Chapter 4

Exercise 4A (p.84)

1. $3! = 3\times 2\times 1$
$= \underline{\underline{6}}$

2. $6! = 6\times 5\times 4\times 3\times 2\times 1$
$= \underline{\underline{720}}$

3. $\frac{7!}{4!} = \frac{7\times 6\times 5\times 4!}{4!}$
$= 7\times 6\times 5$
$= \underline{\underline{210}}$

4. $\frac{10!}{8!} = \frac{10\times 9\times 8!}{8!}$
$= \underline{\underline{90}}$

5. $7! - 6! = 7\times 6! - 6!$
$= 6!\,(7-1)$
$= 6!\times 6$
$= 6\times 6\times 5\times 4\times 3\times 2\times 1$
$= \underline{\underline{4\ 320}}$

6. $4! + 5! = 4! + 5\times 4!$
$= 4!\,(1+5)$
$= 6\times 4\times 3\times 2\times 1$
$= \underline{\underline{144}}$

7. ${}_5C_3 = \frac{5!}{3!(5-3)!}$
$= \frac{5!}{3!2!}$
$= \frac{5\times 4\times 3!}{3!2!}$
$= \underline{\underline{10}}$

8. ${}_6C_5 = \frac{6!}{5!(6-5)!}$
$= \frac{6\times 5!}{5!1!}$
$= \underline{\underline{6}}$

9. ${}_{12}C_9 = \frac{12!}{9!(12-9)!}$
$= \frac{12\times 11\times 10\times 9!}{9!3!}$
$= \underline{\underline{220}}$

10. $\frac{(n+1)!}{(n-2)!} = \frac{(n+1)\times n\times (n-1)\times (n-2)!}{(n-2)!}$
$= \underline{\underline{n(n+1)(n-1)}}$

11. $(n-1)! - (n-2)! = (n-1)\times (n-2)! - (n-2)!$
$= (n-2)!(n-1-1)$
$= \underline{\underline{(n-2)[(n-2)!]}}$

12. ${}_{n+1}C_3 = \frac{(n+1)!}{3!(n+1-3)!}$
$= \frac{(n+1)!}{3!(n-2)!}$
$= \frac{(n+1)n(n-1)(n-2)!}{3!(n-2)!}$
$= \underline{\underline{\frac{1}{6}n(n+1)(n-1)}}$

13. ${}_{n+1}C_n = \frac{(n+1)!}{n!(n+1-n)!}$
$= \frac{(n+1)\times n!}{n!1!}$
$= \underline{\underline{n+1}}$

14. ${}_nC_{n-2} = \frac{n!}{(n-2)![n-(n-2)!]}$
$= \frac{n!}{(n-2)!2!}$
$= \frac{n(n-1)(n-2)!}{2(n-2)!}$
$= \underline{\underline{\frac{1}{2}n(n-1)}}$

Exercise 4B (p.90)

1. $(a+b)^3 = (a)^3 + {}_3C_1(a)^2(b) + {}_3C_2(a)(b)^2 + b^3$
$= \underline{\underline{a^3 + 3a^2b + 3ab^2 + b^3}}$

2. $(x+2y)^5$
$= (x)^5 + {}_5C_1(x)^4(2y) + {}_5C_2(x)^3(2y)^2$
$+ {}_5C_3(x)^2(2y)^3 + {}_5C_4(x)(2y)^4 + (2y)^5$
$= (x)^5 + 5(x)^4(2y) + 10(x)^3(2y)^2$
$+ 10(x)^2(2y)^3 + 5(x)(2y)^4 + (2y)^5$
$= \underline{\underline{x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5}}$

3. $(2x-3)^4$
$= (2x)^4 + {}_4C_1(2x)^3(-3) + {}_4C_2(2x)^2(-3)^2$
$+ {}_4C_3(2x)(-3)^3 + (-3)^4$
$= \underline{\underline{16x^4 - 96x^3 + 216x^2 - 216x + 81}}$

4. $(3x-2y)^6$
$= (3x)^6 + {}_6C_1(3x)^5(-2y) + {}_6C_2(3x)^4(-2y)^2$
$+ {}_6C_3(3x)^3(-2y)^3 + {}_6C_4(3x)^2(-2y)^4$
$+ {}_6C_5(3x)(-2y)^5 + (-2y)^6$
$= \underline{\underline{729x^6 - 2\ 916x^5y + 4\ 860x^4y^2 - 4\ 320x^3y^3}}$
$\underline{\underline{+ 2\ 160x^2y^4 - 576xy^5 + 64y^6}}$

5. $(x - \frac{1}{x})^5$
$= (x)^5 + {}_5C_1(x)^4(-\frac{1}{x}) + {}_5C_2(x)^3(-\frac{1}{x})^2$
$+ {}_5C_3(x)^2(-\frac{1}{x})^3 + {}_5C_4(x)(-\frac{1}{x})^4 + (-\frac{1}{x})^5$
$= x^5 - 5x^4(\frac{1}{x}) + 10x^3(\frac{1}{x})^2 - 10x^2(\frac{1}{x})^3$
$+ 5x(-\frac{1}{x})^4 - \frac{1}{x^5}$
$= \underline{\underline{x^5 - 5x^3 + 10x - 10x^{-1} + 5x^{-3} - x^{-5}}}$

6. $(\frac{x}{2} + \frac{2}{x})^6$
$= (\frac{x}{2})^6 + {}_6C_1(\frac{x}{2})^5(\frac{2}{x}) + {}_6C_2(\frac{x}{2})^4(\frac{2}{x})^2$
$+ {}_6C_3(\frac{x}{2})^3(\frac{2}{x})^3 + {}_6C_4(\frac{x}{2})^2(\frac{2}{x})^4$
$+ {}_6C_5(\frac{x}{2})(\frac{2}{x})^5 + (\frac{2}{x})^6$
$= (\frac{x}{2})^6 + 6(\frac{x}{2})^5(\frac{2}{x}) + 15(\frac{x}{2})^4(\frac{2}{x})^2 + 20(\frac{x}{2})^3(\frac{2}{x})^3$
$+ 15(\frac{x}{2})^2(\frac{2}{x})^4 + 6(\frac{x}{2})(\frac{2}{x})^5 + (\frac{2}{x})^6$
$= \underline{\underline{\frac{1}{64}x^6 + \frac{3}{8}x^4 + \frac{15}{4}x^2 + 20 + \frac{60}{x^2} + \frac{96}{x^4} + \frac{64}{x^6}}}$

7. $(x+2)^8$

The general term in the expansion $= {}_8C_r(x)^{8-r}(2)^r$

$\therefore$ The term in $x^4 = {}_8C_4x^4(2)^4$

$\therefore$ The coefficient of $x^4 = {}_8C_42^4 = \underline{\underline{1\ 120}}$

8. $(3x-2)^7$

The general term in the expansion

$= {}_7C_r(3x)^{7-r}(-2)^r$

$\therefore$ The term in $x^5 = {}_7C_2(3x)^5(-2)^2$

$\therefore$ The coefficient of

$x^5 = {}_7C_2\cdot 3^5(-2)^2 = \underline{\underline{20\ 412}}$

9. $(2x+y)^{10}$

The general term in the expansion

$= {}_{10}C_r(2x)^{10-r}(y)^r$

$\therefore$ The term in $x^3y^7 = {}_{10}C_7(2x)^3y^7$

$\therefore$ The coefficient of $x^3y^7 = {}_{10}C_7\cdot 2^3 = \underline{\underline{960}}$

10. $(\frac{1}{2} - 3x)^8$

The general term in the expansion

$= {}_8C_r(\frac{1}{2})^{8-r}(-3x)^r$

$\therefore$ The term in $x^3 = {}_8C_3(\frac{1}{2})^5(-3x)^3$

$\therefore$ The coefficient of $x^3 = {}_8C_3(\frac{1}{2})^5(-3)^3$

$= \underline{\underline{-\frac{189}{4}}}$

11. The general term in the expansion

$(2-x)^6 = {}_6C_r(2)^{6-r}(-x)^r$

$\therefore$ The term in $x^4 = {}_6C_4(2)^2(-x)^4$

$\therefore$ The coefficient of $x^4 = {}_6C_4(2)^2(-1)^4 = 60$

The general term in the expansion

$(2x-1)^9 = {}_9C_r(2x)^{9-r}(-1)^r$

$\therefore$ The term in $x^4 = {}_9C_5(2x)^4(-1)^5$

$\therefore$ The coefficient of $x^4 = -2\ 016$

$\therefore$ The coefficient of x^4 in the expansion of

$(2-x)^6 - (2x-1)^9 = 60 + 2\ 016 = \underline{\underline{2\ 076}}$

12. The general term in the expansion

$(\frac{1}{3x} - 6x)^6 = {}_6C_r(\frac{1}{3x})^{6-r}(-6x)^r$

$= {}_6C_r(\frac{1}{3})^{6-r}(-6)^rx^{2r-6}$

It is the constant term when $2r-6=0$, i.e $r=3$.

$\therefore$ The constant term $= {}_6C_3(\frac{1}{3})^3(-6)^3 = \underline{\underline{-160}}$

13. The general term in the expansion

$(4x^2 - \frac{1}{2x})^9 = {}_9C_r(4x^2)^{9-r}(-\frac{1}{2x})^r$

$= {}_9C_r(4)^{9-r}(-\frac{1}{2})^rx^{18-3r}$

It is the constant term when $18-3r=0$, i.e $r=6$.

$\therefore$ The constant term $= {}_9C_6(4)^3(-\frac{1}{2})^6 = \underline{\underline{84}}$

14. The general term in the expansion

$(1+x)^{24} = {}_{24}C_r x^r$

$\therefore$ The coefficient of $x^r = {}_{24}C_r$

i.e. $B_r = {}_{24}C_r$

$$\frac{B_{r+2}}{B_r} = \frac{{}_{24}C_{r+2}}{{}_{24}C_r} = \frac{57}{7}$$

$$\frac{\frac{24!}{(r+2)![24-(r+2)!]}}{\frac{24!}{r!(24-r)!}} = \frac{57}{7}$$

$$\frac{(24-r)!}{(r+2)(r+1)(22-r)!} = \frac{57}{7}$$

$$\frac{(24-r)(23-r)}{(r+2)(r+1)} = \frac{57}{7}$$

$57(r^2+3r+2) = 7(552-47r+r^2)$

$50r^2+500r-3\ 750=0$

$r^2+10r-75=0$

$(r+15)(r-5)=0$

$\because$ $r>0$, $r=-15$ (rejected)

$\therefore$ $r=\underline{\underline{5}}$

15. $(1-2x)^9(1+\frac{1}{x})^3$

$=[1+9(-2x)+36(-2x)^2+84(-2x)^3+\cdots]$

$[1+3(\frac{1}{x})+3(\frac{1}{x})^2+(\frac{1}{x})^3]$

$=(1-18x+144x^2-672x^3+\cdots)$

$(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3})$

$\therefore$ The constant term

$=1+(-18)(3)+144(3)+(-672)$

$=\underline{\underline{-293}}$

16. $(1-5x)^3(1+2x)^6$

$=[1+3(-5x)+3(-5x)^2+\cdots]$

$[1+6(2x)+15(2x)^2+\cdots]$

$=(1-15x+75x^2+\cdots)$

$(1+12x+60x^2+\cdots)$

$=1+(-15+12)x+(75-15\times12+60)x^2+\cdots$

$=1-3x-45x^2+\cdots$

$\therefore$ $a=\underline{\underline{-3}}$, $b=\underline{\underline{-45}}$

17. $(2x^2+1)^n=(1+2x^2)^n$

$=1+{}_nC_1(2x^2)+{}_nC_2(2x^2)^2+\cdots$

The coefficient of the third term $=4\cdot{}_nC_2$

$4\cdot{}_nC_2=60$

$\frac{n!}{2!(n-2)!}=15$

$n(n-1)=30$

$n^2-n-30=0$

$(n-6)(n+5)=0$

$n=\underline{\underline{6}}$ or -5 (rejected)

The general term in the expansion

$(1+2x^2)^6={}_6C_r(2x^2)^r$

$\therefore$ The term in $x^8={}_6C_4(2x^2)^4$

$\therefore$ The coefficient of $x^8=\underline{\underline{240}}$

18. $(1+mx^2)^n=1+{}_nC_1(mx^2)+{}_nC_2(mx^2)^2+\cdots$

Comparing coefficients of x^2 and x^4 respectively,

$m\cdot{}_nC_1=14$

$mn=14$ ....(1)

$m^2\cdot{}_nC_2=21m^2$

$m^2\cdot\frac{n(n-1)}{2}=21m^2$

$n^2-n-42=0$

$(n+6)(n-7)=0$

$n=\underline{\underline{7}}$ or -6 (rejected)

Put $n=7$ into (1),

$7m=14$

$m=2$

$\therefore$ $m=\underline{\underline{2}}$

19. The general term in the expansion

$(1+x)^n={}_nC_rx^r$

$\therefore$ The coefficient of $x^4={}_nC_4$

$\therefore$ The coefficient of $x^5={}_nC_5$

$\therefore$ The coefficient of $x^6={}_nC_6$

${}_nC_4+{}_nC_6=2{}_nC_5$

$$\frac{n!}{4!(n-4)!}+\frac{n!}{6!(n-6)!}=\frac{2(n!)}{5!(n-5)!}$$

$$\frac{30}{(n-4)!}+\frac{(n-4)(n-5)}{(n-4)!}=\frac{12(n-4)}{(n-4)!}$$

$30+(n-4)(n-5)=12(n-4)$

$n^2-21n+98=0$

$(n-14)(n-7)=0$

$n=\underline{\underline{7}}$ or $\underline{\underline{14}}$

20. $(1+px)(1+qx)^5$

$=(1+px)[1+5(qx)+10(qx)^2+\cdots]$

$=(1+px)(1+5qx+10q^2x^2+\cdots)$

Comparing the coefficients of x,

$5q+p=-6$ ....(1)

Comparing the coefficients of x^2,

$10q^2+5pq=0$ ......(2)

Put $p=-6-5q$ into (2),

$10q^2+5(-6-5q)q=0$

$10q^2-30q-25q^2=0$

$15q^2+30q=0$

$q(q+2)=0$

$q=\underline{\underline{-2}}$ or 0 (rejected)

Put $q=-2$ into $p=-6-5q$,

$p=-6-5(-2)$

$=\underline{\underline{4}}$

21 – 22. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

Exercise 4C (p.93)

1. $(1+x+3x^2)^3$

$=[1+x(1+3x)]^3$

$=1+3x(1+3x)+3x^2(1+3x)^2+x^3(1+3x)^3$

$=1+3x+9x^2+3x^2(1+6x+\cdots)+x^3(1+\cdots)$

$=\underline{\underline{1+3x+12x^2+19x^3+\cdots}}$

2. $(1+x-2x^2)^6$

$=[1+x(1-2x)]^6$

$=1+6x(1-2x)+15x^2(1-2x)^2$

$+20x^3(1-2x)^3+\cdots$

$=1+6x-12x^2+15x^2(1-4x+\cdots)$

$+20x^3(1+\cdots)$

$=\underline{\underline{1+6x+3x^2-40x^3+\cdots}}$

3. $(1-x-x^2)^5$

$=[1-x(1+x)]^5$

$=1-5x(1+x)+10x^2(1+x)^2$

$-10x^3(1+x)^3+\cdots$

$=1-5x-5x^2+10x^2(1+2x+\cdots)$

$-10x^3(1+\cdots)$

$=\underline{\underline{1-5x+5x^2+10x^3+\cdots}}$

4. $(3-x+x^2)^8$

$=[3-x(1-x)]^8$

$=3^8-8\cdot3^7x(1-x)+28\cdot3^6x^2(1-x)^2$

$-56\cdot3^5x^3(1-x)^3+\cdots$

$=6\ 561-17\ 496x+17\ 496x^2$

$+20\ 412x^2(1-2x+\cdots)-13\ 608x^3(1+\cdots)$

$=\underline{\underline{6\ 561-17\ 496x+37\ 908x^2-54\ 432x^3+\cdots}}$

5. $(1-4x+x^2)^9$

$=[1-x(4-x)]^9$

$=1-9x(4-x)+36x^2(4-x)^2-\cdots$

$=1-36x+9x^2+36x^2(16+\cdots)+\cdots$

$=1-36x+9x^2+576x^2+\cdots$

$=1-36x+585x^2+\cdots$

Coefficient of $x^2=\underline{\underline{585}}$

6. $(2-x+x^2)^{10}$

$=[2-x(1-x)]^{10}$

$=2^{10}-10\cdot2^9x(1-x)+45\cdot2^8x^2(1-x)^2$

$-120\cdot2^7x^3(1-x)^3+\cdots$

$=1\ 024-5\ 120x(1-x)+11\ 520x^2$

$(1-2x+\cdots)-15\ 360x^3(1+\cdots)+\cdots$

$=1\ 024-5\ 120x+5\ 120x^2+11\ 520x^2$

$-23\ 040x^3-15\ 360x^3+\cdots$

$=1\ 024-5\ 120x+16\ 640x^2-38\ 400x^3+\cdots$

Coefficient of $x^3=\underline{\underline{-38\ 400}}$

7. $(1-\frac{1}{3x}+6x)^3$

$=[1-\frac{1}{x}(\frac{1}{3}-6x^2)]^3$

$=1-3\cdot\frac{1}{x}(\frac{1}{3}-6x^2)+3\cdot\frac{1}{x^2}(\frac{1}{3}-6x^2)^2$

$-\frac{1}{x^3}(\frac{1}{3}-6x^2)^3$

$=1-\frac{3}{x}(\frac{1}{3}-6x^2)+\frac{3}{x^2}(\frac{1}{9}-4x^2+36x^4)+\cdots$

Constant term $=1+3(-4)=\underline{\underline{-11}}$

8. $(1-x+2x^2)^n$

$=[1-x(1-2x)]^n$

$=1-{}_nC_1x(1-2x)+{}_nC_2x^2(1-2x)^2+\cdots$

$=1-{}_nC_1x(1-2x)+{}_nC_2x^2(1-4x+4x^2)+\cdots$

$=1-{}_nC_1x+(2{}_nC_1+{}_nC_2)x^2+\cdots$

$$2_nC_1 + {}_nC_2 = 44$$
$$2n + \frac{1}{2}n(n-1) = 44$$
$$n^2 + 3n - 88 = 0$$
$$(n-8)(n+11) = 0$$
$$n = \underline{\underline{8}} \quad \text{or} \quad -11 \text{ (rejected)}$$

9. (a) $(1 - x + 2x^2)^6$

$= [1 - x(1-2x)]^6$

$= 1 - 6x(1-2x) + 15x^2(1-2x)^2 - 20x^3(1-2x)^3 + \cdots$

$= 1 - 6x + 12x^2 + 15x^2(1 - 4x + \cdots) - 20x^3(1 + \cdots)$

$= \underline{\underline{1 - 6x + 27x^2 - 80^3 + \cdots}}$

(b) $(1 - x + 2x^2)^6(1+x)^6$

$= (1 - 6x + 27x^2 - 80^3x^3 + \cdots)(1 + 6x + 15x^2 + 20x^3 + \cdots)$

$= 1 + x^2(27 - 6\times 6 + 15) + x^3(-80 + 27\times 6 - 6\times 15 + 20) + \cdots$

$= 1 + 6x^2 + 12x^3 + \cdots$

$a = \underline{\underline{0}}$, $b = \underline{\underline{6}}$, $c = \underline{\underline{12}}$

10. (a) $(1 + x - 2ax^2)^n$

$= [1 + x(1-2ax)]^n$

$= 1 + {}_nC_1x(1-2ax) + {}_nC_2x^2(1-2ax)^2 + \cdots$

$= 1 + nx(1-2ax) + \frac{1}{2}n(n-1)x^2(1+\cdots) + \cdots$

$= \underline{\underline{1 + nx - [2an - \frac{n(n-1)}{2}]x^2 + \cdots}}$

(b) Coefficient of $x = n = \underline{\underline{7}}$

Comparing the coefficients of x^2,

$$-[2an - \frac{n(n-1)}{2}] = 0$$
$$2a(7) - \frac{7\times 6}{2} = 0$$
$$a = \underline{\underline{\frac{3}{2}}}$$

11. (a) $(1 + x^2 + x^3)^n$

$= [1 + x^2(1+x)]^n$

$= 1 + nx^2(1+x) + \frac{n(n-1)}{2}x^4(1+x)^2 + \frac{n(n-1)(n-2)}{6}x^6(1+x)^3 + \cdots$

$= 1 + nx^2 + nx^3 + \frac{n(n-1)}{2}x^4(1 + 2x + x^2) + \frac{n(n-1)(n-2)}{6}x^6(1 + 3x + \cdots) + \cdots$

$= 1 + nx^2 + nx^3 + \frac{n(n-1)}{2}x^4 + n(n-1)x^5 + \frac{n(n-1)}{2}x^6 + \frac{n(n-1)(n-2)}{6}x^6 + \frac{n(n-1)(n-2)}{2}x^7 + \cdots$

Coefficient of $x^5 = \underline{\underline{n(n-1)}}$

Coefficient of $x^7 = \underline{\underline{\frac{1}{2}n(n-1)(n-2)}}$

(b)
$$\frac{\text{Coefficient of } x^7}{\text{Coefficient of } x^5} = 5$$
$$\frac{\frac{1}{2}n(n-1)(n-2)}{n(n-1)} = 5$$
$$\frac{n-2}{2} = 5$$
$$n = \underline{\underline{12}}$$

12. $(1 + px + qx^2)^4$

$= [1 + x(p+qx)]^4$

$= 1 + 4x(p+qx) + 6x^2(p+qx)^2 + 4x^3(p+qx)^3 + \cdots$

$= 1 + 4px + 4qx^2 + 6x^2(p^2 + 2pqx + \cdots) + 4x^3(p^3 + \cdots) + \cdots$

$= 1 + 4px + 4qx^2 + 6p^2x^2 + 12pqx^3 + 4p^3x^3 + \cdots$

$= 1 + 4px + 2(3p^2 + 2q)x^2 + 4p(p^2 + 3q)x^3 + \cdots$

$$\begin{cases} 2(3p^2 + 2q) = 0 \quad \text{.............(1)} \\ 4p(p^2 + 3q) = -112 \quad \text{......(2)} \end{cases}$$

By (1), $2q = -3p^2$

$$q = -\frac{3}{2}p^2$$

Substitute into (2), $4p[p^2 + (\frac{-3}{2}p^2)(3)] = -112$

$$4p(-\frac{7}{2}p^2) = -112$$
$$-14p^3 = -112$$
$$p^3 = 8$$
$$p = \underline{\underline{2}}$$

$\therefore \quad q = -\frac{3}{2}(4) = \underline{\underline{-6}}$

13. (a) $(1 - tx - x^2)^7$

$= [1 - x(t+x)]^7$

$= 1 - 7x(t+x) + 21x^2(t+x)^2 - 35x^3(t+x)^3 + \cdots$

$= 1 - 7tx - 7x^2 + 21x^2(t^2 + 2tx + \cdots) - 35x^3(t^3 + \cdots) + \cdots$

$= 1 - 7tx - 7x^2 + 21t^2x^2 + 42tx^3 - 35t^3x^3 + \cdots$

$= \underline{\underline{1 - 7tx + 7(3t^2 - 1)x^2}}$
$\underline{\underline{-7t(5t^2 - 6)x^3 + \cdots}}$

(b) By (a), comparing the coefficients of x^3,

$$-91t^2 = -7t(5t^2 - 6)$$
$$13t = 5t^2 - 6$$
$$5t^2 - 13t - 6 = 0$$
$$(5t+2)(t-3) = 0$$
$$t = \underline{\underline{3}}, -\frac{2}{5} \text{ (rejected)}$$

Coefficient of $x^2 = 7(27 - 1)$
$= \underline{\underline{182}}$

14 – 15. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

Revision Exercise 4 (p.95)

1. $(2x - \frac{1}{x})^5$

$= (2x)^5 - 5(2x)^4(\frac{1}{x}) + 10(2x)^3(\frac{1}{x})^2 - 10(2x)^2(\frac{1}{x})^3 + 5(2x)(\frac{1}{x})^4 - (\frac{1}{x})^5$

$= \underline{\underline{32x^5 - 80x^3 + 80x - 40x^{-1} + 10x^{-3} - x^{-5}}}$

2. $(a-b)^3(a+b)^3$

$= (a^2 - b^2)^3$

$= \underline{\underline{a^6 - 3a^4b^2 + 3a^2b^4 - b^6}}$

3. $(x^2 - \frac{1}{x})^4$

$= x^8 - 4(x^2)^3(\frac{1}{x}) + 6(x^2)^2(\frac{1}{x})^2 - 4(x^2)(\frac{1}{x})^3 + (-\frac{1}{x})^4$

$= \underline{\underline{x^8 - 4x^5 + 6x^2 - 4x^{-1} + x^{-4}}}$

4. $(1+2x)^3(1-x)^2$

$= [1 + 3(2x) + 3(2x)^2 + (2x)^3](1 - 2x + x^2)$

$= (1 + 6x + 12x^2 + 8x^3)(1 - 2x + x^2)$

$= 1 + (-2+6)x + (1 - 12 + 12)x^2 + (8 - 24 + 6)x^3 + (12 - 16)x^4 + 8x^5$

$= \underline{\underline{1 + 4x + x^2 - 10x^3 - 4x^4 + 8x^5}}$

5. $(2x - \frac{1}{x})^{12}$

The general term is

${}_{12}C_r(2x)^{12-r}(-x^{-1})^r = {}_{12}C_r(2)^{12-r}(-1)^r x^{12-2r}$

It is the constant term when $12 - 2r = 0$, $\therefore r = 6$

$\therefore$ The constant term $= {}_{12}C_6\ 2^6(-1)^6 = \underline{\underline{59\ 136}}$

6. $(\frac{x^2}{2} - \frac{2}{x})^8$

The general term is

${}_8C_r(\frac{x^2}{2})^{8-r}(-\frac{2}{x})^r = {}_8C_r(-1)^r 2^{2r-8} x^{16-3r}$

Term in x^4 when $16 - 3r = 4$, $\therefore r = 4$

Coefficient of $x^4 = {}_8C_4(-1)^4 2^{8-8} = \underline{\underline{70}}$

7. $(1+x)^{16} = 1 + {}_{16}C_1x + {}_{16}C_2x^2 + \cdots$

Coefficient of the 3rd term $= {}_{16}C_2 = \underline{\underline{120}}$

8. $(2-x)^{15}$

$= 2^{15} - {}_{15}C_1 2^{14}x + \cdots + {}_{15}C_{12}2^3(-x)^{12} + \cdots$

Coefficient of the 13th term $= {}_{15}C_{12}\ 2^3 = \underline{\underline{3\ 640}}$

9. $(2 + x + x^3)^7$

$= [2 + x(1 + x^2)]^7$

$= 2^7 + {}_7C_1 \cdot 2^6 x(1+x^2) + {}_7C_2 \cdot 2^5 x^2(1+x^2)^2 + {}_7C_3 \cdot 2^4 x^3(1+x^2)^3 + \cdots$

$= 128 + 448x(1+x^2) + 672x^2(1+\cdots) + 560x^3(1+\cdots) + \cdots$

$= \underline{\underline{128 + 448x + 672x^2 + 1\ 008x^3 + \cdots}}$

10. $(3 + 2x - x^2)^4$

$= [3 + x(2-x)]^4$

$= 3^4 + 4\cdot 3^3x(2-x) + 6\cdot 3^2x^2(2-x)^2 + 4\cdot 3x^3(2-x)^3 + \cdots$

$= 81 + 108x(2-x) + 54x^2(4 - 4x + \cdots) + 12x^3(8 - \cdots) + \cdots$

$= \underline{\underline{81 + 216x + 108x^2 - 120x^3 + \cdots}}$

11. $(1+2x)^4(1-x)^6$
$= (1+8x+24x^2+32x^3+\cdots)$
$\quad (1-6x+15x^2-20x^3+\cdots)$
$= \underline{\underline{1+2x-9x^2-12x^3+\cdots}}$

12. $(1-2x)^4(1+x)^7$
$= (1-8x+24x^2+\cdots)(1+7x+21x^2+\cdots)$
Coefficient of $x^2 = 21-56+24 = \underline{\underline{-11}}$

13. $(1-\frac{1}{2x}+4x^2)^4$
$= [1-\frac{1}{2x}(1-8x^3)]^4$
$= 1-4(\frac{1}{2x})(1-8x^3)+6(\frac{1}{2x})^2(1-8x^3)^2$
$\quad -4(\frac{1}{2x})^3(1-8x^3)^3+(\frac{1}{2x})^4(1-8x^3)^4$
Constant term $= 1+12 = \underline{\underline{13}}$

14. $(1+x-2x^2)^9(1+x)^4$
$= [1+x(1-2x)]^9(1+x)^4$
$= [1+9x(1-2x)+36x^2(1-2x)^2+84x^3(1-2x)^3$
$\quad +\cdots](1+4x+6x^2+4x^3+\cdots)$
$= [1+9x-18x^2+36x^2(1-4x+\cdots)$
$\quad +84x^3(1+\cdots)+\cdots](1+4x+6x^2+4x^3+\cdots)$
$= (1+9x+18x^2-60x^3)(1+4x+6x^2+4x^3+\cdots)$
Coefficient of $x^3 = 4+9\times 6+18\times 4-60 = \underline{\underline{70}}$

15. $(1+2x)^n = 1+2nx+\frac{1}{2}n(n-1)(2x)^2$
$\quad +\frac{1}{6}n(n-1)(n-2)(2x)^3$
$\quad +\frac{1}{24}n(n-1)(n-2)(n-3)(2x)^4+\cdots$
$\because$ Coefficient of x^3 = Coefficient of x^4
$\therefore \dfrac{2^3\cdot n(n-1)(n-2)}{6} = \dfrac{2^4\cdot n(n-1)(n-2)(n-3)}{24}$
$\frac{4}{3} = \frac{2}{3}(n-3)$
$2 = n-3$
$n = \underline{\underline{5}}$

16. The general term in the expansion
$(2x^2+\frac{1}{2x})^n = {}_nC_r(2x^2)^{n-r}(\frac{1}{2x})^r$
$\quad = {}_nC_r\cdot 2x^{n-2r}x^{2n-3r}$
$\therefore$ The 7th term $= {}_nC_6\cdot 2^{n-12}x^{2n-18}$

It is the constant term when
$2n-18=0$. $\therefore n = \underline{\underline{9}}$
$\therefore$ The value of the 7th term
$= {}_9C_6\cdot 2^{-3} = \underline{\underline{\frac{21}{2}}}$

17. The general term in the expansion
$(x^2+\frac{a}{2x})^7 = {}_7C_r(x^2)^{7-r}(\frac{a}{2x})^r$
$\quad = {}_7C_r(\frac{a}{2})^r x^{14-3r}$
$\therefore A_8 = {}_7C_2(\frac{a}{2})^2 = \frac{21}{4}a^2$
$A_{11} = {}_7C_1(\frac{a}{2}) = \frac{7}{2}a$
$A_8 = 6A_{11}$
$\frac{21}{4}a^2 = 6(\frac{7}{2})a$
$a(a-4)=0$
$a = \underline{\underline{4}}$ or 0 (rejected)

18. $(ax+\frac{2}{x^2})^n = (ax)^n+n(ax)^{n-1}(\frac{2}{x^2})$
$\quad +\frac{n(n-1)}{2}(ax)^{n-2}(\frac{2}{x^2})^2+\cdots$
The third term $= \frac{n(n-1)}{2}(ax)^{n-2}(\frac{4}{x^4})$
$\quad = 2n(n-1)a^{n-2}x^{n-6}$
$\because$ The third term is independent of x.
$\therefore n-6=0$, i.e. $n = \underline{\underline{6}}$
The coefficient of the third term:
$2n(n-1)a^{n-2} = \frac{15}{4}$
$2(6)(5)a^4 = \frac{15}{4}$
$a^4 = \frac{1}{16}$
$a = \underline{\underline{-\frac{1}{2}}}$ or $\frac{1}{2}$ (rejected)

19. $(1+2x)^5(1-x)^n$
$= [1+5(2x)+10(2x)^2+\cdots]$
$\quad [1-nx+\frac{n(n-1)}{2}x^2+\cdots]$
$= (1+10x+40x^2+\cdots)$
$\quad (1-nx+\frac{n^2-n}{2}x^2+\cdots)$
(a) Coefficient of $x = -n+10 = 1$
$n = \underline{\underline{9}}$

(b) Coefficient of $x^2 = \frac{n^2-n}{2}-10n+40$
$= \frac{9^2-9}{2}-90+40$
$= \underline{\underline{-14}}$

20 – 22. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

23. $(1+\frac{x}{2n})^n = 1+n(\frac{x}{2n})+\frac{n(n-1)}{2}(\frac{x}{2n})^2$
$\quad +\frac{1}{6}n(n-1)(n-2)(\frac{x}{2n})^3\cdots$
Coefficient of $x^2 = \frac{n(n-1)}{2}(\frac{1}{2n})^2 = \frac{1}{10}$
$\frac{n-1}{8n} = \frac{1}{10}$
$2n = 10$
$n = \underline{\underline{5}}$
Coefficient of $x^3 = \frac{1}{6}n(n-1)(n-2)(\frac{1}{2n})^3$
$= \frac{1}{6}\times 5\times 4\times 3\times(\frac{1}{10})^3$
$= \underline{\underline{\frac{1}{100}}}$

24. (a) $(1+x+px^2)^q$
$= [1+x(1+px)]^q$
$= 1+qx(1+px)+\frac{1}{2}q(q-1)x^2(1+px)^2$
$\quad +\frac{1}{6}q(q-1)(q-2)x^3(1+px)^3+\cdots$
$= 1+qx+pqx^2+\frac{1}{2}q(q-1)x^2(1+2px$
$\quad +\cdots)+\frac{1}{6}q(q-1)(q-2)x^3(1+\cdots)+\cdots$
$= 1+qx+pqx^2+\frac{q(q-1)}{2}x^2$
$\quad +pq(q-1)x^3+\frac{q(q-1)(q-2)}{6}x^3+\cdots$
$= 1+qx+[pq+\frac{q(q-1)}{2}]x^2$
$\quad +[pq(q-1)+\frac{q(q-1)(q-2)}{6}]x^3+\cdots$
$= \underline{\underline{1+qx+\frac{1}{2}q(2p+q-1)x^2}}$
$\quad \underline{\underline{+\frac{1}{6}q(q-1)(6p+q-2)x^3+\cdots}}$

(b) $\begin{cases} q = \underline{\underline{6}} \text{ ....(1)} \\ \frac{1}{2}q(2p+q-1) = 27 \text{ ....(2)} \end{cases}$
Substitute $q=6$ into (2).
$\frac{1}{2}(6)(2p+6-1) = 27$
$2p+5 = 9$
$p = \underline{\underline{2}}$
Coefficient of $x^3 = \frac{1}{6}(6)(5)(12+6-2)$
$= (5)(16)$
$= \underline{\underline{80}}$

25. (a) $(1-px)^6-(1+x)^n$
$= (1-6px+15p^2x^2+\cdots)$
$\quad -[1+nx+\frac{1}{2}n(n-1)x^2+\cdots]$
$= (-6p-n)x+[15p^2-\frac{1}{2}n(n-1)]x^2+\cdots$
$= \underline{\underline{-(6p+n)x+\frac{1}{2}(30p^2-n^2+n)x^2+\cdots}}$

(b) $-(6p+n) = -17$ ....(1)
$\frac{1}{2}(30p^2-n^2+n) = 50$ ....(2)
By (1), $p = \frac{17-n}{6}$ ....(3)
Put (3) into (2),
$\frac{1}{2}[30(\frac{17-n}{6})^2-n^2+n] = 50$
$5(289-34n+n^2)-6n^2+6n = 600$
$n^2+164n-845 = 0$
$n = \underline{\underline{5}}$ or -169 (rejected)
Put $n=5$ into (3),
$p = \frac{17-5}{6} = \underline{\underline{2}}$

Enrichment 4 (p.97)

1. (a) $(ax+\frac{b}{x})^n$
$= (ax)^n+{}_nC_1(ax)^{n-1}(\frac{b}{x})+{}_nC_2(ax)^{n-2}(\frac{b}{x})^2$
$\quad +{}_nC_3(ax)^{n-3}(\frac{b}{x})^3+{}_nC_4(ax)^{n-4}(\frac{b}{x})^4+\cdots$
$= \underline{\underline{a^nx^n+{}_nC_1a^{n-1}bx^{n-2}+{}_nC_2a^{n-2}b^2x^{n-4}}}$
$\quad \underline{\underline{+{}_nC_3a^{n-3}b^3x^{n-6}+{}_nC_4a^{n-4}b^4x^{n-8}+\cdots}}$

(b) The fifth term is the constant term.

$n-8=0$

$n=\underline{\underline{8}}$

2. (a) $(1+3x)^m+(1+5x)^n$

$=[1+m(3x)+\frac{m(m-1)}{2}(3x)^2+\cdots]+$

$[1+n(5x)+\frac{n(n-1)}{2}(5x)^2+\cdots]$

$=2+(3m+5n)x$

$+[\frac{9m(m-1)}{2}+\frac{25n(n-1)}{2}]x^2+\cdots$

Comparing coefficients of x,

$3m+5n=19$

$\because$ m and n are positive integers.

$\therefore$ $m=\underline{\underline{3}},\ n=\underline{\underline{2}}$

(b) $a=\frac{9m(m-1)}{2}+\frac{25n(n-1)}{2}$

$=\frac{9(3)(2)}{2}+\frac{25(2)}{2}$

$=\underline{\underline{52}}$

3. (a) $(1+x-2x^2)^7$

$=[1+x(1-2x)]^7$

$=1+7x(1-2x)+21x^2(1-2x)^2$

$+35x^3(1-2x)^3+\cdots$

$=1+7x-14x^2+21x^2(1-4x+\cdots)$

$+35x^3(1+\cdots)+\cdots$

$=\underline{\underline{1+7x+7x^2-49x^3+\cdots}}$

(b) $(1+x-2x^2)^7(1+x)^n$

$=(1+7x+7x^2-49x^3+\cdots)[1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+\cdots]$

$=1+(7+n)x+[\frac{n(n-1)}{2}+7n+7]x^2$

$+[\frac{n(n-1)(n-2)}{6}+\frac{7n(n-1)}{2}+7n-49]x^3$

$+\cdots$

By comparing coefficients,

$7+n=10$

$n=\underline{\underline{3}}$

$a=\frac{n(n-1)}{2}+7n+7$

$=\frac{(3)(2)}{2}+7(3)+7$

$=\underline{\underline{31}}$

$b=\frac{n(n-1)(n-2)}{6}+\frac{7n(n-1)}{2}+7n-49$

$=\frac{(3)(2)}{6}+\frac{7(3)(2)}{2}+7(3)-49$

$=\underline{\underline{-6}}$

4. Let the three consecutive coefficients be the coefficients of the $(r-1)$th, rth and $(r+1)$th terms, then

${}_nC_{r-2}=3a$ ....(1)

${}_nC_{r-1}=12a$ ....(2)

${}_nC_r=28a$ ....(3)

$\frac{(1)}{(2)}$: $\frac{{}_nC_{r-2}}{{}_nC_{r-1}}=\frac{3a}{12a}$

$\frac{r-1}{n-r+2}=\frac{3}{12}$

$15r-3n=18$ ....(4)

$\frac{(2)}{(3)}$: $\frac{{}_nC_{r-1}}{{}_nC_r}=\frac{12a}{28a}$

$\frac{r}{n-r+1}=\frac{3}{7}$

$10r-3n=3$ ....(5)

$(4)-(5):5r=15$

$r=3$

Put $r=3$ into (4),

$15(3)-3n=18$

$n=\underline{\underline{9}}$

Put $n=9$ and $r=3$ into (3),

${}_9C_3=28a$

$a=\underline{\underline{3}}$

Classwork 1 (p.84)

1. (a) $5!=5\times4\times3\times2\times1$

$=\underline{\underline{120}}$

(b) $\frac{9!}{7!}=\frac{9\times8\times7!}{7!}$

$=\underline{\underline{72}}$

(c) ${}_7C_3=\frac{7!}{3!(7-3)!}$

$=\frac{7!}{3!4!}$

$=\frac{7\times6\times5\times4!}{3!4!}$

$=\underline{\underline{35}}$

2. (a) ${}_nC_3=\frac{n!}{3!(n-3)!}$

$=\frac{n(n-1)(n-2)n-3!}{6(n-3)!}$

$=\underline{\underline{\frac{1}{6}n(n-1)(n-2)}}$

(b) ${}_nC_{n-3}=\frac{n!}{(n-3)![n-(n-3)]!}$

$=\frac{n!}{(n-3)!3!}$

$=\frac{n(n-1)(n-2)(n-3)!}{(n-3)!3!}$

$=\underline{\underline{\frac{1}{6}n(n-1)(n-2)}}$

(c) ${}_{n+2}C_2=\frac{(n+2)!}{2!(n+2-2)!}$

$=\frac{(n+2)!}{2!n!}$

$=\frac{(n+2)(n+1)n!}{2!n!}$

$=\underline{\underline{\frac{1}{2}(n+1)(n+2)}}$

Classwork 2 (p.89)

1. (a) $(1+2a)^4$

$=1+{}_4C_1(2a)+{}_4C_2(2a)^2$

$+{}_4C_3(2a)^3+(2a)^4$

$=\underline{\underline{1+8a+24a^2+32a^3+16a^4}}$

(b) $(3-b)^4$

$=(3)^4+{}_4C_1(3)^3(-b)+{}_4C_2(3)^2(-b)^2$

$+{}_4C_3(3)(-b)^3+(-b)^4$

$=\underline{\underline{81-108b+54b^2-12b^3+b^4}}$

(c) $(x+2y)^3$

$=(x)^3+{}_3C_1(x)^2(2y)+{}_3C_2(x)(2y)^2+(2y)^3$

$=\underline{\underline{x^3+6x^2y+12xy^2+8y^3}}$

(d) $(y-3x)^3=(y)^3+{}_3C_1(y)^2(-3x)$

$+{}_3C_2(y)(-3x)^2+(-3x)^3$

$=\underline{\underline{y^3-9y^2x+27yx^2-27x^3}}$

2. (a) The general term in the expansion

$={}_7C_r(-2x)^r$

$\therefore$ The term in $x^3={}_7C_3(-2x)^3$

$\therefore$ The coefficient of $x^3={}_7C_3(-2)^3$

$=\underline{\underline{-280}}$

(b) The general term in the expansion

$={}_6C_r(x^2)^{6-r}(2)^r$

$\therefore$ The term in $x^6={}_6C_3(x^2)^3(2)^3$

$\therefore$ The coefficient of $x^6=\underline{\underline{160}}$

(c) The general term in the expansion

$={}_6C_r(3x)^{6-r}(\frac{1}{x})^r$

$={}_6C_r\cdot3^{6-r}x^{6-2r}$

It is the constant term when

$6-2r=0$, $\therefore$ $r=3$

$\therefore$ The constant term $={}_6C_3\cdot3^3=\underline{\underline{540}}$

(d) The general term in the expansion

$={}_9C_r(x^2)^{9-r}(-\frac{2}{x})^r$

$={}_9C_r(x^{18-2r})(-\frac{2}{x})^r$

$={}_9C_r(-2)^rx^{18-3r}$

It is the constant term when

$18-3r=0$. $\therefore$ $r=6$

$\therefore$ The constant term $={}_9C_6(-2)^6=\underline{\underline{5\ 376}}$

Classwork 3 (p.90)

1. (a) $(x^2-1)^9$

$=(-1+x^2)^9$

$=(-1)^9+9(-1)^8(x^2)+36(-1)^7(x^2)^2+\cdots$

$=\underline{\underline{-1+9x^2-36x^4+\cdots}}$

(b) $(2+\frac{1}{x})^4$

$=(2)^4+4(2)^3(\frac{1}{x})+6(2)^2(\frac{1}{x})^2$

$+4(2)(\frac{1}{x})^3+(\frac{1}{x})^4$

$=\underline{\underline{16+\frac{32}{x}+\frac{24}{x^2}+\frac{8}{x^3}+\frac{1}{x^4}}}$

(c) $(x^2-1)^9(2+\frac{1}{x})^4$

$=(-1+9x^2-36x^4+\cdots)$

$(16+\frac{32}{x}+\frac{24}{x^2}+\frac{8}{x^3}+\frac{1}{x^4})$

$\therefore$ The constant term in the expansion

$=(-1)(16)+9(24)+(-36)(1)$

$=\underline{\underline{164}}$

2. $(3x^2+1)^n = (1+3x^2)^n$
$= 1 + {}_nC_1(3x^2) + {}_nC_2(3x^2)^2 + {}_nC_3(3x^2)^3 + \cdots$

Compare coefficient to $a + bx^2 + cx^4 + dx^6 + \cdots$

$b = 3 \cdot {}_nC_1$, $d = 27 \cdot {}_nC_3$

$\because \quad d = 108b$

$27 \cdot {}_nC_3 = 108(3 \cdot {}_nC_1)$

$27 \cdot \dfrac{n!}{3!(n-3)!} = 324 \cdot \dfrac{n!}{(n-1)!}$

$\dfrac{9}{2}(n-1)(n-2) = 324$

$n^2 - 3n + 2 = 72$

$n^2 - 3n - 70 = 0$

$(n+7)(n-10) = 0$

$n = \underline{\underline{10}}$ or -7 (rejected)

$\therefore$ Comparing the coefficients of x,

${}_nC_1 p = -7$

$np = -7$

$p = \dfrac{-7}{n}$ ....(1)

Comparing the coefficients of x^2,

$2 \cdot {}_nC_1 + p^2 \cdot {}_nC_2 = 35$

$2n + p^2 \cdot \dfrac{1}{2}n(n-1) = 35$ ....(2)

Put (1) into (2),

$2n + (\dfrac{-7}{n})^2 \dfrac{1}{2}n(n-1) = 35$

$2n + \dfrac{49}{2} \cdot \dfrac{n-1}{n} = 35$

$4n^2 + 49n - 49 = 70n$

$4n^2 - 21n - 49 = 0$

$(n-7)(4n+7) = 0$

$n = \underline{\underline{7}}$ or $-\dfrac{7}{4}$ (rejected)

Put $n = 7$ into (1), $p = -\dfrac{7}{7} = \underline{\underline{-1}}$

Classwork 4 (p.92)

1. $(1 + x - 3x^2)^6$
$= [1 + x(1-3x)]^6$
$= 1 + 6x(1-3x) + 15x^2(1-3x)^2 + 20x^3(1-3x)^3 + \cdots$
$= 1 + 6x - 18x^2 + 15x^2(1 - 6x + \cdots) + 20x^3(1 + \cdots) + \cdots$
$= 1 + 6x - 18x^2 + 15x^2 - 90x^3 + 20x^3 + \cdots$
$= \underline{\underline{1 + 6x - 3x^2 - 70x^3 + \cdots}}$

2. $(1 + x - 3x^2)^6(1 - \dfrac{1}{x})^3$
$= (1 + 6x - 3x^2 - 70x^3 + \cdots)[1 + 3(-\dfrac{1}{x}) + 3(-\dfrac{1}{x})^2 + (-\dfrac{1}{x})^3]$
$= (1 + 6x - 3x^2 - 70x^3 + \cdots)(1 - \dfrac{3}{x} + \dfrac{3}{x^2} - \dfrac{1}{x^3})$

$\therefore$ The constant term $= 1 + 6(-3) - 3(3) - 70(-1)$
$= \underline{\underline{44}}$

Classwork 5 (p.93)

$(1 + px + 2x^2)^n$
$= [1 + x(p+2x)]^n$
$= 1 + {}_nC_1 x(p+2x) + {}_nC_2 x^2(p+2x)^2 + \cdots$
$= 1 + {}_nC_1 px + 2\,{}_nC_1 x^2 + {}_nC_2 x^2(p^2 + \cdots) + \cdots$
$= 1 + {}_nC_1 px + (2 \cdot {}_nC_1 + p^2 \cdot {}_nC_2)x^2 + \cdots$

CHAPTER 5

Exercise 5A (p.105)

1. (a) $36.9° = 36.9(\dfrac{\pi}{180})$
$= \underline{\underline{0.644}}$ (corr. to 3 sig. fig.)

(b) $132.5° = 132.5(\dfrac{\pi}{180})$
$= \underline{\underline{2.31}}$ (corr. to 3 sig. fig.)

(c) $214° = 214(\dfrac{\pi}{180})$
$= \underline{\underline{3.74}}$ (corr. to 3 sig. fig.)

(d) $316.3° = 316.3(\dfrac{\pi}{180})$
$= \underline{\underline{5.52}}$ (corr. to 3 sig. fig.)

2. (a) $45° = 45(\dfrac{\pi}{180}) = \underline{\underline{\dfrac{\pi}{4}}}$

(b) $90° = 90(\dfrac{\pi}{180}) = \underline{\underline{\dfrac{\pi}{2}}}$

(c) $210° = 210(\dfrac{\pi}{180}) = \underline{\underline{\dfrac{7\pi}{6}}}$

(d) $300° = 300(\dfrac{\pi}{180}) = \underline{\underline{\dfrac{5\pi}{3}}}$

3. (a) $0.21^c = 0.21(\dfrac{180°}{\pi})$
$= \underline{\underline{12.03°}}$ (corr. to 2 d.p.)

(b) $0.546^c = 0.546(\dfrac{180°}{\pi})$
$= \underline{\underline{31.28°}}$ (corr. to 2 d.p.)

(c) $\dfrac{\pi}{8} = \dfrac{\pi}{8}(\dfrac{180°}{\pi})$
$= \underline{\underline{22.5°}}$

(d) $\dfrac{5\pi}{12} = \dfrac{5\pi}{12}(\dfrac{180°}{\pi})$
$= \underline{\underline{75°}}$

4. (a) $\sin\dfrac{\pi}{4} = \underline{\underline{\dfrac{\sqrt{2}}{2}}}$

(b) $\cos\dfrac{\pi}{3} = \underline{\underline{\dfrac{1}{2}}}$

(c) $\sin\dfrac{\pi}{6} = \underline{\underline{\dfrac{1}{2}}}$

(d) $\tan\dfrac{\pi}{4} = \underline{\underline{1}}$

(e) $\cos\dfrac{\pi}{2} = \underline{\underline{0}}$

(f) $\tan\dfrac{\pi}{3} = \underline{\underline{\sqrt{3}}}$

5. (a) $a = \pi - \dfrac{2\pi}{9} = \underline{\underline{\dfrac{7\pi}{9}}}$

(b) $b = \pi - \dfrac{5\pi}{6} = \underline{\underline{\dfrac{\pi}{6}}}$

(c) $c = \dfrac{\pi}{2} - \dfrac{\pi}{8} = \underline{\underline{\dfrac{3\pi}{8}}}$

(d) $d = \dfrac{\pi}{2} - \dfrac{\pi}{3} = \underline{\underline{\dfrac{\pi}{6}}}$

6. $\angle R = \pi - \dfrac{\pi}{3} - \dfrac{\pi}{6} = \underline{\underline{\dfrac{\pi}{2}}}$

7. (a) $33\dfrac{1}{3}$ rev./min. $= \dfrac{100}{3} \cdot \dfrac{2\pi}{60\text{s}}$ rad.
$= \underline{\underline{\dfrac{10\pi}{9}\text{ rad./s}}}$

(b) Time $= \dfrac{\text{angle}}{\text{angular speed}}$
$= 45\pi \times \dfrac{9}{10\pi}$ s
$= \underline{\underline{40.5\text{ s}}}$

8. $30° = 30(\dfrac{\pi}{180})$
$= \dfrac{\pi}{6}$

Distance travelled by the train = arc length
$= (450\text{ m})(\dfrac{\pi}{6})$
$= 75\pi$ m

Time $= \dfrac{\text{distance}}{\text{speed}}$
$= \dfrac{75\pi}{24}$ s
$= \underline{\underline{9.82\text{ s}}}$ (corr. to 3 sig. fig.)

9. $24° = 24(\dfrac{\pi}{180})$
$= \dfrac{2\pi}{15}$

Length of arc $AB = (6.8\text{ cm})(\dfrac{2\pi}{15})$
$= \underline{\underline{2.85\text{ cm}}}$ (corr. to 3 sig. fig.)

Area of sector $OAB = \dfrac{1}{2}(6.8)^2(\dfrac{2\pi}{15})\text{ cm}^2$
$= \underline{\underline{9.68\text{ cm}^2}}$ (corr. to 3 sig. fig.)

10. $120° = 120(\dfrac{\pi}{180})$
$= \dfrac{2\pi}{3}$