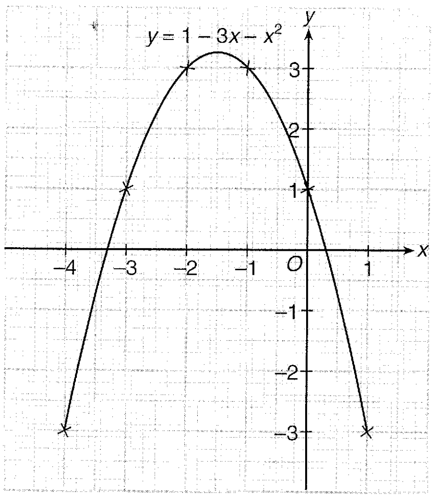
The graph cuts the x-axis at $(-0.5, 0)$ and $(2.0, 0)$.

$\therefore$ The solution is -0.5 or 2.0.

14. (a) $y = 1 - 3x - x^2$

x	-4	-3	-2	-1	0	1
y	-3	1	3	3	1	-3

(b)



The graph cuts the x-axis at $(-3.3, 0)$ and $(0.3, 0)$.

$\therefore$ The solution is -3.3 or 0.3.

15. (a) The graph cuts the x-axis at $(-\frac{1}{2}, 0)$ and $(1, 0)$.

$\therefore$ The solution is $-\frac{1}{2}$ or 1.

(b) $\begin{cases} m + n\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)^2 = 0 \\ m + n(1) - 1^2 = 0 \end{cases}$

$\begin{cases} m - \frac{1}{2}n - \frac{1}{4} = 0 \\ m + n - 1 = 0 \end{cases}$

$\begin{cases} 4m - 2n - 1 = 0 \quad \text{.................. (1)} \\ m + n - 1 = 0 \quad \text{...................... (2)} \end{cases}$

$(1) + (2) \times 2$:

$$4m - 1 + 2m - 2 = 0$$
$$6m = 3$$
$$m = \underline{\underline{\frac{1}{2}}}$$

Putting $m = \frac{1}{2}$ into (2),

$$\frac{1}{2} + n - 1 = 0$$
$$n = 1 - \frac{1}{2}$$
$$= \underline{\underline{\frac{1}{2}}}$$

16. When $x = 0$,

$$y = 2(0)^2 - 0 - 3 = -3$$

$\therefore$ $R = (0, -3)$

When $y = 0$,

$$2x^2 - x - 3 = 0$$
$$(2x - 3)(x + 1) = 0$$
$$x + 1 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$x = -1 \quad \text{or} \quad \frac{3}{2}$$

$\therefore$ $P = (-1, 0)$, $Q = (\frac{3}{2}, 0)$

$PQ = \frac{3}{2} - (-1) = \frac{5}{2}$

$OR = 0 - (-3) = 3$

$\therefore$ Area of $\Delta PQR = \frac{1}{2} \times \frac{5}{2} \times 3$

$= \underline{\underline{\frac{15}{4} \text{ sq. units}}}$

17. (a) $\frac{1}{2}x^2 - \frac{1}{3}x - \frac{1}{6} = 0$

$$\Delta = \left(-\frac{1}{3}\right)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{1}{6}\right)$$
$$= \frac{1}{9} + \frac{1}{3}$$
$$= \frac{4}{9} > 0$$

$\therefore$ The equation has two unequal real roots.

(b) $\sqrt{7}x^2+5x+\sqrt{7}=0$

$\Delta=(5)^2-4(\sqrt{7})(\sqrt{7})$
$=25-28$
$=-3<0$

$\therefore$ The equation has no real roots.

(c) $1.2x^2+6x+7.5=0$

$\Delta=6^2-4(1.2)(7.5)$
$=0$

$\therefore$ The equation has one double real root.

18. (a) $\Delta=(-1)^2-4(k)(-5)$
$=1+20k$

The graph has two distinct x-intercepts if $\Delta>0$,
i.e., $1+20k>0$

$k>-\frac{1}{20}$

(b) $\Delta=(2k-1)^2-4(1)(9)$
$=4k^2-4k+1-36$
$=4k^2-4k-35$

The graph touches the x-axis if $\Delta=0$.
i.e., $4k^2-4k-35=0$
$(2k+5)(2k-7)=0$
$2k+5=0$ or $2k-7=0$
$k=-\frac{5}{2}$ or $\frac{7}{2}$

(c) $\Delta=(6)^2-4(5)(1-3k)$
$=36-20+60k$
$=16+60k$

The graph has no x-intercepts if $\Delta<0$.
i.e., $16+60k<0$
$60k<-16$
$k<-\frac{4}{15}$

Level 2

p.42

19. (a) $0.\dot{2}1\dot{8}=0.218218\cdots$

$0.2\dot{1}\dot{8}=0.21818\cdots$

$0.21\dot{8}=0.21888\cdots$

$\therefore$ $0.21\dot{8}$ has the largest value.

(b) $0.21\dot{8}$
$=0.21+0.00888\cdots$
$=\frac{21}{100}+s$
where $s=0.00888\cdots$
$10s=0.0888\cdots$
$10s-s=0.08$
$s=\frac{0.08}{9}$
$=\frac{8}{900}$

$\therefore$ $0.21\dot{8}=\frac{21}{100}+\frac{8}{900}$
$=\frac{197}{900}$

20. (a) No, it is an irrational number.

(b) $\sin^2 10^\circ+\cos^2 10^\circ=1$

$\sin^2 10^\circ$ and $\cos^2 10^\circ$ are irrational numbers,
but $\sin^2 10^\circ+\cos^2 10^\circ=1$ is a rational number.

(c) $\sqrt{2}\times\sqrt{8}=4$

$\sqrt{2}$ and $\sqrt{8}$ are irrational numbers,
but $\sqrt{2}\times\sqrt{8}=\sqrt{16}=4$ is a rational number.

21. $(9x-14)^2+(9x-14)-20=0$
$(9x-14+5)(9x-14-4)=0$
$(9x-9)(9x-18)=0$
$9x-9=0$ or $9x-18=0$
$x=1$ or 2

22. $2(15y-13)^2-5(15y-13)-3=0$
$[2(15y-13)+1][(15y-13)-3]=0$
$(30y-26+1)(15y-16)=0$
$(30y-25)(15y-16)=0$
$30y-25=0$ or $15y-16=0$
$y=\frac{5}{6}$ or $\frac{16}{15}$

23. $x^2+2x=a^2+2a$
$(x^2-a^2)+(2x-2a)=0$
$(x+a)(x-a)+2(x-a)=0$
$(x-a)(x+a+2)=0$
$x-a=0$ or $x+a+2=0$
$x=a$ or $-a-2$

24. (a) Putting $x=2$ into the equation,

$(2)^2-5\times2+k(1-2)=0$
$4-10-k=0$
$k=-6$

(b) $x^2 - 5x + (-6)(1 - x) = 0$

$x^2 - 5x - 6 + 6x = 0$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x + 3 = 0$ or $x - 2 = 0$

$x = -3$ or 2

$\therefore$ The other root is -3.

25. (a) Putting $x = 3 + \sqrt{2}$ into the equation,

$(3+\sqrt{2})^2 - 6(3+\sqrt{2}) + c = 0$

$9 + 6\sqrt{2} + 2 - 18 - 6\sqrt{2} + c = 0$

$c - 7 = 0$

$c = \underline{\underline{7}}$

(b) $x^2 - 6x + 7 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{8}}{2}$$

$$= \frac{6 + 2\sqrt{2}}{2}$$

$$= 3 \pm \sqrt{2}$$

$\therefore$ The other root is $3 - \sqrt{2}$.

26. (a) $\Delta = (-8)^2 - 4(k)(1)$

$= 64 - 4k$

The equation has equal roots if $\Delta = 0$,

i.e., $64 - 4k = 0$

$k = \underline{\underline{16}}$

(b) $16x^2 - 8x + 1 = 0$

$(4x - 1)^2 = 0$

$4x - 1 = 0$

$x = \underline{\underline{\frac{1}{4}}}$

27. (a) $2x^2 - k(x - k) = 0$

$2x^2 - kx + k^2 = 0$

$\Delta = (-k)^2 - 4(2)(k)^2$

$= k^2 - 8k^2$

$= \underline{\underline{-7k^2}}$

(b) $\because$ $k \neq 0$

$\therefore$ $-7k^2 < 0$ for all real values of k,

i.e., $\Delta < 0$

$\therefore$ The equation has no real roots.

28. (a) Putting $x = 1, y = -9$ into the equation,

$-9 = -(1)^2 + 6(1) + k(1 - 8)$

$-9 = 5 - 7k$

$7k = 14$

$k = \underline{\underline{2}}$

(b) $y = -x^2 + 6x + 2(x - 8)$

$= -x^2 + 8x - 16$

When $y = 0$,

$-x^2 + 8x - 16 = 0$

$x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0$

$x - 4 = 0$

$x = 4$

$\therefore$ The coordinates of A are $(4, 0)$.

(c) Putting $x = h, y = -25$ into the equation,

$-25 = -h^2 + 8h - 16$

$h^2 - 8h - 9 = 0$

$(h + 1)(h - 9) = 0$

$h + 1 = 0$ or $h - 9 = 0$

$h = \underline{\underline{-1 \text{ or } 9}}$

29. (a) $\Delta = 2^2 - 4(m - 1)(2m - 3)$

$= 4 - 4(2m^2 - 2m - 3m + 3)$

$= 4 - 4(2m^2 - 5m + 3)$

$= 4(1 - 2m^2 + 5m - 3)$

$= 4(-2m^2 + 5m - 2)$

The equation has equal roots if $\Delta = 0$,

i.e., $-2m^2 + 5m - 2 = 0$

$2m^2 - 5m + 2 = 0$

$(2m - 1)(m - 2) = 0$

$2m - 1 = 0$ or $m - 2 = 0$

$m = \underline{\underline{\frac{1}{2} \text{ or } 2}}$

(b) When $m = \frac{1}{2}$, the equation becomes

$\left(\frac{1}{2} - 1\right)x^2 + 2x + \left(2\left(\frac{1}{2}\right) - 3\right) = 0$

$-\frac{1}{2}x^2 + 2x - 2 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$x - 2 = 0$

$x = \underline{\underline{2}}$

When $m = 2$, the equation becomes

$(2 - 1)x^2 + 2x + (2 \times 2 - 3) = 0$

$x^2 + 2x + 1 = 0$

$(x + 1)^2 = 0$

$x + 1 = 0$

$x = \underline{\underline{-1}}$

30. (a) $\Delta = [-(2k + 1)]^2 - 4(k + 2)(k - 3)$

$= 4k^2 + 4k + 1 - 4(k^2 - k - 6)$

$= 4k^2 + 4k + 1 - 4k^2 + 4k + 24$

$= \underline{\underline{8k + 25}}$

(b) (i) The equation has two unequal real roots if $\Delta > 0$.

$8k + 25 > 0$

$k > -\dfrac{25}{8}$

(ii) The equation has two equal real roots if $\Delta = 0$.

$8k + 25 = 0$

$k = -\dfrac{25}{8}$

(iii) The equation has no real roots if $\Delta < 0$.

$8k + 25 < 0$

$k < -\dfrac{25}{8}$

31. (a) The graph touches the x-axis if $\Delta = 0$,

i.e., $6^2 - 4(3)[-(k-1)] = 0$

$36 + 12(k-1) = 0$

$12(k-1) = -36$

$k - 1 = -3$

$k = -2$

(b) $y = 3x^2 + 6x - (-2 - 1)$

$= 3x^2 + 6x + 3$

When $y = 0$,

$3x^2 + 6x + 3 = 0$

$3(x+1)^2 = 0$

$x + 1 = 0$

$x = -1$

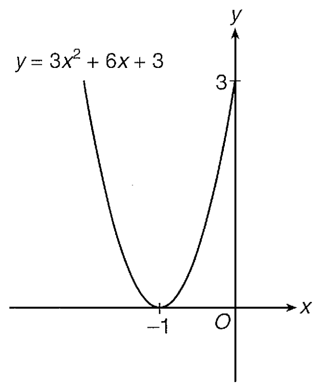
$\therefore$ $P = (-1, 0)$

(c) When $x = 0$,

$y = 3(0)^2 + 6(0) + 3 = 3$

$\therefore$ $q = 3$

(d)



32. (a) When $t = 5$,

$s = 10(5) + 50(5)^2 = 1300$

$\therefore$ The space shuttle is 1300 m above the ground after 5 seconds.

(b) When $s = 3000$,

$10t + 50t^2 = 3000$

$5t^2 + t - 300 = 0$

$t = \dfrac{-1 \pm \sqrt{(1)^2 - 4(5)(-300)}}{2(5)}$

$= \dfrac{-1 \pm \sqrt{6001}}{10}$

$= \dfrac{-1 + \sqrt{6001}}{10}$ or $\dfrac{-1 - \sqrt{6001}}{10}$ (rejected)

$= 7.65$

$\therefore$ It takes 7.65 seconds.

33. Let $AP = x$ cm.

$\therefore$ $PB = (x - 12)$ cm

$AP \times PB = AB^2$

$x(x - 12) = 12^2$

$x^2 - 12x - 144 = 0$

$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-144)}}{2 \times 1}$

$= \dfrac{12 \pm \sqrt{720}}{2}$

$= \dfrac{12 \pm 12\sqrt{5}}{2}$

$= 6 + 6\sqrt{5}$ or $6 - 6\sqrt{5}$ (rejected)

$\therefore$ $AP = (6 + 6\sqrt{5})$ cm

34. $b^2 - 3b + 1 = 0$

$b(b^2 - 3b + 1) = 0$

$b^3 - 3b^2 + b = 0$

$\therefore$ $b^3 - 3b^2 + b + 4 = 0 + 4$

$= 4$

35. (a) $a^2 + 3a - 2 = 0$

$a^2 + 3a = 2$

$b^2 + 3b - 2 = 0$

$b^2 + 3b = 2$

(b) $(a^2 + 3a) - (b^2 + 3b) = 2 - 2$

$(a^2 - b^2) + (3a - 3b) = 0$

$(a + b)(a - b) + 3(a - b) = 0$

$(a - b)(a + b + 3) = 0$

$a - b = 0$ or $a + b + 3 = 0$

$a = b$ (rejected) or $a + b = -3$

36 – 38 (HKCEE Questions)

Extended Question

p.44

39. (a) Putting $n = 10$,

$T_{10} = \dfrac{10 \times (10 + 1)}{2}$

$= 55$

(b)
$$\frac{n(n+1)}{2} = 231$$
$$n^2 + n = 462$$
$$n^2 + n - 462 = 0$$
$$(n-21)(n+22) = 0$$
$$n - 21 = 0 \quad \text{or} \quad n + 22 = 0$$
$$n = 21 \quad \text{or} \quad -22 \text{ (rejected)}$$
∴ 231 is the 21st triangular number.

(c)
$$T_{n-1} + T_n = \frac{(n-1)(n-1+1)}{2} + \frac{n(n+1)}{2}$$
$$= \frac{(n-1)n}{2} + \frac{n(n+1)}{2}$$
$$= \frac{n(n-1+n+1)}{2}$$
$$= \frac{2n^2}{2}$$
$$= \underline{\underline{n^2}}$$

(d)
$$n^2 = 484$$
$$n = \pm\sqrt{484}$$
$$n = \pm 22$$
$$n = 22 \ (\because n > 0)$$
$$T_{21} = \frac{21 \times (21+1)}{2} = 231$$
$$T_{22} = \frac{22 \times (22+1)}{2} = 253$$
∴ The triangular numbers are 231 and 253.

Multiple-choice Questions

p.45

1. B

2. C
$$(x-1)(x-2) = 2$$
$$x^2 - 3x + 2 = 2$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$x = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = \underline{\underline{0 \text{ or } 3}}$$

3. B
$$(x-p)(x+p) = x - p$$
$$(x-p)(x+p-1) = 0$$
$$x - p = 0 \quad \text{or} \quad x + p - 1 = 0$$
$$x = \underline{\underline{p \text{ or } 1-p}}$$

4. D

5. A

6. B
$$x^2 + 2x - 5 = 0$$
$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)}$$
$$= \underline{\underline{-1 \pm \sqrt{6}}}$$
∴ The roots are unequal.

7. C

Since 3 is a root of

$x^2 + px - 18 = 0$, ... (*)

Putting $x = 3$ into (*), we have
$$(3)^2 + 3p - 18 = 0$$
$$3p - 9 = 0$$
$$3p = 9$$
$$p = 3$$
Putting $p = 3$ into (*),
$$x^2 + 3x - 18 = 0$$
$$(x+6)(x-3) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = \underline{\underline{-6}} \quad \text{or} \quad 3 \text{ (rejected)}$$

8. B

9. D
$$(2x^2 - 4x + 5) + k(x-1) = 0$$
$$2x^2 - (4-k)x + (5-k) = 0$$
∵ The equation has equal roots,

∴ $\Delta = (4-k)^2 - 4(2)(5-k) = 0$
$$16 + k^2 - 8k - 40 + 8k = 0$$
$$k^2 - 24 = 0$$
$$k = \underline{\underline{\pm 2\sqrt{6}}}$$

10. C

Consider the discriminant of the equation:
$$\Delta = (6)^2 - 4(1)(m)$$
$$= 36 - 4m$$
If $m > 9$, then $36 - 4m < 0$,

i.e., $\Delta < 0$.

∴ The equation $x^2 + 6x + m = 0$ has no real roots when $m > 9$.

11. A

Consider the discriminant of the equation:
$$\Delta = b^2 - 4ac$$
If the equation have two unequal real roots, Δ must be greater than zero.
$$\Delta = b^2 - 4ac > 0.$$
$4ac < b^2$ .... (*)

Since b^2 attains the minimum value of zero when $b = 0$, we can rewrite (*) as
$$4ac < 0$$
$$\underline{\underline{ac < 0}}$$