• To find the equation of the tangent to a circle at a point on the circle. • To find the equations of the tangents to a circle with given slope. • To find the equations of the tangents to a circle from an external point. • To find the length of tangents from an external point.	12.3 Equations of Tangents to a Circle A. Tangent to a circle at a point on the circle B. Tangents with given slope C. Tangents from a given external point D. Length of the tangent	5		
• To recognize cases of touching of two circles.	12.4 Touching of Two Circles	2	Exercise 12B (p.308)	
• To find the family of concentric circles with given centre. • To find the family of circles passing through the intersection(s) of a line and a circle. • To find the family of circles through the intersection(s) of two circles.	12.5 Families of Circles A. Concentric circles B. Circles passing through the intersection(s) of a line and a circle C. Circles passing through the intersection(s) of two circles	4	Exercise 12C (p.316)	
• To remind students the essential knowledge in the chapter.	Chapter Summary	1	Revision Exercise 12 (p.320) Enrichment 12 (p.323)	

18

CHAPTER 1

Exercise 1A (p.8)

1. (a) $x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = \underline{\underline{1}}$

(b) $x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = \underline{\underline{5}}$ or $\underline{\underline{-1}}$

(c) $3x^2 - 7x + 4 = 0$

$(3x-4)(x-1) = 0$

$x = \underline{\underline{\frac{4}{3}}}$ or $\underline{\underline{1}}$

(d) $8x^2 = 6x - 1$

$8x^2 - 6x + 1 = 0$

$(4x-1)(2x-1) = 0$

$x = \underline{\underline{\frac{1}{4}}}$ or $\underline{\underline{\frac{1}{2}}}$

(e) $56 - 10x - 6x^2 = 0$

$6x^2 + 10x - 56 = 0$

$3x^2 + 5x - 28 = 0$

$(3x-7)(x+4) = 0$

$x = \underline{\underline{\frac{7}{3}}}$ or $\underline{\underline{-4}}$

(f) $2(5x^2 - 1) - x = 0$

$10x^2 - 2 - x = 0$

$10x^2 - x - 2 = 0$

$(5x+2)(2x-1) = 0$

$x = \underline{\underline{-\frac{2}{5}}}$ or $\underline{\underline{\frac{1}{2}}}$

2. (a) $x^2 + 6x + 5 = 0$

$x^2 + 6x + (\frac{6}{2})^2 - (\frac{6}{2})^2 + 5 = 0$

$(x+3)^2 - 9 + 5 = 0$

$(x+3)^2 = 4$

$x + 3 = \pm 2$

$x = \underline{\underline{-5}}$ or $\underline{\underline{-1}}$

(b) $2x^2 - 5x - 8 = 0$

$x^2 - \frac{5}{2}x - 4 = 0$

$x^2 - \frac{5}{2}x + (\frac{5}{4})^2 - (\frac{5}{4})^2 - 4 = 0$

$(x - \frac{5}{4})^2 - \frac{25}{16} - 4 = 0$

$(x - \frac{5}{4})^2 - \frac{89}{16} = 0$

$(x - \frac{5}{4})^2 - (\frac{\sqrt{89}}{4})^2 = 0$

$(x - \frac{5}{4} - \frac{\sqrt{89}}{4})(x - \frac{5}{4} + \frac{\sqrt{89}}{4}) = 0$

$x = \underline{\underline{\frac{5+\sqrt{89}}{4}}}$ or $\underline{\underline{\frac{5-\sqrt{89}}{4}}}$

3. (a) $2x^2 - 5x - 6 = 0$

Let $a = 2$, $b = -5$, $c = -6$.

Using the formula,

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-6)}}{2(2)}$

$= \frac{5 \pm \sqrt{25+48}}{4}$

$= \underline{\underline{\frac{5 \pm \sqrt{73}}{4}}}$

(b) $35 - 100x + x^2 = 0$

$x^2 - 100x + 35 = 0$

Let $a = 1$, $b = -100$, $c = 35$.

Using the formula,

$x = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(1)(35)}}{2(1)}$

$= \frac{100 \pm \sqrt{10\ 000 - 140}}{2}$

$= \frac{100 \pm \sqrt{9\ 860}}{2}$

$= \underline{\underline{50 \pm \sqrt{2\ 465}}}$

(c) $2x^2 + 4x = 5 + 5x$

$2x^2 - x - 5 = 0$

Let $a = 2$, $b = -1$, $c = -5$.

Using the formula,

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-5)}}{2(2)}$

$= \frac{1 \pm \sqrt{1+40}}{2(2)}$

$= \underline{\underline{\frac{1 \pm \sqrt{41}}{4}}}$

(d) $2x^2 + 4x = -3$

$2x^2 + 4x + 3 = 0$

Let $a = 2$, $b = 4$, $c = 3$.

Using the formula,

$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(3)}}{2(2)} = \frac{-4 \pm \sqrt{-8}}{4}$

$\therefore$ $\underline{\underline{\text{The equation has no real solution.}}}$

(e)
$$\frac{x^2}{3}-1=\frac{x}{6}$$
$$2x^2-6=x$$
$$2x^2-x-6=0$$
Let $a=2$, $b=-1$, $c=-6$.
Using the formula,
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-6)}}{2(2)}=\frac{1\pm\sqrt{49}}{4}=\frac{1\pm7}{4}=\frac{-3}{2}\text{ or }2$$

(f)
$$3x^2+5x-1=x(x+2)$$
$$3x^2+5x-1=x^2+2x$$
$$2x^2+3x-1=0$$
Let $a=2$, $b=3$, $c=-1$.
Using the formula,
$$x=\frac{-3\pm\sqrt{3^2-4(2)(-1)}}{2(2)}=\frac{-3\pm\sqrt{17}}{4}$$

4.
$$(3x+7)(x-1)=4(x-1)$$
$$(3x+7)(x-1)-4(x-1)=0$$
$$(x-1)(3x+7-4)=0$$
$$(x-1)(3x+3)=0$$
$$x=1\text{ or }-1$$

5.
$$5(2x+1)^2+12(2x+1)-9=0$$
$$[5(2x+1)-3][(2x+1)+3]=0$$
$$(10x+5-3)(2x+1+3)=0$$
$$(10x+2)(2x+4)=0$$
$$x=-\frac{1}{5}\text{ or }-2$$

6.
$$(x^2+5x)^2-8(x^2+5x)-84=0$$
$$(x^2+5x+6)(x^2+5x-14)=0$$
$$(x+2)(x+3)(x-2)(x+7)=9$$
$$x=-3,\ -2,\ 2,\ -7$$

7.
$$x^{\frac{2}{3}}-13x^{\frac{1}{3}}+36=0$$
$$(x^{\frac{1}{3}}-9)(x^{\frac{1}{3}}-4)=0$$
$$x^{\frac{1}{3}}=9\text{ or }4$$
$$x=729\text{ or }64$$

8.
$$x^{-2}-x^{-1}-56=0$$
$$(x^{-1}-8)(x^{-1}+7)=0$$
$$x^{-1}=8\text{ or }-7$$
$$x=\frac{1}{8}\text{ or }-\frac{1}{7}$$

9.
$$2x-9x^{\frac{1}{2}}+4=0$$
$$(2x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-4)=0$$
$$x^{\frac{1}{2}}=\frac{1}{2}\text{ or }4$$
$$x=\frac{1}{4}\text{ or }16$$

10.
$$2y=5+\frac{3}{y}$$
$$2y^2=5y+3$$
$$2y^2-5y-3=0$$
$$(2y+1)(y-3)=0$$
$$y=-\frac{1}{2}\text{ or }3$$

11.
$$\frac{3x^2-11x-40}{2x+1}=x-5$$
$$3x^2-11x-40=2x^2-9x-5$$
$$x^2-2x-35=0$$
$$(x-7)(x+5)=0$$
$$x=7\text{ or }-5$$

12. (a)
$$x^2+8px+1=x^2+8px+\left(\frac{8p}{2}\right)^2-\left(\frac{8p}{2}\right)^2+1$$
$$=(x+4p)^2-16p^2+1$$
$$=(x+4p)^2+(1-16p^2)$$
$\therefore\ l=4p,\ m=1-16p^2$

(b)
$$x^2-px+2p=x^2-px+\left(\frac{p}{2}\right)^2-\left(\frac{p}{2}\right)^2+2p$$
$$=\left(x-\frac{p}{2}\right)^2-\frac{p^2}{4}+2p$$
$$=\left(x-\frac{p}{2}\right)^2+\left(2p-\frac{p^2}{4}\right)$$
$\therefore\ l=-\frac{p}{2},\ m=2p-\frac{p^2}{4}$

13. Since $\frac{5}{2}$ is a root of $2x^2-3x-a=0$,
$$\therefore\ 2\left(\frac{5}{2}\right)^2-3\left(\frac{5}{2}\right)-a=0$$
$$a=5$$

14. Since -2 is a root of $2x^2+6x+k=0$,
$$2(-2)^2+6(-2)+k=0$$
$$k=4$$
Substitute $k=4$ into the given equation,
$$2x^2+6x+4=0$$
$$x^2+3x+2=0$$
$$(x+1)(x+2)=0$$
$\therefore\ x=-1$ or -2
$\therefore$ The other root of the equation is -1.

15.
$$2x^2-5x+3=2\left(x^2-\frac{5}{2}x+\frac{3}{2}\right)$$
$$=2\left[x^2-\frac{5}{2}x+\left(\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2+\frac{3}{2}\right]$$
$$=2\left(x-\frac{5}{4}\right)^2+2\left(-\frac{1}{16}\right)$$
$$=2\left(x-\frac{5}{4}\right)^2-\frac{1}{8}$$

16. (a) Since 2 is a root of
$m^2x^2+2(2m-5)x+8=0$.
$$m^2(2)^2+2(2m-5)(2)+8=0$$
$$4m^2+8m-20+8=0$$
$$4m^2+8m-12=0$$
$$m^2+2m-3=0$$
$$(m-1)(m+3)=0$$
$$m=1\text{ or }-3$$

(b) When $m=1$,
$$x^2+2(2-5)x+8=0$$
$$x^2-6x+8=0$$
$$(x-2)(x-4)=0$$
$$x=2\text{ or }4$$
$\therefore$ When $m=1$, the other root $=4$

When $m=-3$,
$$9x^2+2(-6-5)x+8=0$$
$$9x^2-22x+8=0$$
$$(9x-4)(x-2)=0$$
$$x=2\text{ or }\frac{4}{9}$$
$\therefore$ When $m=-3$, the other root $=\frac{4}{9}$

17. (a)
$$\frac{x^2+3x+1}{2x^2-5x+2}=l$$
$$x^2+3x+1=2lx^2-5lx+2l$$
$$(2l-1)x^2-(5l+3)x+(2l-1)=0$$

(b) Since $-\frac{3}{4}$ is a root of (*),
$$\frac{(-\frac{3}{4})^2+3(-\frac{3}{4})+1}{2(-\frac{3}{4})^2-5(-\frac{3}{4})+2}=l$$
$$l=-\frac{1}{10}$$

(c) Substitute $l=-\frac{1}{10}$ into the equation obtain in **(a)**,
$$\left[2\left(-\frac{1}{10}\right)-1\right]x^2-\left[5\left(-\frac{1}{10}\right)+3\right]x+\left[2\left(-\frac{1}{10}\right)-1\right]=0$$
$$-\frac{12}{10}x^2-\frac{25}{10}x-\frac{12}{10}=0$$
$$12x^2+25x+12=0$$
$$(3x+4)(4x+3)=0$$
$$x=-\frac{4}{3}\text{ or }-\frac{3}{4}$$
$\therefore$ The other root of equation is $-\frac{4}{3}$

Exercise 1B (p.13)

1. $kx^2-(3k-5)x+14=0$
Let $a=k$, $b=-(3k-5)$, $c=14$
$$D=[-(3k-5)]^2-4(k)(14)$$
$$=9k^2-30k+25-56k$$
$$=9k^2-86k+25$$

2. $(2k+1)x^2-2k=(k+4)x$
$(2k+1)x^2-(k+4)x-2k=0$
Let $a=2k+1$, $b=-(k+4)$, $c=-2k$
$$D=[-(k+4)]^2-4(2k+1)(-2k)$$
$$=k^2+8k+16+16k^2+8k$$
$$=17k^2+16k+16$$

3. $x^2-kx+16=0$
$$D=k^2-4(16)=0$$
$$k^2=64$$
$$k=\pm8$$

4. $x^2-k(x-1)=0$
$$x^2-kx+k=0$$
$$\therefore\ D=k^2-4k=0$$
$$k(k-4)=0$$
$$k=0\text{ or }4$$

5. $kx^2 - 2x + 1 = 0$

$D = (-2)^2 - 4k = 0$

$4 - 4k = 0$

$4k = 4$

$k = \underline{\underline{1}}$

6. $x^2 - kx + k + 3 = 0$

$D = (-k)^2 - 4(k+3) = 0$

$k^2 - 4k - 12 = 0$

$(k-6)(k+2) = 0$

$k = \underline{\underline{6}}$ or $\underline{\underline{-2}}$

7. $x^2 - 2ax + a^2 - b^2 - 2bc - c^2 = 0$

$D = (-2a)^2 - 4(a^2 - b^2 - 2bc - c^2)$

$= 4a^2 - 4a^2 + 4b^2 + 8bc + 4c^2$

$= 4b^2 + 8bc + 4c^2$

$= 4(b^2 + 2bc + c^2)$

$= 4(b+c)^2$

$= [2(b+c)]^2$

which is a perfect square for any integers b and c.

$\therefore$ Roots are rational.

8. $px^2 + 2qx - p + 2q = 0$

$D = (2q)^2 - 4p(-p + 2q)$

$= 4q^2 + 4p^2 - 8pq$

$= 4(p^2 - 2pq + q^2)$

$= 4(p-q)^2$

$= [2(p-q)]^2$

which is the square of a rational number as p and q are rational.

$\therefore$ Roots are rational.

9. $x^2 - (a+b+c)x + a(b+c) = 0$

$D = (a+b+c)^2 - 4a(b+c)$

$= (a+a)^2 - 4a \cdot a$

$= 4a^2 - 4a^2$

$= 0$

$\therefore$ The equation has equal roots.

10. $(m^2 + n^2)x^2 - 2(m+n)x + 2 = 0$

$D = [-2(m+n)]^2 - 4(m^2 + n^2)(2)$

$= 4(m+n)^2 - 8(m^2 + n^2)$

$= 4(m^2 + 2mn + n^2) - 8m^2 - 8n^2$

$= 4m^2 + 8mn + 4n^2 - 8m^2 - 8n^2$

$= -4m^2 + 8mn - 4n^2$

$= -4(m^2 - 2mn + n^2)$

$= -4(m-n)^2$

If $m \neq n$, $D < 0$.

$\therefore$ The equation has unreal roots.

11. $x^2 + 2ax + a^2 - b^2 - c^2 = 0$

$D = (2a)^2 - 4(a^2 - b^2 - c^2)$

$= 4a^2 - 4a^2 + 4b^2 + 4c^2$

$= 4(b^2 + c^2)$

If $b \neq 0$ and $c \neq 0$, $D > 0$.

$\therefore$ The equation has unequal real roots.

12. $(x-a)(x-b) = c$

$x^2 - (a+b)x + ab - c = 0$

$D = (a+b)^2 - 4(ab - c)$

$= a^2 + 2ab + b^2 - 4ab + 4c$

$= a^2 - 2ab + b^2 + 4c$

$= (a-b)^2 + 4c$

$\because$ $(a-b)^2 \geq 0$ and $c > 0$

$\therefore$ $(a-b)^2 + 4c > 0$

$\therefore$ The roots of the equation are real.

13. $x^2 - 2(a+3)x + (11a+3) = 0$

$D = [-2(a+3)]^2 - 4(11a+3)$

$= 0$

$4(a^2 + 6a + 9) - 4(11a + 3) = 0$

$4(a^2 - 5a + 6) = 0$

$a^2 - 5a + 6 = 0$

$(a-2)(a-3) = 0$

$a = \underline{\underline{2}}$ or $\underline{\underline{3}}$

When $a = 2$, the equation is

$x^2 - 10x + 25 = 0$

$(x-5)^2 = 0$

$x = 5$

$\therefore$ When $a = 2$, $x = \underline{\underline{5}}$

When $a = 3$, the equation is

$x^2 - 12x + 36 = 0$

$(x-6)^2 = 0$

$x = 6$

$\therefore$ When $a = 3$, $x = \underline{\underline{6}}$

14. $x^2 + 2(a+2)x + (5a+24) = 0$

$D = [2(a+2)]^2 - 4(5a+24) = 0$

$4(a^2 + 4a + 4) - 4(5a + 24) = 0$

$4(a^2 - a - 20) = 0$

$a^2 - a - 20 = 0$

$(a-5)(a+4) = 0$

$a = \underline{\underline{5}}$ or $\underline{\underline{-4}}$

When $a = 5$, the equation is

$x^2 + 14x + 49 = 0$

$(x+7)^2 = 0$

$x = -7$

$\therefore$ When $a = 5$, $x = \underline{\underline{-7}}$

When $a = -4$, the equation is

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

$\therefore$ When $a = -4$, $x = \underline{\underline{2}}$

15. $x^2 + (m+n)x + (m^2 - n^2) = 0$

$D = (m+n)^2 - 4(m^2 - n^2) = 0$

$m^2 + 2mn + n^2 - 4(m^2 - n^2) = 0$

$-3m^2 + 2mn + 5n^2 = 0$

$(-3m + 5n)(m+n) = 0$

$m = \underline{\underline{\frac{5n}{3}}}$ or $\underline{\underline{-n}}$

16. (a) $(c-a)x^2 - 2(a-b)x + (b-c) = 0$

$D = [-2(a-b)]^2 - 4(c-a)(b-c)$

$= 4(a-b)^2 - 4(c-a)(b-c)$

$= 4[(a-b)^2 - (c-a)(b-c)]$

$= 4[a^2 - 2ab + b^2 - (bc - c^2 - ab + ac)]$

$= 4(a^2 + b^2 + c^2 - ab - bc - ac)$

$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$

$= 2(a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2)$

$= 2[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$\because$ a, b and c are not all equal.

$\therefore$ $2[(a-b)^2 + (b-c)^2 + (c-a)^2] > 0$

$\therefore$ The equation has unequal real roots.

(b) If $a = 1$, $b = 3$, the equation becomes

$(c-1)x^2 - 2(-2)x + (3-c) = 0$

$(c-1)x^2 + 4x + (3-c) = 0$

$D = 16 - 4(c-1)(3-c)$

$= 16 - 4(3c - c^2 - 3 + c)$

$= 4[4 - (-c^2 + 4c - 3)]$

$= 4(4 + c^2 - 4c + 3)$

$\because$ $4(c^2 - 4c + 7)$

$= 4[(c^2 - 4c + 4) + 3]$

$= 4[(c-2)^2 + 3] > 0$

Hence, if (*) has unequal real roots, c can be any real numbers. But for (*) to be a quadratic equation, $a \neq c$.

$\therefore$ c can be any real number other than 1.

Exercise 1C (p.19)

1. (a) $x^2 - 4x + 1 = 0$

Sum of the roots $= -\frac{-4}{1} = \underline{\underline{4}}$

Product of the roots $= \frac{1}{1} = \underline{\underline{1}}$

(b) $48x^2 = 22x + 15$

$48x^2 - 22x - 15 = 0$

Sum of the roots $= \frac{22}{48} = \underline{\underline{\frac{11}{24}}}$

Product of the roots $= -\frac{15}{48} = \underline{\underline{-\frac{5}{16}}}$

(c) $(4x+7)^2 = 4x^2$

$16x^2 + 56x + 49 - 4x^2 = 0$

$12x^2 + 56x + 49 = 0$

Sum of the roots $= -\frac{56}{12} = \underline{\underline{-\frac{14}{3}}}$

Product of the roots $= \underline{\underline{\frac{49}{12}}}$

(d) $6 - 5x - 25x^2 = 0$

$25x^2 + 5x - 6 = 0$

Sum of the roots $= -\frac{5}{25} = \underline{\underline{-\frac{1}{5}}}$

Product of the roots $= \underline{\underline{-\frac{6}{25}}}$

2. $3x^2 - 5x - 1 = 0$

$\alpha + \beta = \frac{5}{3}$ and $\alpha\beta = -\frac{1}{3}$

(a) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \frac{25}{9} + \frac{2}{3}$

$= \underline{\underline{\frac{31}{9}}}$

(b) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}$

$=\dfrac{\frac{31}{9}}{-\frac{1}{3}}$

$=\underline{\underline{-\dfrac{31}{3}}}$

(c) $(1-\alpha)(1-\beta)=1-(\alpha+\beta)+\alpha\beta$

$=1-\dfrac{5}{3}-\dfrac{1}{3}$

$=\underline{\underline{-1}}$

(d) $\alpha\beta^2+\alpha^2\beta=\alpha\beta(\alpha+\beta)$

$=-\dfrac{1}{3}(\dfrac{5}{3})$

$=\underline{\underline{-\dfrac{5}{9}}}$

3. $x^2+px+q=0$

$\alpha+\beta=-p$ and $\alpha\beta=q$

(a) $\alpha^2-\alpha\beta+\beta^2=\alpha^2+2\alpha\beta+\beta^2-3\alpha\beta$

$=(\alpha+\beta)^2-3\alpha\beta$

$=(-p)^2-3q$

$=\underline{\underline{p^2-3q}}$

(b) $\alpha^3\beta+\alpha\beta^3=\alpha\beta(\alpha^2+\beta^2)$

$=\alpha\beta[(\alpha+\beta)^2-2\alpha\beta]$

$=q[(-p)^2-2q]$

$=\underline{\underline{p^2q-2q^2}}$

(c) $\dfrac{1}{2\alpha+\beta}+\dfrac{1}{\alpha+2\beta}$

$=\dfrac{\alpha+2\beta+2\alpha+\beta}{(2\alpha+\beta)(\alpha+2\beta)}$

$=\dfrac{3(\alpha+\beta)}{2\alpha^2+4\alpha\beta+\alpha\beta+2\beta^2}$

$=\dfrac{3(\alpha+\beta)}{2(\alpha^2+2\alpha\beta+\beta^2)+\alpha\beta}$

$=\dfrac{3(\alpha+\beta)}{2(\alpha+\beta)^2+\alpha\beta}$

$=\dfrac{3(-p)}{2(-p)^2+q}$

$=\underline{\underline{\dfrac{-3p}{2p^2+q}}}$

(d) $\alpha^4+\beta^4=\alpha^4+2\alpha^2\beta^2+\beta^4-2\alpha^2\beta^2$

$=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2$

$=[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2$

$=[(-p)^2-2q]^2-2q^2$

$=(p^2-2q)^2-2q^2$

$=p^4-4p^2q+4q^2-2q^2$

$=\underline{\underline{p^4-4p^2q+2q^2}}$

4. (a) Sum of the roots $=4+(-5)=-1$

Product of the roots $=4(-5)=-20$

One required equation is $\underline{\underline{x^2+x-20=0}}$.

(b) Sum of the roots $=-\dfrac{3}{2}+\dfrac{2}{3}=-\dfrac{5}{6}$

Product of the roots $=(-\dfrac{3}{2})(\dfrac{2}{3})=-1$

One required equation is $x^2+\dfrac{5}{6}x-1=0$,

i.e. $\underline{\underline{6x^2+5x-6=0}}$.

(c) Sum of the roots $=-1+\sqrt{5}+(-1-\sqrt{5})$

$=-2$

Product of the roots $=(-1+\sqrt{5})(-1-\sqrt{5})$

$=(-1)^2-(\sqrt{5})^2$

$=-4$

One required equation is $\underline{\underline{x^2+2x-4=0}}$.

(d) Sum of the roots $=\dfrac{-5+3\sqrt{2}}{2}+\dfrac{-5-3\sqrt{2}}{2}$

$=\dfrac{-10}{2}$

$=-5$

Product of the roots $=(\dfrac{-5+3\sqrt{2}}{2})(\dfrac{-5-3\sqrt{2}}{2})$

$=\dfrac{(-5)^2-(3\sqrt{2})^2}{4}$

$=\dfrac{7}{4}$

One required equation is $x^2+5x+\dfrac{7}{4}=0$,

i.e. $\underline{\underline{4x^2+20x+7=0}}$.

5. $x^2-4x+1=0$

$\alpha+\beta=4$, $\alpha\beta=1$

(a) Sum of the roots

$=2\alpha+1+2\beta+1=2(\alpha+\beta)+2$

$=10$

Product of the roots

$=(2\alpha+1)(2\beta+1)$

$=4\alpha\beta+2(\alpha+\beta)+1$

$=13$

One required equation is $\underline{\underline{x^2-10x+13=0}}$.

(b) Sum of the roots $=\dfrac{3}{\alpha}+\dfrac{3}{\beta}=3(\dfrac{\alpha+\beta}{\alpha\beta})=12$

Product of the roots $=(\dfrac{3}{\alpha})(\dfrac{3}{\beta})=9(\dfrac{1}{\alpha\beta})=9$

One required equation is $\underline{\underline{x^2-12x+9=0}}$.

(c) Sum of the roots

$=\alpha^2+\dfrac{1}{\alpha}+\beta^2+\dfrac{1}{\beta}$

$=(\alpha+\beta)^2-2\alpha\beta+\dfrac{\alpha+\beta}{\alpha\beta}$

$=18$

Product of the roots

$=(\alpha^2+\dfrac{1}{\alpha})(\beta^2+\dfrac{1}{\beta})$

$=\alpha^2\beta^2+\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}+\dfrac{1}{\alpha\beta}$

$=2+\dfrac{\alpha^3+\beta^3}{\alpha\beta}$

$=2+(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$=2+4[(\alpha+\beta)^2-3\alpha\beta]$

$=2+4(16-3)$

$=54$

One required equation is $\underline{\underline{x^2-18x+54=0}}$.

(d) Sum of the roots

$=\alpha-\beta+\beta-\alpha=0$

Product of the roots

$=(\alpha-\beta)(\beta-\alpha)$

$=\alpha\beta-\alpha^2-\beta^2+\alpha\beta$

$=-(\alpha^2-2\alpha\beta+\beta^2)$

$=-[(\alpha+\beta)^2-4\alpha\beta]$

$=-12$

One required equation is $\underline{\underline{x^2-12=0}}$.

6. Let the other root be α.

Sum of the roots $=2-\sqrt{5}+\alpha=4$

$\alpha=2+\sqrt{5}$

$\therefore$ The other root of the equation is $\underline{\underline{2+\sqrt{5}}}$.

Product of the roots $=l=(2-\sqrt{5})(2+\sqrt{5})$

$=(2)^2-(\sqrt{5})^2$

$=\underline{\underline{-1}}$

7. $8x^2-ax+9=0$

Let α, 2α be the roots.

Sum of the roots $=\alpha+2\alpha=3\alpha=\dfrac{a}{8}$ ....(1)

Product of the roots $=\alpha(2\alpha)=2\alpha^2=\dfrac{9}{8}$ ......(2)

Solving (2), $\alpha=\pm\dfrac{3}{4}$.

When $\alpha=\dfrac{3}{4}$, $a=\underline{\underline{18}}$.

When $\alpha=-\dfrac{3}{4}$, $a=\underline{\underline{-18}}$

$\therefore$ When $a=18$, the roots are $\underline{\underline{\dfrac{3}{4},\dfrac{3}{2}}}$

When $a=-18$, the roots are $\underline{\underline{-\dfrac{3}{4},-\dfrac{3}{2}}}$

8. $12x^2+mx-5=0$

Sum of the roots $=-\dfrac{m}{12}$

Product of the roots $=\dfrac{-5}{12}$

$-\dfrac{m}{12}-(-\dfrac{5}{12})=\dfrac{11}{6}$

$-\dfrac{m}{12}+\dfrac{5}{12}=\dfrac{11}{6}$

$m=\underline{\underline{-17}}$

The equation becomes

$12x^2-17x-5=0$

$(4x+1)(3x-5)=0$

$x=\underline{\underline{-\dfrac{1}{4}}}$ or $\underline{\underline{\dfrac{5}{3}}}$

9. $kx^2+4x+k^2-21=0$

Product of the roots $=\dfrac{k^2-21}{k}$

$=4$

$k^2-4k-21=0$

$(k-7)(k+3)=0$

$k=\underline{\underline{7}}$ or $\underline{\underline{-3}}$

10. $(kx-1)^2=k$

$k^2x^2-2kx+1-k=0$

Product of the roots $=\dfrac{1-k}{k^2}=\dfrac{-3}{16}$

$16-16k=-3k^2$

$3k^2-16k+16=0$

$(3k-4)(k-4)=0$

$k=\underline{\underline{\dfrac{4}{3}}}$ or $\underline{\underline{4}}$

11. (a) $x^2+a(3a-5)x=2(x+4a)$

$x^2+a(3a-5)x-2(x+4a)=0$

$x^2+a(3a-5)x-2x-8a=0$

$x^2+[a(3a-5)-2]x-8a=0$ ....(*)

Let α, $-\alpha$ be the roots of (*).

Sum of the roots

$=-[a(3a-5)-2]=\alpha-\alpha=0$

$3a^2-5a-2=0$

$(a-2)(3a+1)=0$

$a=\underline{\underline{2}}$ or $\underline{\underline{-\frac{1}{3}}}$

(b) If $a>0$, by **(a)**, $a=2$.

The equation becomes

$x^2+2x=2(x+8)$

$x^2+2x=2x+16$

$x^2=16$

$x=\underline{\underline{\pm4}}$

12. α and β are the roots of $x^2+px-5=0$.

Sum of the roots $=\alpha+\beta=-p$ ....(1)

Product of the roots $=\alpha\beta=-5$ ....(2)

α^2 and β^2 are the roots of $x^2-19x+q=0$,

Sum of the roots $=\alpha^2+\beta^2=19$ ....(3)

Product of the roots $=\alpha^2\beta^2=q$ ....(4)

By (3), $\alpha^2+\beta^2=19$

$(\alpha+\beta)^2-2\alpha\beta=19$

By (1) and (2),

$(-p)^2-2(-5)=19$

$p^2+10=19$

$p^2=9$

$p=\underline{\underline{\pm3}}$

By (2) and (4),

$(-5)^2=q$

$q=\underline{\underline{25}}$

13. α and β are roots of $x^2-5x+k=0$,

Sum of the roots $=\alpha+\beta=5$ ....(1)

Product of the roots $=\alpha\beta=k$ ....(2)

$\frac{1}{2\alpha+1}$ and $\frac{1}{2\beta+1}$ are roots of $35x^2+nx+1=0$,

Sum of the roots

$=\frac{1}{2\alpha+1}+\frac{1}{2\beta+1}=-\frac{n}{35}$ ....(3)

Product of the roots

$=(\frac{1}{2\alpha+1})(\frac{1}{2\beta+1})=\frac{1}{35}$ ....(4)

By (4), $(\frac{1}{2\alpha+1})(\frac{1}{2\beta+1})=\frac{1}{35}$

$\frac{1}{(2\alpha+1)(2\beta+1)}=\frac{1}{35}$

$\frac{1}{4\alpha\beta+2(\alpha+\beta)+1}=\frac{1}{35}$

By (1) and (2), $\frac{1}{4k+2(5)+1}=\frac{1}{35}$

$\frac{1}{4k+11}=\frac{1}{35}$

$4k+11=35$

$k=\underline{\underline{6}}$

By (3), $\frac{1}{2\alpha+1}+\frac{1}{2\beta+1}=-\frac{n}{35}$

$\frac{2\beta+1+2\alpha+1}{(2\alpha+1)(2\beta+1)}=-\frac{n}{35}$

$\frac{2(\alpha+\beta)+2}{(2\alpha+1)(2\beta+1)}=-\frac{n}{35}$

By (1) and (4), $\frac{2(5)+2}{35}=-\frac{n}{35}$

$n=\underline{\underline{-12}}$

14. Let α and $\alpha+1$ be the roots of $x^2-px+q=0$.

Sum of the roots $=\alpha+\alpha+1=2\alpha+1=p$ .....(1)

Product of the roots $=\alpha(\alpha+1)=q$ ....(2)

By (1), $\alpha=\frac{1}{2}(p-1)$

Put $\alpha=\frac{1}{2}(p-1)$ into (2),

$\frac{1}{2}(p-1)[\frac{1}{2}(p-1)+1]=q$

$\frac{1}{2}(p-1)\frac{1}{2}(p+1)=q$

$p^2-1=4q$

$p^2-4q-1=0$

15. Let α and $\alpha-2$ be the roots of $x^2+px+q=0$.

Sum of the roots

$=\alpha+\alpha-2=2\alpha-2=-p$ ....(1)

Product of the roots

$=\alpha(\alpha-2)=q$ ....(2)

By (1), $\alpha=\frac{1}{2}(2-p)$

Put $\alpha=\frac{1}{2}(2-p)$ into (2),

$\frac{1}{2}(2-p)(1-\frac{p}{2}-2)=q$

$\frac{1}{2}(2-p)(-\frac{p}{2}-1)=q$

$\frac{1}{2}[-(p-2)][-\frac{1}{2}(p+2)]=q$

$\frac{1}{4}(p-2)(p+2)=q$

$p^2-4=4q$

$p^2=4+4q$

16. Let α and β be the roots of $ax^2+bx+c=0$.

Sum of the roots $=\alpha+\beta=-\frac{b}{a}$ ....(1)

Product of the roots $=\alpha\beta=\frac{c}{a}$ ....(2)

$\because\ \frac{\alpha}{\beta}=\frac{m}{n}$, $\therefore\ \alpha=\frac{m}{n}\beta$ ....(3)

Put (3) into (1), $\frac{m}{n}\beta+\beta=-\frac{b}{a}$

$(\frac{m+n}{n})\beta=-\frac{b}{a}$

$\beta=-\frac{bn}{a(m+n)}$

Put (3) into (2), $(\frac{m}{n}\beta)\beta=\frac{c}{a}$

$\beta^2=\frac{cn}{am}$

$\frac{b^2n^2}{a^2(m+n)^2}=\frac{cn}{am}$

$ab^2mn^2=a^2cn(m+n)^2$

$mnb^2=\frac{a^2cn(m+n)^2}{an}$

$=(m+n)^2ac$

17. Let α and β be the roots of $ax^2+bx+c=0$.

Sum of the roots $=\alpha+\beta=-\frac{b}{a}$ ....(1)

Product of the roots $=\alpha\beta=\frac{c}{a}$ ....(2)

$\alpha^2+\beta^2=4$ ....(3)

By (3), $\alpha^2+2\alpha\beta+\beta^2-2\alpha\beta=4$

$(\alpha+\beta)^2-2\alpha\beta=4$

$\frac{b^2}{a^2}-2(\frac{c}{a})=4$

$b^2-2ac=4a^2$

$b^2=2ac+4a^2$

18. (a) Given α and β are roots of $ax^2+bx+c=0$.

Sum of the roots $=\alpha+\beta=-\frac{b}{a}$

Product of the roots $=\alpha\beta=\frac{c}{a}$

$\therefore\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\frac{b^2}{a^2}-\frac{2c}{a}$

$=\frac{1}{a^2}(b^2-2ac)$

$\alpha^2\beta^2=\frac{c^2}{a^2}$

A quadratic equation with roots α^2 and β^2 is

$x^2-\frac{1}{a^2}(b^2-2ac)x+\frac{c^2}{a^2}=0$

$a^2x^2-(b^2-2ac)x+c^2=0$

(b) α and β are roots of $2x^2-4x-1=0$

By **(a)**, take $a=2$, $b=-4$, $c=-1$

A quadratic equation with roots α^2 and β^2 is

$4x^2-[16-2(2)(-1)]x+(-1)^2=0$

$4x^2-20x+1=0$

By **(a)**, take $a=4$, $b=-20$, $c=1$.

A quadratic equation with roots α^4 and β^4 is

$16x^2-[20^2-2(4)(1)]x+1=0$

$\underline{\underline{16x^2-392x+1=0}}$

19. (a) α and β are the roots of $2x^2+ax+b=0$.

Sum of the roots $=\alpha+\beta=\underline{\underline{-\frac{a}{2}}}$

Product of the roots $=\alpha\beta=\underline{\underline{\frac{b}{2}}}$

(b) $(\alpha-1)$ and $(\beta-1)$ are the roots of $2x^2+mx+n=0$.

Sum of the roots

$=(\alpha-1)+(\beta-1)=-\frac{m}{2}$ ....(1)

Product of the roots

$=(\alpha-1)(\beta-1)=\frac{n}{2}$ ....(2)

By (1), $\alpha+\beta-2=-\frac{m}{2}$

By **(a)**, $-\frac{a}{2}-2=-\frac{m}{2}$

$\frac{a}{2}+2=\frac{m}{2}$

$\underline{\underline{m=a+4}}$

By (2), $(\alpha-1)(\beta-1)=\frac{n}{2}$

$\alpha\beta-(\alpha+\beta)+1=\frac{n}{2}$

By **(a)**, $\frac{b}{2}-(-\frac{a}{2})+1=\frac{n}{2}$

$b+a+2=n$

$\underline{\underline{n=a+b+2}}$

(c) **(i)** α and β are the roots of $4x^2+3x-5=0$.

Sum of the roots $=\alpha+\beta=-\frac{3}{4}$

Product of the roots $=\alpha\beta=-\frac{5}{4}$

$(\alpha-1)$ and $(\beta-1)$ are the roots of $4x^2+cx+d=0$.

$(\alpha-1)+(\beta-1)=-\frac{c}{4}$

$(\alpha+\beta)-2=-\frac{c}{4}$

$-\frac{3}{4}-2=-\frac{c}{4}$

$\therefore\quad c=\underline{\underline{11}}$

$(\alpha-1)(\beta-1)=\frac{d}{4}$

$\alpha\beta-(\alpha+\beta)+1=\frac{d}{4}$

$-\frac{5}{4}-(-\frac{3}{4})+1=\frac{d}{4}$

$d=\underline{\underline{2}}$

(ii) α and β are the roots of $2x^2-3x-1=0$.

Sum of the roots $=\alpha+\beta=\frac{3}{2}$

Product of the roots $=\alpha\beta=-\frac{1}{2}$

$(\alpha-2)$ and $(\beta-2)$ are the roots of $2x^2+ex+f=0$.

Sum of the roots $=(\alpha-2)+(\beta-2)$

$=-\frac{e}{2}$ ....(1)

Product of the roots $=(\alpha-2)(\beta-2)$

$=\frac{f}{2}$ ....(2)

By (1), $\alpha+\beta-4=-\frac{e}{2}$

$\frac{3}{2}-4=-\frac{e}{2}$

$e=\underline{\underline{5}}$

By (2), $(\alpha-2)(\beta-2)=\frac{f}{2}$

$\alpha\beta-2(\alpha+\beta)+4=\frac{f}{2}$

$-\frac{1}{2}-2(\frac{3}{2})+4=\frac{f}{2}$

$f=\underline{\underline{1}}$

Exercise 1D (p.28)

1. **(a)** $4x^2-8x+7$

$=4(x^2-2x+\frac{7}{4})$

$=4[x^2-2x+(\frac{2}{2})^2-(\frac{2}{2})^2+\frac{7}{4}]$

$=4(x-1)^2+3$

$\therefore\quad a=\underline{\underline{-1}},\ b=\underline{\underline{3}}$

(b) For all real values of x,

$(x-1)^2\ge 0$

$4(x-1)^2\ge 0$

$4(x-1)^2+3\ge 0+3$

$4x^2-8x+7\ge 3$

$\therefore$ The minimum value of $4x^2-8x+7$ is $\underline{\underline{3}}$.

2. **(a)** $3x^2-12x+14$

$=3(x^2-4x+\frac{14}{3})$

$=3[x^2-4x+(\frac{4}{2})^2-(\frac{4}{2})^2+\frac{14}{3}]$

$=3(x-2)^2+2$

$\therefore\quad p=\underline{\underline{-2}},\ q=\underline{\underline{2}}$

(b) For all real values of x,

$(x-2)^2\ge 0$

$3(x-2)^2\ge 0$

$3(x-2)^2+2\ge 0+2$

$3x^2-12x+14\ge 2$

$\frac{1}{3x^2-12x+14}\le\frac{1}{2}$

$\frac{8}{3x^2-12x+14}\le\frac{8}{2}$

$\frac{8}{3x^2-12x+14}\le 4$

$\therefore$ The maximum value of $\frac{8}{3x^2-12x+14}$ is $\underline{\underline{4}}$.

3. **(a)** $3x^2-6x+10$

$=3(x^2-2x+\frac{10}{3})$

$=3[x^2-2x+(\frac{2}{2})^2-(\frac{2}{2})^2+\frac{10}{3}]$

$=\underline{\underline{3(x-1)^2+7}}$

(b) For all real values of x,

$(x-1)^2\ge 0$

$3(x-1)^2\ge 0$

$3(x-1)^2+7\ge 7$

$\frac{1}{3(x-1)^2+7}\le\frac{1}{7}$

$\frac{5}{3x^2-6x+10}\le\frac{5}{7}$

$-\frac{5}{3x^2-6x+10}\ge-\frac{5}{7}$

$1-\frac{5}{3x^2-6x+10}\ge 1-\frac{5}{7}$

$=\frac{2}{7}$

$\therefore$ The given expression is always greater than or equal to $\underline{\underline{\frac{2}{7}}}$.

4. $-3x^2+2x-7=-3(x^2-\frac{2}{3}x+\frac{7}{3})$

$=-3[x^2-\frac{2}{3}x+(\frac{2}{6})^2-(\frac{2}{6})^2+\frac{7}{3}]$

$=-3(x-\frac{1}{3})^2-\frac{20}{3}$

For all real values of x,

$(x-\frac{1}{3})^2\ge 0$

$-3(x-\frac{1}{3})^2\le 0$

$-3(x-\frac{1}{3})^2-\frac{20}{3}\le\frac{-20}{3}$

$-3x^2+2x-7\le-\frac{20}{3}$

$\therefore\quad -3x^2+2x-7$ is always negative for all real values of x.

5. The graph of $y=ax^2-8x-4$ lies below the x-axis for all values of x, i.e. $a<0$ and $D<0$.

$D=(-8)^2-4a(-4)<0$

$64+16a<0$

$\underline{\underline{a<-4}}$

6. Consider $2x^2+2ax-(b+1)=0$.............. $(*_1)$

Let α and β be the roots of $(*_1)$.

Sum of the roots $=\alpha+\beta=-\frac{2a}{2}=-a$

Product of the roots $=\alpha\beta=-\frac{(b+1)}{2}$

Consider $x^2+\frac{b-a}{2}x-(b-1)=0$ .... $(*_2)$

Since $(*_1)$ and $(*_2)$ cut the same points on the points on the x-axis, α and β are also the roots of $(*_2)$.

Sum of the roots $=\alpha+\beta=-(\frac{b-a}{2})$

Product of the roots $=\alpha\beta=-(b-1)$

$-\frac{(b+1)}{2}=-(b-1)$

$b+1=2b-2$

$b=\underline{\underline{3}}$

$-a=-(\frac{b-a}{2})$

$2a=b-a$

$3a=b=3$

$a=\underline{\underline{1}}$

7. **(a)** $f(x)=-x^2+qx+r=-(x^2-qx-r)$

$=-[x^2-qx+(\frac{q}{2})^2-(\frac{q}{2})^2-r]$

$=\underline{\underline{-(x-\frac{q}{2})^2+(\frac{q^2}{4}+r)}}$

(b) The maximum value of $f(x)$ occurs when $x=3$.

$\therefore\quad 3-\frac{q}{2}=0$

$q=\underline{\underline{6}}$

(c) $f(x)\le 0$

$-(x-\frac{q}{2})^2+(\frac{q^2}{4}+r)\le 0$

Form **(b)**, $\frac{q^2}{4}+r=0$

$\frac{6^2}{4}+r=0$

$r=\underline{\underline{-9}}$

8. **(a)** y-intercept $=k$

x-intercept $=1,\ 3$

$\underline{\underline{f(x)=\frac{k}{3}(x-1)(x-3)}}$

(b) $f(x)=\frac{k}{3}(x-1)(x-3)$

$=\frac{k}{3}(x^2-4x+3)$

$=\frac{k}{3}[x^2-4x+(\frac{4}{2})^2-(\frac{4}{2})^2+3]$

$=\frac{k}{3}(x-2)^2-\frac{k}{3}$

For all real values of x,

$$(x-2)^2 \ge 0$$
$$\frac{k}{3}(x-2)^2 \ge 0$$
$$\frac{k}{3}(x-2)^2 - \frac{k}{3} \ge 0 - \frac{k}{3}$$
$$\therefore \quad f(x) \ge -\frac{k}{3}$$

$\therefore$ The least value of $f(x)$ is $\underline{\underline{-\frac{k}{3}}}$.

(c) If $f(x) \ge -2$,

from **(b)**, $-\frac{k}{3} = -2$

$$k = 6$$
$$f(x) = \frac{k}{3}(x-1)(x-3)$$
$$= \frac{6}{3}(x-1)(x-3)$$
$$= 2(x-1)(x-3)$$
$$= \underline{\underline{2x^2 - 8x + 6}}$$

9. (a) $\begin{cases} y = x + m \text{.......................(1)} \\ y = x^2 + 4x - 3m \text{..........(2)} \end{cases}$

$P(x_1, y_1)$ and $Q(x_2, y_2)$ are points of intersection,

By (1) and (2), $x + m = x^2 + 4x - 3m$

$$\underline{\underline{x^2 + 3x - 4m = 0}}$$

(b) If P and Q coincide, $x^2 + 3x - 4m = 0$ has two equal real roots, i.e. $D = 0$.

$$3^2 - 4(-4m) = 0$$
$$m = \underline{\underline{-\frac{9}{16}}}$$

10. (a) Consider $y = a(x+2)^2 + 1$.

$$y = a(x^2 + 4x + 4) + 1$$
$$y = ax^2 + 4ax + 4a + 1$$
$$D = (4a)^2 - 4(a)(4a+1)$$
$$= 16a^2 - 16a^2 - 4a$$
$$= -4a$$

If $a < 0$, $D > 0$

$\therefore$ The graph of $y = a(x+2)^2 + 1$ cuts the x-axis at two distinct points if $a < 0$.

(b) From **(a)**,

$y = ax^2 + 4ax + 4a + 1$......(*)

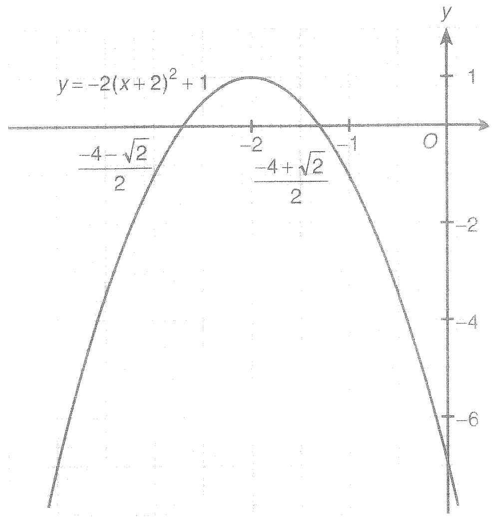
α and β are roots of $ax^2 + 4ax + 4a + 1 = 0$.

Sum of the roots $= \alpha + \beta = -\frac{4a}{a} = -4$

Product of the roots $= \alpha\beta = \frac{4a+1}{a}$

If
$$\alpha^2 + \beta^2 = 9$$
$$(\alpha+\beta)^2 - 2\alpha\beta = 9$$
$$(-4)^2 - 2\left(\frac{4a+1}{a}\right) = 9$$
$$16 - \frac{8a+2}{a} = 9$$
$$-a - 2 = 0$$
$$a = \underline{\underline{-2}}$$

(c)



Exercise 1E (p.34)

1. $|-2+7| - |9-15| - |-3|$
$$= |5| - |-6| - |-3|$$
$$= 5 - 6 - 3$$
$$= \underline{\underline{-4}}$$

2. $|4-9| + |7-16| - |12-5|$
$$= |-5| + |-9| - |7|$$
$$= 5 + 9 - 7$$
$$= \underline{\underline{7}}$$

3. $f(x) = |2x-5| + |x|$

$$f(0) = |2(0)-5| + |0| = |-5| = \underline{\underline{5}}$$
$$f(1) = |2(1)-5| + |1| = |-3| + |1| = 3 + 1 = \underline{\underline{4}}$$
$$f(-3) = |2(-3)-5| + |-3|$$
$$= |-11| + |-3|$$
$$= 11 + 3$$
$$= \underline{\underline{14}}$$

4. $g(x) = \frac{x}{|x|}$

$$g(-6) = \frac{-6}{|-6|} = \frac{-6}{6} = \underline{\underline{-1}}$$
$$g(-3) = \frac{-3}{|-3|} = \frac{-3}{3} = \underline{\underline{-1}}$$
$$g(7) = \frac{7}{|7|} = \frac{7}{7} = \underline{\underline{1}}$$

5. $|3x+7| = 5$

$3x + 7 = 5$ or $3x + 7 = -5$

$x = \underline{\underline{-\frac{2}{3}}}$ or $x = \underline{\underline{-4}}$

6. $|-3x| = -6$

Since any absolute value should be non-negative

$\therefore$ The equation has $\underline{\underline{\text{no solution}}}$

7. $|x+1| = |3x-2|$

$x + 1 = 3x - 2$ or $x + 1 = -(3x-2)$

$3 = 2x$ or $x + 1 = -3x + 2$

$x = \frac{3}{2}$ or $4x = 1$

$x = \underline{\underline{\frac{3}{2}}}$ or $x = \underline{\underline{\frac{1}{4}}}$

8. $|x+1| = 3x - 2$

$x + 1 = 3x - 2$ or $x + 1 = -3x + 2$

$x = \frac{3}{2}$ or $x = \frac{1}{4}$

But $3x - 2$ should be non-negative.

$\because$ $3(\frac{1}{4}) - 2 < 0$, $\therefore$ $x = \frac{1}{4}$ is not a solution.

$\because$ $3(\frac{3}{2}) - 3 > 0$, $\therefore$ $x = \underline{\underline{\frac{3}{2}}}$

9. $\left|\frac{2x-1}{5}\right| = 9$

$|2x-1| = 45$

$2x - 1 = 45$ or $2x - 1 = -45$

$2x = 46$ or $2x = -44$

$x = \underline{\underline{23}}$ or $x = \underline{\underline{-22}}$

10. $\left|\frac{6-x}{3+x}\right| = \frac{1}{2}$

$\frac{6-x}{3+x} = \frac{1}{2}$ or $\frac{6-x}{3+x} = -\frac{1}{2}$

$12 - 2x = 3 + x$ or $12 - 2x = -3 - x$

$9 = 3x$ or $15 = x$

$x = \underline{\underline{3}}$ or $x = \underline{\underline{15}}$

11. $3|x+4| = 4x - 6$

$3(x+4) = 4x - 6$ or $3(x+4) = -4x + 6$

$3x + 12 = 4x - 6$ or $3x + 12 = -4x + 6$

$x = 18$ or $x = -\frac{6}{7}$

For the same reason as question 8, $x = \underline{\underline{18}}$.

12. $|x^2 - 5x + 5| = 1$

$x^2 - 5x + 5 = 1$ or $x^2 - 5x + 5 = -1$

$x^2 - 5x + 4 = 0$ or $x^2 - 5x + 6 = 0$

$(x-1)(x-4) = 0$ or $(x-2)(x-3) = 0$

$x = 1, 4$ or $x = 2, 3$

$\therefore$ $x = \underline{\underline{1, 2, 3, 4}}$

13. $|x^2 - x - 4| = 2$

$x^2 - x - 4 = 2$ or $x^2 - x - 4 = -2$

$x^2 - x - 6 = 0$ or $x^2 - x - 2 = 0$

$(x-3)(x+2) = 0$ or $(x-2)(x+1) = 0$

$x = 3, -2$ or $x = 2, -1$

$\therefore$ $x = \underline{\underline{-1, -2, 2, 3}}$

14. $(2x-1)^2 + |2x-1| - 12 = 0$

$|2x-1|^2 + |2x-1| - 12 = 0$

$(|2x-1| - 3)(|2x-1| + 4) = 0$

$|2x-1| = 3$ or $|2x-1| = -4$ (rejected)

$2x - 1 = 3$ or $2x - 1 = -3$

$x = \underline{\underline{2}}$ or $x = \underline{\underline{-1}}$

15. $2(x+3)^2 - 3|x+3| - 14 = 0$

$2|x+3|^2 - 3|x+3| - 14 = 0$

$(2|x+3| - 7)(|x+3| + 2) = 0$

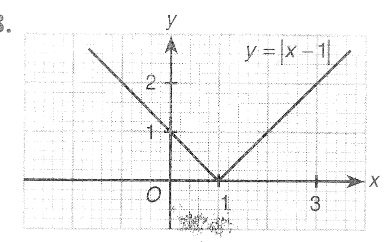
$2|x+3| = 7$ or $|x+3| = -2$ (rejected)

$2(x+3) = 7$ or $2(x+3) = -7$

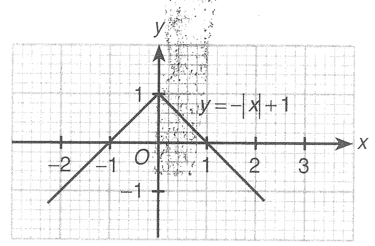
$x = \underline{\underline{\frac{1}{2}}}$ or $x = \underline{\underline{-\frac{13}{2}}}$

16 – 17. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

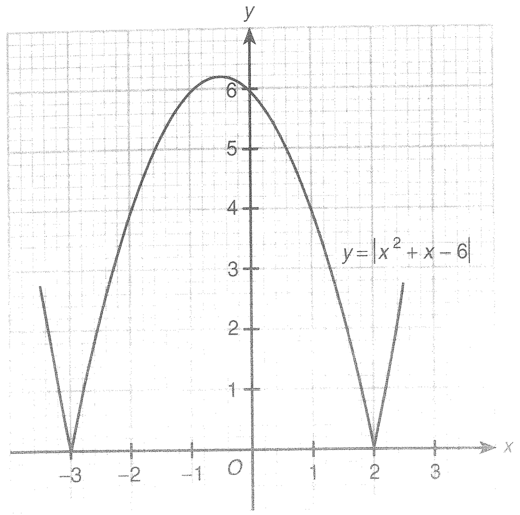
18.



19.



20.



Revision Exercise 1 (p.36)

1. $2(3x^2+5)=19x$
$6x^2-19x+10=0$
$(3x-2)(2x-5)=0$
$x=\frac{2}{3}$ or $\frac{5}{2}$

2. $4(x-1)^2+5(x-1)-6=0$
$[4(x-1)-3][(x-1)+2]=0$
$(4x-7)(x+1)=0$
$x=\frac{7}{4}$ or -1

3. The equation $x^2-2kx+36=0$ has equal roots.
$D=(-2k)^2-4(1)(36)=0$
$4k^2-144=0$
$k^2=36$
$k=\pm 6$

4. The equation $(6k+1)x^2-2(2k-3)x+1=0$ has equal roots.
$D=[-2(2k-3)]^2-4(6k+1)=0$
$4(4k^2-12k+9)-4(6k+1)=0$
$4k^2-12k+9-6k-1=0$
$4k^2-18k+8=0$
$2k^2-9k+4=0$
$(2k-1)(k-4)=0$
$k=\frac{1}{2}$ or 4

5. α and β are roots of $2x^2-5x-4=0$.
Sum of the roots $=\alpha+\beta=\frac{5}{2}$

Product of the roots $=\alpha\beta=-\frac{4}{2}=-2$

(a) $\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$
$=-2(\frac{5}{2})$
$=-5$

(b) $(\alpha-3\beta)(3\alpha-\beta)=3\alpha^2-\alpha\beta-9\alpha\beta+3\beta^2$
$=3(\alpha^2+\beta^2)-10\alpha\beta$
$=3[(\alpha+\beta)^2-2\alpha\beta]-10\alpha\beta$
$=3[(\frac{5}{2})^2-2(-2)]-10(-2)$
$=50\frac{3}{4}$

6. α and β are roots of $x^2-2x-1=0$.
Sum of the roots $=\alpha+\beta=2$
Product of the roots $=\alpha\beta=-1$

(a) $\alpha+2\beta+\beta+2\alpha=3\alpha+3\beta$
$=3(\alpha+\beta)$
$=3(2)$
$=6$
$(\alpha+2\beta)(\beta+2\alpha)=2\alpha^2+5\alpha\beta+2\beta^2$
$=2(\alpha^2+\beta^2)+5\alpha\beta$
$=2[(\alpha+\beta)^2-2\alpha\beta]+5\alpha\beta$
$=2[(2)^2-2(-1)]+5(-1)$
$=7$
One required equation is $x^2-6x+7=0$.

(b) $\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$
$=-1(2)$
$=-2$
$(\alpha^2\beta)(\alpha\beta^2)=\alpha^3\beta^3$
$=(\alpha\beta)^3$
$=(-1)^3$
$=-1$
One required equation is $x^2+2x-1=0$.

7. $y=3x^2+12x+7+k(x^2-1)$
$=3x^2+12x+7+kx^2-k$
$=(3+k)x^2+12x+7-k$
$D=12^2-4(7-k)(3+k)$
$=144-4(21+4k-k^2)$
$=144-84-16k+4k^2$
$=4k^2-16k+60$
$=4(k^2-4k+15)$
$=4[k^2-4k+(\frac{4}{2})^2-(\frac{4}{2})^2+15]$
$=4(k-2)^2+44$

$\because$ $(k-2)^2\ge 0$, $\therefore$ $D>0$
$\therefore$ The graph of $y=3x^2+12x+7+k(x^2-1)$ cuts the x-axis for any value of k.

8. α and β are roots of $2x^2-3x-5=0$.
Sum of the roots $=\alpha+\beta=\frac{3}{2}$
Product of the roots $=\alpha\beta=-\frac{5}{2}$
$\alpha+2$ and $\beta+2$ are roots of $2x^2+px+q=0$.
Sum of the roots $=(\alpha+2)+(\beta+2)=-\frac{p}{2}$ ......(1)
Product of the roots $=(\alpha+2)(\beta+2)=\frac{q}{2}$ ....(2)
By (1), $\alpha+\beta+4=-\frac{p}{2}$
$\frac{3}{2}+4=-\frac{p}{2}$
$p=-11$
By (2), $(\alpha+2)(\beta+2)=\frac{q}{2}$
$\alpha\beta+2(\alpha+\beta)+4=\frac{q}{2}$
$-\frac{5}{2}+2(\frac{3}{2})+4=\frac{q}{2}$
$q=9$

9. (a) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=335$
$5(\alpha^2-\alpha\beta+\beta^2)=335$
$\alpha^2+\beta^2-\alpha\beta=67$......(*)
$(\alpha+\beta)^2-2\alpha\beta-\alpha\beta=67$
$5^2-3\alpha\beta=67$
$\alpha\beta=-14$
By (*), $\alpha^2+\beta^2=67+\alpha\beta$
$=67-14$
$=53$

(b) α^2 and β^2 are roots of equation
Sum of the roots $=\alpha^2+\beta^2=53$
Product of the roots $=\alpha^2\beta^2=(\alpha\beta)^2$
$=(-14)^2$
$=196$
One required equation is $x^2-53x+196=0$.

10. (a) Let α, $-\alpha$ be the roots of
$x^2+(k^2+14k+45)x+2k-5=0$.
Sum of the roots $=\alpha+(-\alpha)$
$=0$
$=-(k^2+14k+45)$
$k^2+14k+45=0$
$(k+5)(k+9)=0$
$k=-5$ or -9

(b) When $k=-5$,
the equation becomes
$x^2+[(-5)^2+14(-5)+45]x+2(-5)-5=0$
$x^2-15=0$
$x=\pm\sqrt{15}$

When $k=-9$,
the equation becomes
$x^2+[(-9)^2+14(-9)+45]x+2(-9)-5=0$
$x^2-23=0$
$x=\pm\sqrt{23}$

11. (a) $x^2-(k+3)x+(k^2-3k+6)=0$
$D=[-(k+3)]^2-4(k^2-3k+6)=0$
$k^2+6k+9-4k^2+12k-24=0$
$-3k^2+18k-15=0$
$k^2-6k+5=0$
$(k-5)(k-1)=0$
$k=1$ or 5

(b) $2kx^2+4(4k-5)x+(2k+5)=0$
$D=[4(4k-5)]^2-4(2k)(2k+5)=0$
$16(16k^2-40k+25)-(16k^2+40k)=0$
$256k^2-640k+400-16k^2-40k=0$
$240k^2-680k+400=0$
$6k^2-17k+10=0$
$(6k-5)(k-2)=0$
$k=\frac{5}{6}$ or 2

12 – 16. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

17. α and β are roots of $ax^2+bx+c=0$.
Sum of the roots $=\alpha+\beta=-\frac{b}{a}$
Product of the roots $=\alpha\beta=\frac{c}{a}$

(a) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$
$=(\alpha+\beta)[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta]$
$=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$
$=-\frac{b}{a}[(\frac{-b}{a})^2-3(\frac{c}{a})]$
$=-\frac{b}{a}(\frac{b^2-3ac}{a^2})$
$=\frac{-b(b^2-3ac)}{a^3}$

$\alpha^3\beta^3=(\alpha\beta)^3=\frac{c^3}{a^3}$

A quadratic equation with roots α^3 and β^3 is

$x^2+\frac{b(b^2-3ac)}{a^3}x+\frac{c^3}{a^3}=0$,

i.e. $a^3x^2+b(b^2-3ac)x+c^3=0$

(b) By **(a)**, take $a=1$, $b=-3$, $c=-2$.
One required equation is

$(1)x^2+(-3)[(-3)^2-3(1)(-2)]x+(-2)^3=0$,

i.e. $x^2-45x-8=0$

18. α and β are roots of $x^2+ax+b=0$.
Sum of the roots $=\alpha+\beta=-a$
Product of the roots $=\alpha\beta=b$

(a) (i) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$
$=(\alpha+\beta)[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta]$
$=-a[(-a)^2-3b]$
$=-a(a^2-3b)$

(ii) $(\alpha-\beta^2+1)(\beta-\alpha^2+1)$
$=\alpha\beta-\alpha^3+\alpha-\beta^3+\alpha^2\beta^2-\beta^2+\beta-\alpha^2+1$
$=\alpha\beta-(\alpha^3+\beta^3)+(\alpha+\beta)-(\alpha^2+\beta^2)+(\alpha\beta)^2+1$
$=\alpha\beta-(\alpha^3+\beta^3)+(\alpha+\beta)-[(\alpha+\beta)^2-2\alpha\beta]+(\alpha\beta)^2+1$
$=b+a(a^2-3b)-a-(a^2-2b)+b^2+1$
$=b+a^3-3ab-a-a^2+2b+b^2+1$
$=b^2+3b-3ab+a^3-a^2-a+1$
$=b^2-3(a-1)b+a^3+1-(a^2+a)$
$=b^2-3(a-1)b+(a+1)(a^2-a+1)-a(a+1)$
$=b^2-3(a-1)b+(a+1)(a^2-2a+1)$
$=b^2-3(a-1)b+(a-1)^2(a+1)$

(b) If one root of the equation plus 1 is equal to the square of the other, i.e. $\alpha+1=\beta^2$ or $\beta+1=\alpha^2$, then
$(\alpha+1-\beta^2)(\beta+1-\alpha^2)=0$.
From **(a)(ii)**,
$b^2-3(a-1)b+(a-1)^2(a+1)=0$

19. $f(x)=ax^2+bx+c$

(a) α and β are roots of $f(x)-x=0$.
$\therefore$ $f(\alpha)-\alpha=0$ $f(\alpha)=\alpha$
$f(\beta)-\beta=0$ $f(\beta)=\beta$
For the equation, $f[f(x)]-x=0$
If $x=\alpha$, $f[f(\alpha)]-\alpha=f(\alpha)-\alpha$
$=\alpha-\alpha$
$=0$

If $x=\beta$, $f[f(\beta)]-\beta=f(\beta)-\beta$
$=\beta-\beta$
$=0$

$\therefore$ α, β are also the roots of $f[f(x)]-x=0$

(b) For $f(x)=x^2-3x+2$,
We set $f(x)-x=0$,
i.e. $x^2-4x+2=0$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)}}{2}$
$=\frac{4\pm\sqrt{8}}{2}$
$=2\pm\sqrt{2}$

By **(a)**, $f[f(x)]-x=0$ has roots $2\pm\sqrt{2}$.
$f[f(x)]-x=0$
$(x^2-3x+2)^2-3(x^2-3x+2)+2-x=0$
$x^4-6x^3+13x^2-12x+4-3x^2+9x-6+2-x=0$
$x^4-6x^3+10x^2-4x=0$
$x(x^3-6x^2+10x-4)=0$
$x(x-2)(x^2-4x+2)=0$

$\therefore$ $x=0, 2$ or $2\pm\sqrt{2}$

20. (a) Let α be the common root of the two equatio[n]

$\begin{cases}3\alpha^2+a\alpha+b=0 & \text{.........(1)}\\ 3\alpha^2+b\alpha+a=0 & \text{.........(2)}\end{cases}$

(1) − (2), $(a-b)\alpha=a-b$
$\alpha=1$ $(\because\ a\neq b)$

(b) As $\alpha=1$, $3+a+b=0$
$a+b=-3$
a and b are roots of $x^2+hx+k=0$
Sum of the roots $=a+b=-h$
$-3=-h$
$h=3$

Product of the roots $=ab=k$
Since h, k are positive integers and $a+b=-$[3]
$k=ab=(-1)(-2)=2$ is the only solution.
$\therefore$ $k=2$

21. No solution is provided for the H.K.C.E.[E.] question because of the copyright reasons.

Enrichment 1 (p.38)

1. (a) $x^2-6acx+a^2(9c^2-4b^2)=0$
$x^2-6acx+[a(3c+2b)\cdot a(3c-2b)]=0$
$[x-a(3c+2b)][x-a(3c-2b)]=0$
$x=a(3c\pm 2b)$

(b) $(1-l^2)x^2-2mx+m^2=0$
$(1+l)(1-l)x^2-2mx+m^2=0$
$[(1+l)x-m][(1-l)x-m]=0$
$x=\frac{m}{1+l}$ or $\frac{m}{1-l}$

2. $x^{\frac{4}{3}}-5x^{\frac{2}{3}}+4=0$
$(x^{\frac{2}{3}}-4)(x^{\frac{2}{3}}-1)=0$
$x^{\frac{2}{3}}=4$ or $x^{\frac{2}{3}}=1$
$x=\pm 8$ or $x=\pm 1$

3. $2^{2x+2}=9\cdot 2^x-2$
$2^{2x}\cdot 2^2=9\cdot 2^x-2$
$4\cdot 2^{2x}-9\cdot 2^x+2=0$
$(2^x-2)(4\cdot 2^x-1)=0$
$2^x=2$ or $2^x=\frac{1}{4}$
$x=1$ or $x=-2$

4. $\sqrt{x+9}+11=x$
$\sqrt{x+9}=x-11$
$x+9=x^2-22x+121$
$x^2-23x+112=0$
$(x-16)(x-7)=0$
$x=16$ or 7
Check: When $x=16$,
L.H.S. $=\sqrt{16+9}+11$
$=16$
$=$R.H.S.
When $x=7$,
L.H.S. $=\sqrt{7+9}+11$
$=15$
$\neq$R.H.S.
$\therefore$ $x=16$

5. $(1-\sqrt{x})^2+(1-\sqrt{x})-6=0$
$[(1-\sqrt{x})-2][(1-\sqrt{x})+3]=0$
$1-\sqrt{x}=2$ or $1-\sqrt{x}=-3$
$\sqrt{x}=-1$ (rejected) or $\sqrt{x}=4$
$\therefore$ $x=16$

6. $f(x)=x^2-6x-\frac{3(1+k)}{k}$, $g(x)=3x$
$kf(x)=g(x)$
$kx^2-6kx-3(1+k)=3x$
$kx^2-6kx-3-3k-3x=0$
$kx^2-(6k+3)x-(3+3k)=0$

Since $kf(x)=g(x)$ has equal roots,
$D=[-(6k+3)]^2-4k[-(3+3k)]=0$
$36k^2+36k+9+12k+12k^2=0$
$48k^2+48k+9=0$
$16k^2+16k+3=0$
$(4k+3)(4k+1)=0$
$k=-\frac{3}{4}$ or $-\frac{1}{4}$

7. $ax^2+bx+c=0$ and $px^2+qx+r=0$ have one root in common.
Let the common root be α.

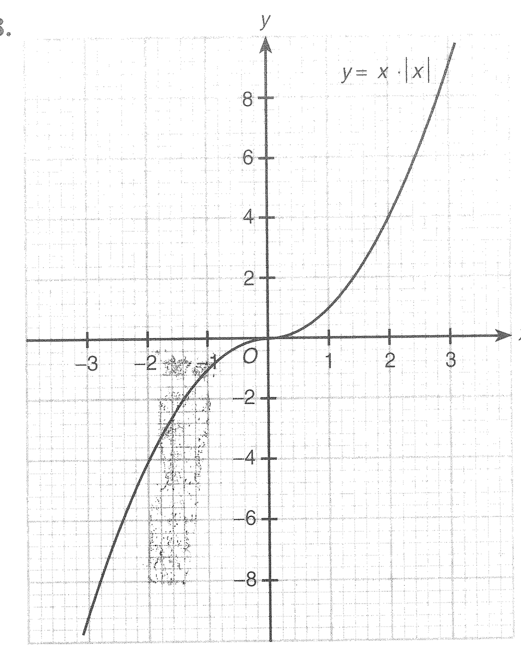
$\begin{cases}a\alpha^2+b\alpha+c=0 & \text{......(1)}\\ p\alpha^2+q\alpha+r=0 & \text{......(2)}\end{cases}$

(1) × p − (2) × a,
$bp\alpha+cp-aq\alpha-ar=0$
$\alpha(bp-aq)=ar-cp$
$\alpha=\frac{ar-cp}{bp-aq}$

(1) × q − (2) × b,
$aq\alpha^2+cq-bp\alpha^2-br=0$
$\alpha^2(aq-bp)=br-cq$
$\alpha^2=\frac{br-cq}{aq-bp}$

$\therefore$ $(\frac{ar-cp}{bp-aq})^2=\frac{br-cq}{aq-bp}$
$\frac{(ar-cp)^2}{aq-bp}=br-cq$
$(br-cq)(aq-bp)=(ar-cp)^2$

8.



Classwork 1 (p.3)

1. $3x^2-14x+8=0$
$(x-4)(3x-2)=0$
$x=\underline{\underline{4}}$ or $\underline{\underline{\frac{2}{3}}}$

2. $6x^2-13x-5=0$
$(2x-5)(3x+1)=0$
$x=\underline{\underline{\frac{5}{2}}}$ or $\underline{\underline{-\frac{1}{3}}}$

3. $(y-3)^2-10(y-3)-56=0$
$[(y-3)+4][(y-3)-14]=0$
$y-3=-4$ or $y-3=14$
$y=\underline{\underline{-1}}$ or $y=\underline{\underline{17}}$

4. $y^{\frac{1}{2}}-3y^{\frac{1}{4}}+2=0$
$(y^{\frac{1}{4}}-2)(y^{\frac{1}{4}}-1)=0$
$y^{\frac{1}{4}}=2$ or $y^{\frac{1}{4}}=1$
$y=\underline{\underline{16}}$ or $y=\underline{\underline{1}}$

Classwork 2 (p.6)

1. $x^2+4x+2=0$
$x^2+4x+(\frac{4}{2})^2-(\frac{4}{2})^2+2=0$
$(x+2)^2-2=0$
$(x+2)^2-(\sqrt{2})^2=0$
$(x+2-\sqrt{2})(x+2+\sqrt{2})=0$
$x=\underline{\underline{-2\pm\sqrt{2}}}$

2. $x^2-3x+1=0$
$x^2-3x+(\frac{3}{2})^2-(\frac{3}{2})^2+1=0$
$(x-\frac{3}{2})^2-\frac{5}{4}=0$
$(x-\frac{3}{2})^2-(\frac{\sqrt{5}}{2})^2=0$
$(x-\frac{3}{2}+\frac{\sqrt{5}}{2})(x-\frac{3}{2}-\frac{\sqrt{5}}{2})=0$
$x=\underline{\underline{\frac{3\pm\sqrt{5}}{2}}}$

3. $3x^2-8x+1=0$
$x^2-\frac{8}{3}x+\frac{1}{3}=0$
$x^2-\frac{8}{3}x+(\frac{8}{6})^2-(\frac{8}{6})^2+\frac{1}{3}=0$
$(x-\frac{4}{3})^2-\frac{13}{9}=0$
$(x-\frac{4}{3})^2-(\frac{\sqrt{13}}{3})^2=0$
$(x-\frac{4}{3}+\frac{\sqrt{13}}{3})(x-\frac{4}{3}-\frac{\sqrt{13}}{3})=0$
$x=\underline{\underline{\frac{4\pm\sqrt{13}}{3}}}$

4. $x^2+2x-m=0$
$x^2+2x+(\frac{2}{2})^2-(\frac{2}{2})^2-m=0$
$(x+1)^2-(1+m)=0$
$(x+1)^2-(\sqrt{1+m})^2=0$
$(x+1-\sqrt{1+m})(x+1+\sqrt{1+m})=0$
$x=\underline{\underline{-1\pm\sqrt{1+m}}}$

5. $x^2-4n+1=x^2-4n+(\frac{4n}{2})^2-(\frac{4n}{2})^2+1$
$=(x-2n)^2-4n^2+1$
$=\underline{\underline{(x-2n)^2+1-4n^2}}$
$\therefore\ a=\underline{\underline{-2n}},\ b=\underline{\underline{1-4n^2}}$

Classwork 3 (p.8)

1. Let $a=2$, $b=-7$, $c=4$.
Using the formula,
$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(4)}}{2(2)}=\underline{\underline{\frac{7\pm\sqrt{17}}{4}}}$

2. $9+3x-4x^2=0$
$-4x^2+3x+9=0$
Let $a=-4$, $b=3$, $c=9$.
Using the formula,
$x=\frac{-3\pm\sqrt{3^2-4(-4)(9)}}{2(-4)}=\frac{-3\pm\sqrt{153}}{-8}=\underline{\underline{\frac{3\pm3\sqrt{17}}{8}}}$

3. $5x+12=3x^2$
$3x^2-5x-12=0$
Let $a=3$, $b=-5$, $c=-12$.
Using the formula,
$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-12)}}{2(3)}$
$x=\frac{5\pm\sqrt{169}}{6}$
$x=\underline{\underline{3}}$ or $x=\underline{\underline{-\frac{4}{3}}}$

4. Let $a=3$, $b=4$, $c=5$.
Using the formula,
$x=\frac{-4\pm\sqrt{4^2-4(3)(5)}}{2(3)}=\frac{-4\pm\sqrt{-44}}{6}$
The equation has $\underline{\underline{\text{no real solution}}}$.

5. Since 4 is a root of $3x^2+ax+8=0$.
$\therefore\ 3(4)^2+a(4)+8=0$
$4a=-56$
$a=\underline{\underline{-14}}$
Substitute $a=-14$ into the given equation.
$3x^2-14x+8=0$
$(x-4)(3x-2)=0$
$\therefore\ x=4$ or $\frac{2}{3}$
$\therefore$ The other root of the equation is $\underline{\underline{\frac{2}{3}}}$.

Classwork 4 (p.9)

$5x^2+9x-2=0$
$D=9^2-4(5)(-2)$
$=\underline{\underline{121}}$
$=11^2$
D is a perfect square.

$x^2+8x+16=0$
$D=8^2-4(16)$
$=\underline{\underline{0}}$
D is a perfect square.

$2x^2-7x+3=0$
$D=(-7)^2-4(2)(3)$
$=\underline{\underline{25}}$
$=5^2$
D is a perfect square.

$2x^2-5x+4=0$
$D=(-5)^2-4(2)(4)$
$=\underline{\underline{-7}}$
D is not a perfect square.

$9x^2-30x+25=0$
$D=(-30)^2-4(9)(25)$
$=\underline{\underline{0}}$
D is a perfect square.

Classwork 5 (p.11)

1. $2x^2-x-21=0$
$D=(-1)^2-4(2)(-21)$
$=169$
$=13^2$
$\because\ D>0$ and D is a perfect square.
$\therefore$ The equation has $\underline{\underline{\text{two rational roots}}}$.

2. $9-5x+x^2=0$
$x^2-5x+9=0$
$D=(-5)^2-4(1)(9)$
$=-11$
$\because\ D<0$
$\therefore$ The equation has $\underline{\underline{\text{unreal roots}}}$.

3. $x^2-2x=5$
$x^2-2x-5=0$
$D=(-2)^2-4(-5)$
$=24$
$\because\ D>0$ and D is not a perfect square.
$\therefore$ The equation has $\underline{\underline{\text{two real roots}}}$.

4. $4x^2-28x+49=0$
$D=(-28)^2-4(4)(49)$
$=0$
$\because\ D=0$
$\therefore$ The equation has $\underline{\underline{\text{two equal rational roots}}}$.

Classwork 6 (p.11)

1. $x^2+8x+5k=0$
$D=8^2-4(1)(5k)$
$=\underline{\underline{64-20k}}$

2. $2x^2-kx-(k-3)=0$
$D=(-k)^2-4(2)[-(k-3)]$
$=k^2+8(k-3)$
$=\underline{\underline{k^2+8k-24}}$

Classwork 7 (p.13)

1. $x^2-(2m-1)x-2m=0$
$D=[-(2m-1)]^2-4(-2m)$
$=4m^2-4m+1+8m$
$=4m^2+4m+1$
$=(2m+1)^2$,
which is the square of a rational number.
Therefore the roots of the given equation are rational.

2. $qx^2+(p+3q)x+2p=0$

$D=(p+3q)^2-4(q)(2p)$
$=p^2+6pq+9q^2-8pq$
$=p^2-2pq+9q^2$
$=p^2-2pq+q^2+8q^2$
$=(p-q)^2+8q^2$

$\because$ $(p-q)^2\ge 0$ and $q^2>0$
$\therefore$ $D>0$
$\therefore$ The equation has real roots.

3. (a) $x^2-kx+(k+3)=0$ ....(*)

$D=(-k)^2-4(k+3)$
$=\underline{\underline{k^2-4k-12}}$

(b) If (*) has equal roots, $D=0$.
From **(a)**, $(k+2)(k-6)=0$
$k=\underline{\underline{-2}}$ or $\underline{\underline{6}}$

(c) For $k=-2$, (*) becomes
$x^2+2x+1=0$
$(x+1)^2=0$
$x=\underline{\underline{-1}}$

Classwork 8 (p.14)

1. $x^2-3x+1=0$
Sum of the roots $=\underline{\underline{3}}$
Product of the roots $=\underline{\underline{1}}$

2. $x^2+5x-3=0$
Sum of the roots $=\underline{\underline{-5}}$
Product of the roots $=\underline{\underline{-3}}$

3. $2x^2-4x-7=0$
Sum of the roots $=\underline{\underline{2}}$
Product of the roots $=\underline{\underline{-\frac{7}{2}}}$

4. $5x^2+(3k-1)x+2k=0$
Sum of the roots $=-\frac{3k-1}{5}$
$=\underline{\underline{\frac{1-3k}{5}}}$
Product of the roots $=\underline{\underline{\frac{2k}{5}}}$

Classwork 9 (p.16)

1. $2x^2+3x-7=0$
Sum of the roots $=\alpha+\beta=-\frac{3}{2}$
Product of the roots $=\alpha\beta=-\frac{7}{2}$

(a) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$=(-\frac{3}{2})^2-2(-\frac{7}{2})$
$=\underline{\underline{\frac{37}{4}}}$

(b) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$
$=\frac{\frac{37}{4}}{-\frac{7}{2}}$
$=\underline{\underline{-\frac{37}{14}}}$

(c) $(\alpha+2\beta+1)(\beta+2\alpha+1)$
$=\alpha\beta+2\alpha^2+\alpha+2\beta^2+4\alpha\beta+2\beta+\beta+2\alpha+1$
$=5\alpha\beta+2(\alpha^2+\beta^2)+3(\alpha+\beta)+1$
$=5(-\frac{7}{2})+2(\frac{37}{4})+3(-\frac{3}{2})+1$
$=\underline{\underline{-\frac{5}{2}}}$

2. α and β are the roots of $x^2+px+q=0$.
Sum of the roots $=\alpha+\beta=-p$
Product of the roots $=\alpha\beta=q$
$(p\alpha-q)^2+(p\beta-q)^2$
$=p^2\alpha^2-2pq\alpha+q^2+p^2\beta^2-2pq\beta+q^2$
$=p^2(\alpha^2+\beta^2)-2pq(\alpha+\beta)+2q^2$
$=p^2[(\alpha+\beta)^2-2\alpha\beta]-2pq(\alpha+\beta)+2q^2$
$=p^2(p^2-2q)-2pq(-p)+2q^2$
$=p^4-2p^2q+2p^2q+2q^2$
$=p^4+2q^2$

Classwork 10 (p.18)

1. α and β are the roots of $x^2-x-7=0$.
Sum of the roots $=\alpha+\beta=1$
Product of the roots $=\alpha\beta=-7$
$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=\alpha+\beta+\frac{\beta+\alpha}{\alpha\beta}$
$=1+\frac{1}{-7}$
$=\frac{6}{7}$

$(\alpha+\frac{1}{\alpha})(\beta+\frac{1}{\beta})$
$=\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}$
$=\alpha\beta+\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{1}{\alpha\beta}$
$=\alpha\beta+\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}+\frac{1}{\alpha\beta}$
$=-7+\frac{1-2(-7)}{-7}+\frac{1}{-7}$
$=-\frac{65}{7}$

One required equation is
$x^2-\frac{6}{7}x-\frac{65}{7}=0$, i.e.
$\underline{\underline{7x^2-6x-65=0}}$

2. Let α and α^2 be the two roots.
Product of the roots $=\alpha(\alpha^2)=27$
$\alpha^3=27$
$\alpha=\underline{\underline{3}}$
Sum of the roots $=\alpha+\alpha^2=-q$
$3+3^2=-q$
$q=\underline{\underline{-12}}$

3. Let α and 2α be the two roots.
Product of the roots $=\alpha(2\alpha)=r$
Sum of the roots $=\alpha+2\alpha=-2q$

$\begin{cases}2\alpha^2=r & \text{.........(1)}\\ 3\alpha=-2q & \text{.........(2)}\end{cases}$

$(2):\ \alpha=\frac{-2q}{3}$
Put $\alpha=\frac{-2q}{3}$ into (1),
$2(-\frac{2q}{3})^2=r$
$\frac{8q^2}{9}=r$
$8q^2=9r$

Classwork 11 (p.24)

1. (a) $3x^2-2x-1=3(x^2-\frac{2}{3}x-\frac{1}{3})$
$=3[x^2-\frac{2}{3}x+(\frac{2}{6})^2-(\frac{2}{6})^2-\frac{1}{3}]$
$=\underline{\underline{3(x-\frac{1}{3})^2-\frac{4}{3}}}$

(b) For all real values of x,
$(x-\frac{1}{3})^2\ge 0$
$3(x-\frac{1}{3})^2\ge 0$
$3(x-\frac{1}{3})^2-\frac{4}{3}\ge 0-\frac{4}{3}$
$3(x-\frac{1}{3})^2-\frac{4}{3}\ge -\frac{4}{3}$

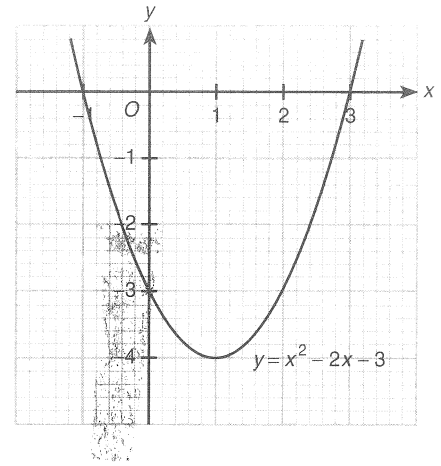
$\therefore$ The least value of $3x^2-2x-1$ is $\underline{\underline{-\frac{4}{3}}}$

2. (a) $-4x^2+2x-1$
$=-4(x^2-\frac{1}{2}x+\frac{1}{4})$
$=-4[x^2-\frac{1}{2}x+(\frac{1}{4})^2-(\frac{1}{4})^2+\frac{1}{4}]$
$=\underline{\underline{-4(x-\frac{1}{4})^2-\frac{3}{4}}}$

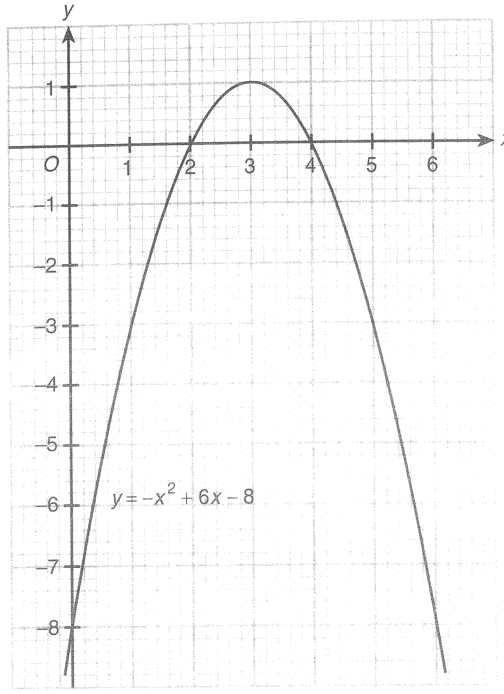
(b) For all real values of x,
$(x-\frac{1}{4})^2\ge 0$
$-4(x-\frac{1}{4})^2\le 0$
$-4(x-\frac{1}{4})^2-\frac{3}{4}\le 0-\frac{3}{4}$
$-4x^2+2x-1\le -\frac{3}{4}$
$4x^2-2x+1\ge \frac{3}{4}$

Classwork 12 (p.28)

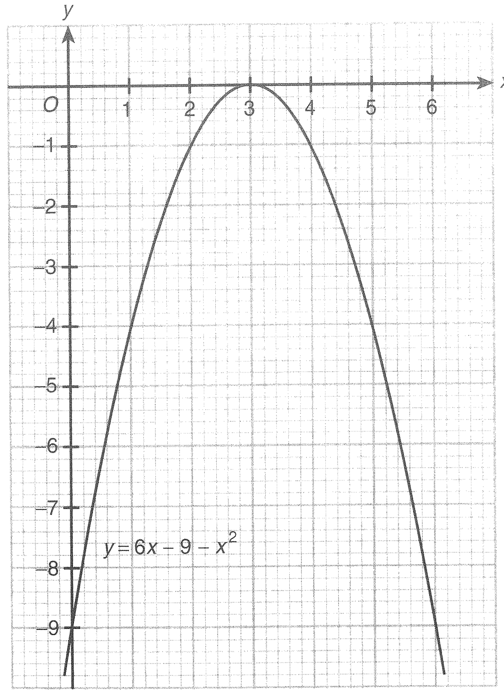
1. (a)



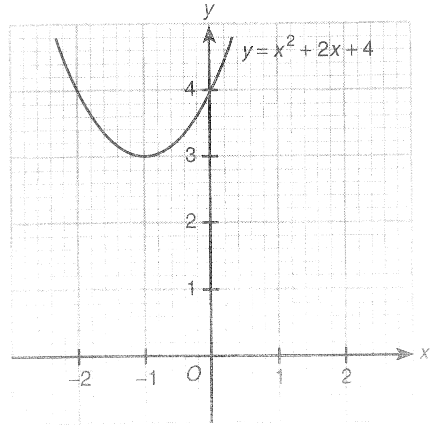
(b)



(c)



(d)



2. Since $y = x^2 + (2a+1)x + a^2$ intersects the x-axis at two distinct points, $D > 0$.

$D = (2a+1)^2 - 4a^2 > 0$

$4a^2 + 4a + 1 - 4a^2 > 0$

$a > -\frac{1}{4}$

3. Consider the discriminant of $x^2 - 2mx + m(m+3)$

$D = (-2m)^2 - 4m(m+3)$

$= 4m^2 - 4m^2 - 12m$

$= -12m$

$m > 0$, $-12m < 0$

$\because$ $D < 0$ and the graph opens upwards

$\therefore$ $x^2 - 2mx + m(m+3)$ is always positive for any real values of x if $m > 0$.

Classwork 13 (p.32)

1. (a) $|5+2x| = 3$

$5+2x = 3$ or $5+2x = -3$

$x = -1$ or $x = -4$

(b) $|2+3x| = |6-2x|$

$2+3x = 6-2x$ or $2+3x = -(6-2x)$

$5x = 4$ or $2+3x = -6+2x$

$x = \frac{4}{5}$ or $x = -8$

2. (a) $\left|\frac{2x+1}{3-x}\right| = \frac{1}{4}$

$\frac{2x+1}{3-x} = \frac{1}{4}$ or $\frac{2x+1}{3-x} = -\frac{1}{4}$

$4(2x+1) = 3-x$ or $4(2x+1) = x-3$

$8x+4 = 3-x$ or $8x+4 = x-3$

$x = -\frac{1}{9}$ or $x = -1$

(b) $|2x+1| = 9+3x$

$2x+1 = 9+3x$ or $2x+1 = -9-3x$

$x = -8$ or $x = -2$

But $9+3x \geq 0$

$\therefore$ $x = -2$

Classwork 14 (p.32)

1. $|x^2+3x-4| = 6$

$x^2+3x-4 = 6$ or $x^2+3x-4 = -6$

$x^2+3x-10 = 0$ or $x^2+3x+2 = 0$

$(x+5)(x-2) = 0$ or $(x+1)(x+2) = 0$

$x = -5, 2$ or $x = -1, -2$

$\therefore$ $x = -5, -2, -1$ or 2

2. $3(x+1)^2 - 7|x+1| + 2 = 0$

Let $a = |x+1| \geq 0$

$a^2 = |x+1|^2$

$= (x+1)^2$

$\therefore$ The equation becomes

$3a^2 - 7a + 2 = 0$

$(a-2)(3a-1) = 0$

$a = 2$ or $a = \frac{1}{3}$

$\because$ $|x+1| = 2$

$x+1 = 2$ or $x+1 = -2$

$x = 1$ or $x = -3$

$\because$ $|x+1| = \frac{1}{3}$

$x+1 = \frac{1}{3}$ or $x+1 = -\frac{1}{3}$

$x = -\frac{2}{3}$ or $x = -\frac{4}{3}$

$\therefore$ $x = -3, -\frac{2}{3}, -\frac{4}{3}$ or 1

Classwork 15 (p.34)