2. $(3x^2+1)^n = (1+3x^2)^n$

$= 1 + {}_nC_1(3x^2) + {}_nC_2(3x^2)^2 + {}_nC_3(3x^2)^3 + \cdots$

Compare coefficient to $a + bx^2 + cx^4 + dx^6 + \cdots$

$b = 3 \cdot {}_nC_1,\ d = 27 \cdot {}_nC_3$

$\because\ d = 108b$

$27 \cdot {}_nC_3 = 108(3 \cdot {}_nC_1)$

$27 \cdot \frac{n!}{3!(n-3)!} = 324 \cdot \frac{n!}{(n-1)!}$

$\frac{9}{2}(n-1)(n-2) = 324$

$n^2 - 3n + 2 = 72$

$n^2 - 3n - 70 = 0$

$(n+7)(n-10) = 0$

$n = \underline{\underline{10}}$ or -7 (rejected)

Classwork 4 (p.92)

1. $(1+x-3x^2)^6$

$= [1 + x(1-3x)]^6$

$= 1 + 6x(1-3x) + 15x^2(1-3x)^2 + 20x^3(1-3x)^3 + \cdots$

$= 1 + 6x - 18x^2 + 15x^2(1 - 6x + \cdots) + 20x^3(1 + \cdots) + \cdots$

$= 1 + 6x - 18x^2 + 15x^2 - 90x^3 + 20x^3 + \cdots$

$= \underline{\underline{1 + 6x - 3x^2 - 70x^3 + \cdots}}$

2. $(1+x-3x^2)^6(1-\frac{1}{x})^3$

$= (1 + 6x - 3x^2 - 70x^3 + \cdots)[1 + 3(-\frac{1}{x}) + 3(-\frac{1}{x})^2 + (-\frac{1}{x})^3]$

$= (1 + 6x - 3x^2 - 70x^3 + \cdots)(1 - \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3})$

$\therefore$ The constant term $= 1 + 6(-3) - 3(3) - 70(-1)$

$= \underline{\underline{44}}$

Classwork 5 (p.93)

$(1 + px + 2x^2)^n$

$= [1 + x(p+2x)]^n$

$= 1 + {}_nC_1x(p+2x) + {}_nC_2x^2(p+2x)^2 + \cdots$

$= 1 + {}_nC_1px + 2{}_nC_1x^2 + {}_nC_2x^2(p^2 + \cdots) + \cdots$

$= 1 + {}_nC_1px + (2 \cdot {}_nC_1 + p^2 \cdot {}_nC_2)x^2 + \cdots$

$\therefore$ Comparing the coefficients of x,

${}_nC_1p = -7$

$np = -7$

$p = \frac{-7}{n}$ ....(1)

Comparing the coefficients of x^2,

$2 \cdot {}_nC_1 + p^2 \cdot {}_nC_2 = 35$

$2n + p^2 \cdot \frac{1}{2}n(n-1) = 35$ ....(2)

Put (1) into (2).

$2n + (\frac{-7}{n})^2 \frac{1}{2}n(n-1) = 35$

$2n + \frac{49}{2} \cdot \frac{n-1}{n} = 35$

$4n^2 + 49n - 49 = 70n$

$4n^2 - 21n - 49 = 0$

$(n-7)(4n+7) = 0$

$n = \underline{\underline{7}}$ or $-\frac{7}{4}$ (rejected)

Put $n = 7$ into (1), $p = -\frac{7}{7} = \underline{\underline{-1}}$

CHAPTER 5

Exercise 5A (p.105)

1. (a) $36.9° = 36.9(\frac{\pi}{180})$

$= \underline{\underline{0.644}}$ (corr. to 3 sig. fig.)

(b) $132.5° = 132.5(\frac{\pi}{180})$

$= \underline{\underline{2.31}}$ (corr. to 3 sig. fig.)

(c) $214° = 214(\frac{\pi}{180})$

$= \underline{\underline{3.74}}$ (corr. to 3 sig. fig.)

(d) $316.3° = 316.3(\frac{\pi}{180})$

$= \underline{\underline{5.52}}$ (corr. to 3 sig. fig.)

2. (a) $45° = 45(\frac{\pi}{180}) = \underline{\underline{\frac{\pi}{4}}}$

(b) $90° = 90(\frac{\pi}{180}) = \underline{\underline{\frac{\pi}{2}}}$

(c) $210° = 210(\frac{\pi}{180}) = \underline{\underline{\frac{7\pi}{6}}}$

(d) $300° = 300(\frac{\pi}{180}) = \underline{\underline{\frac{5\pi}{3}}}$

3. (a) $0.21^c = 0.21(\frac{180°}{\pi})$

$= \underline{\underline{12.03°}}$ (corr. to 2 d.p.)

(b) $0.546^c = 0.546(\frac{180°}{\pi})$

$= \underline{\underline{31.28°}}$ (corr. to 2 d.p.)

(c) $\frac{\pi}{8} = \frac{\pi}{8}(\frac{180°}{\pi})$

$= \underline{\underline{22.5°}}$

(d) $\frac{5\pi}{12} = \frac{5\pi}{12}(\frac{180°}{\pi})$

$= \underline{\underline{75°}}$

4. (a) $\sin\frac{\pi}{4} = \underline{\underline{\frac{\sqrt{2}}{2}}}$

(b) $\cos\frac{\pi}{3} = \underline{\underline{\frac{1}{2}}}$

(c) $\sin\frac{\pi}{6} = \underline{\underline{\frac{1}{2}}}$

(d) $\tan\frac{\pi}{4} = \underline{\underline{1}}$

(e) $\cos\frac{\pi}{2} = \underline{\underline{0}}$

(f) $\tan\frac{\pi}{3} = \underline{\underline{\sqrt{3}}}$

5. (a) $a = \pi - \frac{2\pi}{9} = \underline{\underline{\frac{7\pi}{9}}}$

(b) $b = \pi - \frac{5\pi}{6} = \underline{\underline{\frac{\pi}{6}}}$

(c) $c = \frac{\pi}{2} - \frac{\pi}{8} = \underline{\underline{\frac{3\pi}{8}}}$

(d) $d = \frac{\pi}{2} - \frac{\pi}{3} = \underline{\underline{\frac{\pi}{6}}}$

6. $\angle R = \pi - \frac{\pi}{3} - \frac{\pi}{6} = \underline{\underline{\frac{\pi}{2}}}$

7. (a) $33\frac{1}{3}$ rev/min $= \frac{100}{3} \cdot \frac{2\pi \text{ rad}}{60\text{s}}$

$= \underline{\underline{\frac{10\pi}{9} \text{ rad/s}}}$

(b) Time $= \frac{\text{angle}}{\text{angular speed}}$

$= 45\pi \times \frac{9}{10\pi}$ s

$= \underline{\underline{40.5 \text{ s}}}$

8. $30° = 30(\frac{\pi}{180})$

$= \frac{\pi}{6}$

Distance travelled by the train = arc length

$= (450 \text{ m})(\frac{\pi}{6})$

$= 75\pi$ m

Time $= \frac{\text{distance}}{\text{speed}}$

$= \frac{75\pi}{24}$ s

$= \underline{\underline{9.82 \text{ s}}}$ (corr. to 3 sig. fig.)

9. $24° = 24(\frac{\pi}{180})$

$= \frac{2\pi}{15}$

Length of arc $AB = (6.8 \text{ cm})(\frac{2\pi}{15})$

$= \underline{\underline{2.85 \text{ cm}}}$ (corr. to 3 sig. fig.)

Area of sector $OAB = \frac{1}{2}(6.8)^2(\frac{2\pi}{15})$ cm^2

$= \underline{\underline{9.68 \text{ cm}^2}}$ (corr. to 3 sig. fig.)

10. $120° = 120(\frac{\pi}{180})$

$= \frac{2\pi}{3}$

(a) Radius $= \dfrac{42.6}{(\frac{2\pi}{3})}$ cm

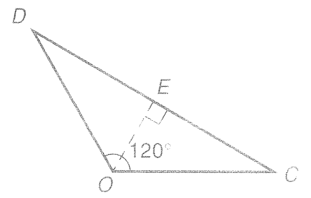
$= \underline{\underline{20.3 \text{ cm}}}$ (corr. to 3 sig. fig.)

(b) Area of sector OCD

$= \dfrac{1}{2}(\dfrac{42.6}{\frac{2\pi}{3}})^2(\dfrac{2\pi}{3})$ cm^2

$= \underline{\underline{433 \text{ cm}^2}}$ (corr. to 3 sig. fig.)

(c)



$\angle EOC = \angle EOD$

$= 60°$

$EC = OC \sin 60°$

$= 20.3 \sin 60°$ cm

$CD = EC + DE$

$= 2EC$

$= 2 \times (20.3 \sin 60°)$ cm

$= \underline{\underline{35.2 \text{ cm}}}$ (corr. to 3 sig. fig.)

11. (a) (i) Area of $\Delta OAB = \underline{\underline{\dfrac{1}{2}r^2 \sin\theta}}$

(ii) Area of sector $OAB = \underline{\underline{\dfrac{1}{2}r^2\theta}}$

(iii) $BC = r\tan\theta$

Area of $\Delta OBC = \dfrac{1}{2}r(r\tan\theta)$

$= \underline{\underline{\dfrac{1}{2}r^2 \tan\theta}}$

(b) Area of $\Delta OAB <$ Area of sector $OAB <$ Area of ΔOBC

$\dfrac{1}{2}r^2 \sin\theta < \dfrac{1}{2}r^2\theta < \dfrac{1}{2}r^2 \tan\theta$

$\underline{\underline{\sin\theta < \theta < \tan\theta}}$

12. (a) $x + x + x\theta = 10$

$\theta = \dfrac{10 - 2x}{x}$

$= \underline{\underline{\dfrac{10}{x} - 2}}$

(b) $A = \dfrac{1}{2}x^2\theta$

$= \dfrac{1}{2}x^2(\dfrac{10}{x} - 2)$

$= 5x - x^2$

(c) $A = 5x - x^2$

$= -(x^2 - 5x)$

$= -[x^2 - 5x + (\frac{5}{2})^2 - (\frac{5}{2})^2]$

$= -(x - \frac{5}{2})^2 + \dfrac{25}{4}$

$(x - \frac{5}{2})^2 \geq 0$

$-(x - \frac{5}{2})^2 \leq 0$

$-(x - \frac{5}{2})^2 + \dfrac{25}{4} \leq 0 + \dfrac{25}{4}$

$A \leq \dfrac{25}{4}$

$\therefore$ The greatest value of A is $\underline{\underline{\dfrac{25}{4}}}$.

13. $OD = OE = a$

Area of $\Delta ABC = \dfrac{1}{2}(2a)^2 \sin\dfrac{\pi}{3} = \sqrt{3}a^2$

Area of $\Delta ADO = \dfrac{1}{2}a^2 \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{4}a^2$

Area of $\Delta BOE =$ Area of $\Delta ADO = \dfrac{\sqrt{3}}{4}a^2$

Area of sector $DOE = \dfrac{1}{2}a^2(\dfrac{\pi}{3}) = \dfrac{\pi}{6}a^2$

$\therefore$ Area of the shaded region

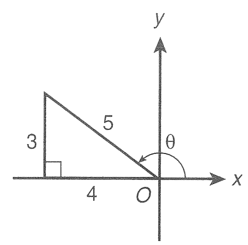
$= \sqrt{3}a^2 - 2(\dfrac{\sqrt{3}}{4})a^2 - \dfrac{\pi}{6}a^2$

$= \dfrac{\sqrt{3}}{2}a^2 - \dfrac{\pi}{6}a^2$

$= \underline{\underline{\dfrac{a^2}{6}(3\sqrt{3} - \pi)}}$

Exercise 5B (p.115)

1.



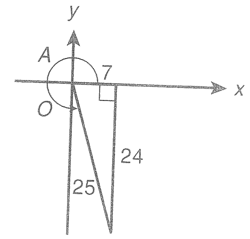
$\sin\theta = \underline{\underline{\dfrac{3}{5}}}$, $\cos\theta = \underline{\underline{-\dfrac{4}{5}}}$, $\tan\theta = \underline{\underline{-\dfrac{3}{4}}}$,

$\sec\theta = \underline{\underline{-\dfrac{5}{4}}}$, $\csc\theta = \underline{\underline{\dfrac{5}{3}}}$

2. $\csc A + \cot A = -\dfrac{25}{24} - \dfrac{7}{24}$

$= -\dfrac{32}{24}$

$= \underline{\underline{-\dfrac{4}{3}}}$



3. (a) $\sin 120° = \sin(180° - 60°) = \sin 60° = \underline{\underline{\dfrac{\sqrt{3}}{2}}}$

(b) $\cos 210° = \cos(180° + 30°) = -\cos 30° = \underline{\underline{-\dfrac{\sqrt{3}}{2}}}$

(c) $\cos 300° = \cos(360° - 60°) = \cos 60° = \underline{\underline{\dfrac{1}{2}}}$

(d) $\tan 120° = \tan(180° - 60°) = -\tan 60° = \underline{\underline{-\sqrt{3}}}$

(e) $\tan 315° = \tan(360° - 45°) = -\tan 45° = \underline{\underline{-1}}$

4. (a) $\cos\dfrac{2\pi}{3} = \cos(\pi - \dfrac{\pi}{3}) = -\cos\dfrac{\pi}{3} = \underline{\underline{-\dfrac{1}{2}}}$

(b) $\tan\dfrac{5\pi}{4} = \tan(\pi + \dfrac{\pi}{4}) = \tan\dfrac{\pi}{4} = \underline{\underline{1}}$

(c) $\sin\dfrac{3\pi}{2} = \underline{\underline{-1}}$

(d) $\cos\pi = \underline{\underline{-1}}$

(e) $\tan 2\pi = \underline{\underline{0}}$

5. (a) $\cos(-120°)$

$= \cos 120° = \cos(180° - 60°) = -\cos 60° = \underline{\underline{-\dfrac{1}{2}}}$

(b) $\sin(-315°) = -\sin 315°$

$= -\sin(360° - 45°)$

$= -(-\sin 45°)$

$= \underline{\underline{\dfrac{\sqrt{2}}{2}}}$

(c) $\tan(-\dfrac{3\pi}{4}) = -\tan\dfrac{3\pi}{4}$

$= -\tan(\pi - \dfrac{\pi}{4})$

$= -(-\tan\dfrac{\pi}{4})$

$= \underline{\underline{1}}$

(d) $\sec(-\dfrac{\pi}{3}) = \sec\dfrac{\pi}{3} = \underline{\underline{2}}$

(e) $\sin(-210°) = -\sin 210°$

$= -\sin(180° + 30°)$

$= -(-\sin 30°)$

$= \underline{\underline{\dfrac{1}{2}}}$

6. $\dfrac{\cos(360° - A)\sin(90° - A)\tan(A - 180°)}{\sin(-A)\sin(180° + A)\cot(-A)}$

$= \dfrac{\cos A \cos A \tan A}{(-\sin A)(-\sin A)(-\cot A)}$

$= \underline{\underline{-1}}$

7. $\dfrac{\tan(\frac{3\pi}{2} - A) + \cot(A + \pi)}{1 + \tan(A - \frac{\pi}{2})\cot(\pi - A)} = \dfrac{\cot A + \cot A}{1 + (-\cot A)(-\cot A)}$

$= \dfrac{2\cot A}{1 + \cot^2 A} = \dfrac{2\cot A}{\csc^2 A}$

$= \dfrac{2\cos A}{\sin A} \cdot \dfrac{\sin^2 A}{1}$

$= \underline{\underline{2\sin A \cos A}}$

8. $\dfrac{\sin(A + C)\cos(B + C) - \cos A \sin B}{\cos(B + C)\cos B + \cos A \cos(A + C)}$

$= \dfrac{\sin(180° - B)\cos(180° - A) - \cos A \sin B}{\cos(180° - A)\cos B + \cos A \cos(180° - B)}$

$= \dfrac{\sin B(-\cos A) - \cos A \sin B}{(-\cos A)\cos B + \cos A(-\cos B)}$

$= \dfrac{-2\cos A \sin B}{-2\cos A \cos B}$

$= \underline{\underline{\tan B}}$

9. $\dfrac{1 - \cos A}{\sin A} = \dfrac{1 - \cos A}{\sin A} \cdot \dfrac{1 + \cos A}{1 + \cos A}$

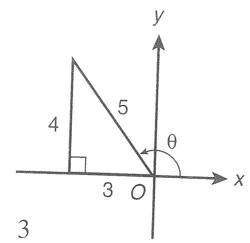
$= \dfrac{1 - \cos^2 A}{\sin A(1 + \cos A)}$

$= \dfrac{\sin^2 A}{\sin A(1 + \cos A)}$

$= \dfrac{\sin A}{1 + \cos A}$

10. $\dfrac{\tan A}{1 + \tan^2 A} = \dfrac{\tan A}{\sec^2 A} = \dfrac{\sin A}{\cos A}\cos^2 A = \sin A \cos A$

11. $\cos\theta = -\dfrac{3}{5}$ (quadrant II)



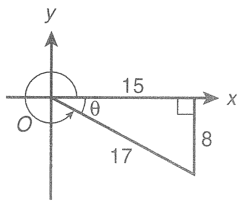
(a) $\sin\theta = \underline{\underline{\dfrac{4}{5}}}$

(b) $\tan\theta = \underline{\underline{-\dfrac{4}{3}}}$

(c) $\sin(270° - \theta) = -\cos\theta = \underline{\underline{\dfrac{3}{5}}}$

(d) $\tan(90° + \theta) = -\cot\theta = \underline{\underline{\dfrac{3}{4}}}$

12. $\tan\theta = -\frac{8}{15}$ (quadrant IV)



$\therefore\ \sin\theta = -\frac{8}{17}$,

$\cos\theta = \frac{15}{17}$

$$\frac{2\sin(\pi+\theta)-\sin(\frac{\pi}{2}-\theta)}{\sec(2\pi-\theta)+2\tan(\pi+\theta)} = \frac{-2\sin\theta-\cos\theta}{\sec\theta+2\tan\theta} = \frac{-2(-\frac{8}{17})-\frac{15}{17}}{\frac{17}{15}+2(-\frac{8}{15})} = \frac{1}{17}\cdot\frac{15}{1} = \underline{\underline{\frac{15}{17}}}$$

13. $\sin\theta(\tan\theta+\cot\theta)$
$= \sin\theta(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta})$
$= \sin\theta(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta})$
$= \frac{1}{\cos\theta}$
$= \sec\theta$

14. $\frac{1-\sec x}{1+\sec x}+\frac{1-\cos x}{1+\cos x}$
$= \frac{1-\frac{1}{\cos x}}{1+\frac{1}{\cos x}}+\frac{1-\cos x}{1+\cos x}$
$= \frac{\cos x-1}{\cos x+1}+\frac{1-\cos x}{1+\cos x}$
$= 0$

Exercise 5C (p.119)

1. $4\sin^2 x = 1$
$4\sin^2 x - 1 = 0$
$(2\sin x+1)(2\sin x-1) = 0$
$\sin x = -\frac{1}{2}$ or $\sin x = \frac{1}{2}$
$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$
$\therefore\ x = \underline{\underline{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}}$

2. $2\cos^2\theta = 1+\sin\theta$
$2(1-\sin^2\theta) = 1+\sin\theta$
$2\sin^2\theta+\sin\theta-1 = 0$
$(\sin\theta+1)(2\sin\theta-1) = 0$
$\sin\theta = -1$ or $\sin\theta = \frac{1}{2}$
$\theta = \frac{3\pi}{2}$ or $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
$\therefore\ \theta = \underline{\underline{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}}$

3. $\tan\theta+2\cot\theta = 3$
$\tan^2\theta+2 = 3\tan\theta$ $(\tan\theta \neq 0)$
$\tan^2\theta-3\tan\theta+2 = 0$
$(\tan\theta-1)(\tan\theta-2) = 0$
$\tan\theta = 1$ or $\tan\theta = 2$
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ or $\theta = 1.11, 4.25$ (corr. to 2 d.p.)
$\therefore\ \theta = \underline{\underline{\frac{\pi}{4}, 1.11, \frac{5\pi}{4}, 4.25}}$

4. $2\sec^2\theta = 5\tan\theta$
$2(1+\tan^2\theta) = 5\tan\theta$
$2\tan^2\theta-5\tan\theta+2 = 0$
$(2\tan\theta-1)(\tan\theta-2) = 0$
$\tan\theta = \frac{1}{2}$ or $\tan\theta = 2$
$\theta = 0.46, 3.61$ or $\theta = 1.11, 4.25$ (corr. to 2 d.p.)
$\therefore\ \theta = \underline{\underline{0.46, 1.11, 3.61, 4.25}}$

5. $\sin^2\theta+2\sin\theta\cos\theta = 3\cos^2\theta$
$\tan^2\theta+2\tan\theta-3 = 0$
$(\tan\theta-1)(\tan\theta+3) = 0$
$\tan\theta = 1$ or $\tan\theta = -3$
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ or $\theta = 1.89, 5.05$ (corr. to 2 d.p.)
$\therefore\ \theta = \underline{\underline{\frac{\pi}{4}, 1.89, \frac{5\pi}{4}, 5.03}}$

6. $7\sin\theta\cos\theta+\cos^2\theta = 2$
$7\tan\theta+1 = 2\sec^2\theta$
$7\tan\theta+1 = 2(1+\tan^2\theta)$
$2\tan^2\theta-7\tan\theta+1 = 0$
$\tan\theta = \frac{7\pm\sqrt{7^2-4(2)}}{4} = \frac{7\pm\sqrt{41}}{4}$
$\tan\theta = \frac{7+\sqrt{41}}{4}$ or $\tan\theta = \frac{7-\sqrt{41}}{4}$
$\theta = 1.28, 4.42$ (corr. to 2 d.p.) or $\theta = 0.15, 3.29$ (corr. to 2 d.p.)
$\therefore\ \theta = \underline{\underline{0.15, 1.28, 3.29, 4.42}}$

7. $\sin\theta-\sqrt{3}\cos\theta = \sqrt{2}$
$\sin\theta-\sqrt{2} = \sqrt{3}\cos\theta$
$\sin^2\theta-2\sqrt{2}\sin\theta+2 = 3\cos^2\theta$
$4\sin^2\theta-2\sqrt{2}\sin\theta-1 = 0$
$\sin\theta = -\frac{2\sqrt{2}-\sqrt{24}}{8}$ or $\sin\theta = -\frac{2\sqrt{2}+\sqrt{24}}{8}$
$\theta = 3.40, 6.02$ (corr. to 2 d.p.) or $\theta = 1.31, 1.83$ (corr. to 2 d.p.)

Check:

(i) When $\theta = 3.40$,
L.H.S. $= \sin 3.40-\sqrt{3}\cos 3.40 = \sqrt{2}$ = R.H.S.
$\therefore$ 3.40 is a solution.

(ii) When $\theta = 6.02$,
L.H.S. $= \sin 6.02-\sqrt{3}\cos 6.02 = -1.93 \neq$ R.H.S.
$\therefore$ 6.02 is not a solution.

(iii) When $\theta = 1.31$,
L.H.S. $= \sin 1.31-\sqrt{3}\cos 1.31 = 0.5 \neq$ R.H.S.
$\therefore$ 1.31 is not a solution.

(iv) When $\theta = 1.83$,
L.H.S. $= \sin 1.83-\sqrt{3}\cos 1.83 = \sqrt{2}$ = R.H.S.
$\therefore$ 1.83 is a solution.

$\theta = \underline{\underline{1.83, 3.40}}$

8. $2(\sin\theta+\cos\theta) = \sqrt{3}+1$
$4(\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta) = 3+2\sqrt{3}+1$
$4(1+2\sin\theta\cos\theta) = 4+2\sqrt{3}$
$1+2\sin\theta\cos\theta = 1+\frac{\sqrt{3}}{2}$
$2\sin\theta\cos\theta = \frac{\sqrt{3}}{2}$
$4\tan\theta = \sqrt{3}\sec^2\theta$
$4\tan\theta = \sqrt{3}(1+\tan^2\theta)$
$\sqrt{3}\tan^2\theta-4\tan\theta+\sqrt{3} = 0$
$(\sqrt{3}\tan\theta-1)(\tan\theta-\sqrt{3}) = 0$
$\tan\theta = \frac{1}{\sqrt{3}}$ or $\tan\theta = \sqrt{3}$
$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$ or $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

Check:

(i) When $\theta = \frac{\pi}{6}$,
L.H.S. $= 2(\frac{1}{2}+\frac{\sqrt{3}}{2}) = 1+\sqrt{3}$ = R.H.S.
$\therefore\ \frac{\pi}{6}$ is a solution.

(ii) When $\theta = \frac{7\pi}{6}$,
L.H.S. $= 2(-\frac{1}{2}-\frac{\sqrt{3}}{2}) = -1-\sqrt{3} \neq$ R.H.S.
$\therefore\ \frac{7\pi}{6}$ is not a solution.

(iii) When $\theta = \frac{\pi}{3}$,
L.H.S. $= 2(\frac{\sqrt{3}}{2}+\frac{1}{2}) = 1+\sqrt{3}$ = R.H.S.
$\therefore\ \frac{\pi}{3}$ is a solution.

(iv) When $\theta = \frac{4\pi}{3}$,
L.H.S. $= 2(\frac{-\sqrt{3}}{2}-\frac{1}{2}) = -\sqrt{3}-1 \neq$ R.H.S.
$\therefore\ \frac{4\pi}{3}$ is not a solution.

$\theta = \underline{\underline{\frac{\pi}{6}, \frac{\pi}{3}}}$

9. $6\tan^2\theta-4\sin^2\theta = 1$
$6\sin^2\theta-4\sin^2\theta\cos^2\theta-\cos^2\theta = 0$
$6(1-\cos^2\theta)-4(1-\cos^2\theta)\cos^2\theta-\cos^2\theta = 0$
$4\cos^4\theta-11\cos^2\theta+6 = 0$
$(4\cos^2\theta-3)(\cos^2\theta-2) = 0$
$\cos^2\theta = \frac{3}{4}$ or $\cos^2\theta = 2$ (rejected)
$\cos\theta = \pm\frac{\sqrt{3}}{2}$
$\theta = \underline{\underline{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}}$

10. $\tan^2\theta = 2+4\cos^2\theta$
$\sin^2\theta = 2\cos^2\theta+4\cos^4\theta$
$4\cos^4\theta+3\cos^2\theta-1 = 0$
$(4\cos^2\theta-1)(\cos^2\theta+1) = 0$
$\cos^2\theta = \frac{1}{4}$ or $\cos^2\theta = -1$ (rejected)
$\cos\theta = \pm\frac{1}{2}$
$\theta = \underline{\underline{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}}$

Exercise 5D (p.126)

1. (a) $\sin 450° = \sin(360°+90°)$
$= \sin 90°$
$= \underline{\underline{1}}$

(b) $\cot\frac{43}{4}\pi = \cot(10\pi + \frac{3\pi}{4})$
$= \cot\frac{3\pi}{4}$
$= -\cot\frac{\pi}{4}$
$= \underline{\underline{-1}}$

(c) $\sec 1\,110^\circ = \sec(3 \times 360^\circ + 30^\circ)$
$= \sec 30^\circ$
$= \underline{\underline{\frac{2\sqrt{3}}{3}}}$

(d) $\csc 480^\circ = \csc(360^\circ + 120^\circ)$
$= \csc 120^\circ$
$= \csc 60^\circ$
$= \underline{\underline{\frac{2\sqrt{3}}{3}}}$

(e) $\tan\frac{26\pi}{3} = \tan(8\pi + \frac{2\pi}{3})$
$= \tan\frac{2\pi}{3}$
$= -\tan\frac{\pi}{3}$
$= \underline{\underline{-\sqrt{3}}}$

2. (a) $\cos(360k^\circ \pm 120^\circ)$
$= \cos(360k^\circ + 120^\circ)$ or $\cos(360k^\circ - 120^\circ)$
$= \cos 120^\circ$ or $\cos(-120^\circ)$
$= -\cos 60^\circ$ or $\cos 120^\circ$
$= \underline{\underline{-\frac{1}{2}}}$

(b) $\tan(k\pi + \frac{1}{4}\pi) = \tan\frac{\pi}{4} = \underline{\underline{1}}$

(c) $\sin[180k^\circ + (-1)^k 45^\circ]$
If k is even, let $k = 2m$, then
$\sin[180k^\circ + (-1)^k 45^\circ]$
$= \sin[180(2m)^\circ + (-1)^{2m} 45^\circ]$
$= \sin[180(2m)^\circ + 45^\circ]$
$= \sin 45^\circ$
$= \frac{\sqrt{2}}{2}$
If k is odd, let $k = 2m + 1$, then
$\sin[180k^\circ + (-1)^k 45^\circ]$
$= \sin[180(2m+1)^\circ + (-1)^{2m+1} 45^\circ]$
$= \sin[360m^\circ + (180^\circ - 45^\circ)]$
$= \sin(180^\circ - 45^\circ)$
$= \sin 45^\circ$
$= \frac{\sqrt{2}}{2}$

Combining the above results,
$\sin[180k^\circ + (-1)^k 45^\circ] = \underline{\underline{\frac{\sqrt{2}}{2}}}$

(d) $\sec(2k\pi + \frac{\pi}{6}) = \sec\frac{\pi}{6} = \underline{\underline{\frac{2\sqrt{3}}{3}}}$

(e) $\csc[k\pi + (-1)^k \frac{5\pi}{6}]$
If k is even, let $k = 2m$, then
$\csc[2m\pi + (-1)^{2m}\frac{5\pi}{6}] = \csc\frac{5\pi}{6} = \csc\frac{\pi}{6} = 2$
If k is odd, let $k = 2m + 1$, then
$\csc[(2m+1)\pi + (-1)^{2m+1}\frac{5\pi}{6}]$
$= \csc[2m\pi + \pi + (-1)\frac{5\pi}{6}]$
$= \csc(\pi - \frac{5\pi}{6})$
$= \csc\frac{\pi}{6}$
$= 2$
Combining the above results,
$\csc[k\pi + (-1)^k \frac{5\pi}{6}] = \underline{\underline{2}}$

3. $y = 3\cos\theta$
$\because$ $-1 \le \cos\theta \le 1$ for all values of θ.
When $\cos\theta = 1$, $3\cos\theta$ is maximum.
$\therefore$ The maximum value is $\underline{\underline{3}}$.
When $\cos\theta = -1$, $3\cos\theta$ is minimum.
$\therefore$ The minimum value is $\underline{\underline{-3}}$.

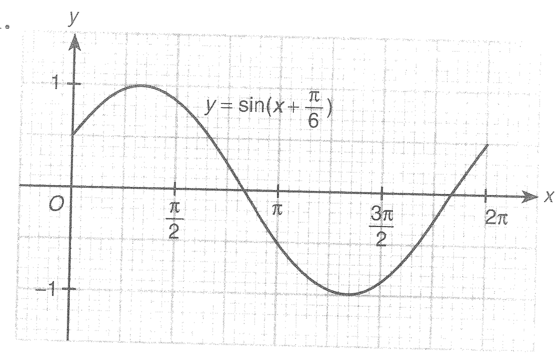
4. $y = 1 - \frac{1}{2}\sin(2x + \frac{\pi}{3})$
$\because$ $-1 \le \sin(2x + \frac{\pi}{3}) \le 1$ for all values of x.
When $\sin(2x + \frac{\pi}{3}) = -1$, y is maximum.
$\therefore$ The maximum value is $\underline{\underline{\frac{3}{2}}}$.
When $\sin(2x + \frac{\pi}{3}) = 1$, y is minimum.
$\therefore$ The minimum value is $\underline{\underline{\frac{1}{2}}}$.

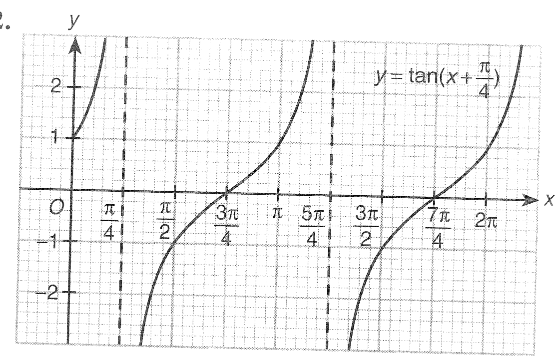
5. $3x^2 - 2x + k = 0$
Sum of the roots $= \sin\theta + \cos\theta = \frac{2}{3}$ ....(1)
Product of the roots $= \sin\theta\cos\theta = \frac{k}{3}$ ....(2)

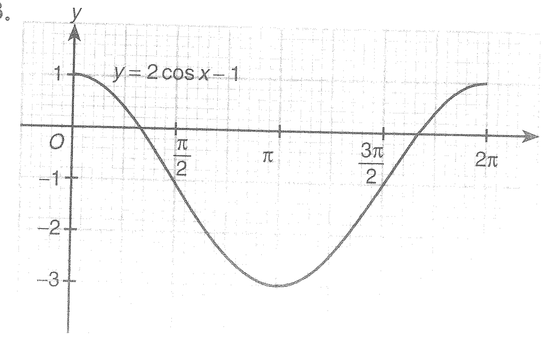
From (1),
$\sin\theta + \cos\theta = \frac{2}{3}$
$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \frac{4}{9}$
$1 + 2(\frac{k}{3}) = \frac{4}{9}$
$k = \underline{\underline{-\frac{5}{6}}}$

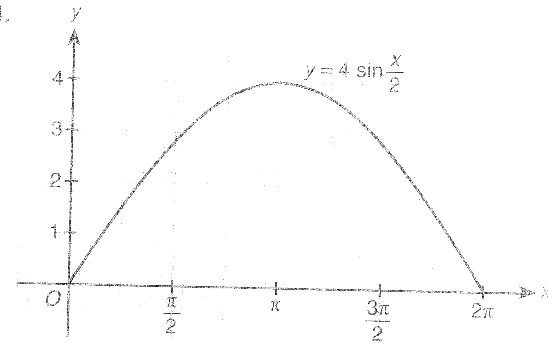
6. $x^2 + 2x + k = 0$
$\because$ $\sin\theta$ is a root of the equation.
$\therefore$ $\sin^2\theta + 2\sin\theta + k = 0$
$\sin\theta = \frac{-2 \pm \sqrt{4 - 4k}}{2}$
$\sin\theta = -1 \pm \sqrt{1 - k}$
$\because$ $-1 \le \sin\theta \le 1$
$\therefore$ $-1 \le -1 \pm \sqrt{1 - k} \le 1$
$0 \le \pm\sqrt{1 - k} \le 2$
$0 \le 1 - k \le 4$
$-1 \le -k \le 3$
$\underline{\underline{-3 \le k \le 1}}$

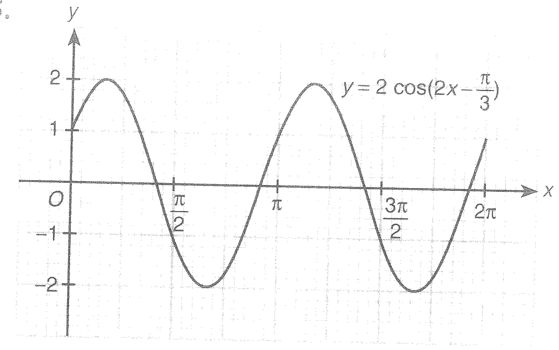
Exercise 5E (p.132)

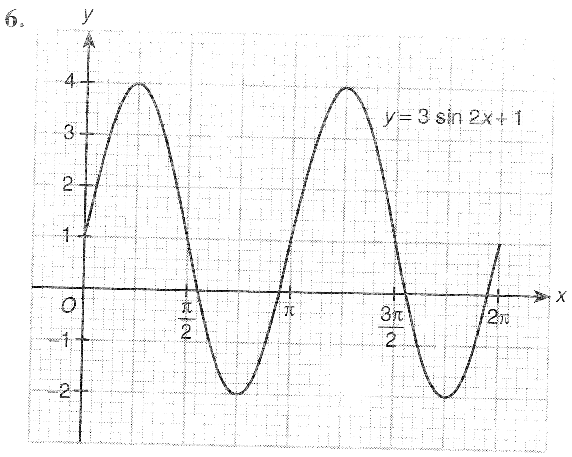
1.


2.


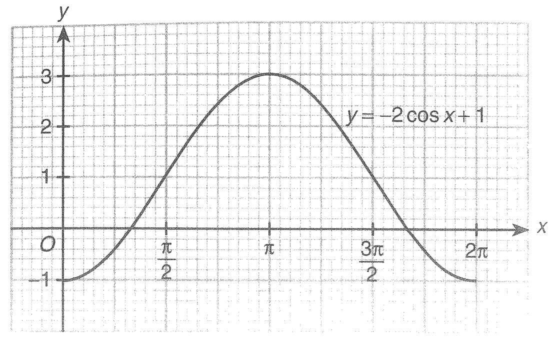
3.


4.


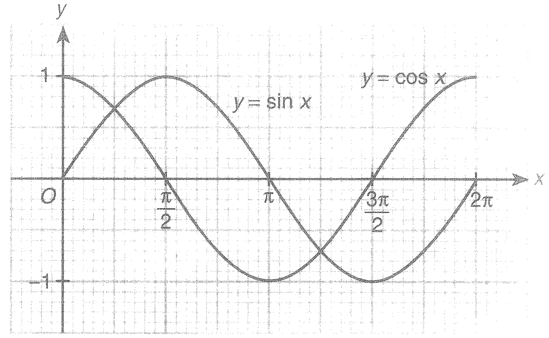
5.


6.


7.



8.



When $\frac{3\pi}{2} \le \theta \le 2\pi$, i.e. in quadrant IV, $\sin\theta$ and $\cos\theta$ both increase as θ increases.

9. By observation,

amplitude $= \frac{5-(-3)}{2} = 4$

distance shift along y-axis = 1 unit upwards

period $= \pi$

distance shift along x-axis = 0

$\therefore$ The given curve represents

$y = 4\sin\frac{x}{2} + 1$

$\therefore$ $A = 4, B = 1, m = 2, \phi = 0$

10. By observation,

amplitude $= \frac{2-(-2)}{2} = 2$

distance shift along y-axis = 0

period $= 3\pi$

distance shift along x-axis = 0

$\therefore$ The given curve represents

$y = 2\cos\frac{2}{3}x$

$\therefore$ $A = 2, B = 0, m = \frac{2}{3}, \phi = 0$

Revision Exercise 5 (p.136)

1. Area bounded

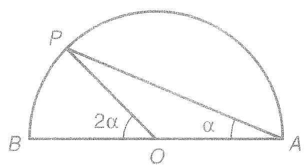
= area of equilateral triangle − areas of 3 sectors

$= \frac{1}{2}(2r)(\sqrt{3}r) - 3(\frac{1}{2}r^2 \cdot \frac{\pi}{3})$

$= \sqrt{3}r^2 - \frac{1}{2}\pi r^2$

$= (\sqrt{3} - \frac{\pi}{2})r^2$

2.



Let O be the centre.

Join OP, then $\angle POB = 2\alpha$.

Let r be the radius, then

$$\frac{1}{2}r^2(2\alpha) + \frac{1}{2}r^2\sin(\pi - 2\alpha) = \frac{1}{2}(\frac{1}{2}\pi r^2)$$

$$r^2\alpha + \frac{1}{2}r^2\sin 2\alpha = \frac{1}{4}\pi r^2$$

$$2\alpha + \sin 2\alpha = \frac{\pi}{2}$$

3. $[\tan(270° + A)\tan(180° + A)] \div [\sin(90° + A)\cos(360° - A) + \cos(A - 90°)\sin(180° + A)]$

$= \frac{(-\cot A)(\tan A)}{(\cos A)(\cos A) + (\sin A)(-\sin A)}$

$= \frac{-1}{\cos^2 A - \sin^2 A}$

$= \frac{1}{\sin^2 A - \cos^2 A}$

4. $2\cos\pi + 3\cos^2(\pi - A) - 1 + \tan 2\pi \sin\frac{\pi}{2}$

$= 2(-1) + 3\cos^2 A - 1 + 0(1)$

$= -3 + 3\cos^2 A$

$= -3\sin^2 A$

5. $\sin^4 x + \cos^4 x$

$= (\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x)$
$\quad - 2\sin^2 x\cos^2 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$= 1 - 2\sin^2 x\cos^2 x$

6. $\sec^2\theta + \csc^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$

$= \frac{1}{\cos^2\theta\sin^2\theta}$

$= \sec^2\theta\csc^2\theta$

7. $(A\sec x + B\tan x)^2 - (A\tan x + B\sec x)^2$

$= (A^2\sec^2 x + 2AB\sec x\tan x + B^2\tan^2 x)$
$\quad - (A^2\tan^2 x + 2AB\tan x\sec x + B^2\sec^2 x)$

$= A^2(\sec^2 x - \tan^2 x) + B^2(\tan^2 x - \sec^2 x)$

$= A^2 - B^2$

8. (a) $\tan(180k° + \alpha) = \tan\alpha$

(b) $\cos(360k° \pm \alpha)$

$= \cos(360k° + \alpha)$ or $\cos(360k° - \alpha)$

$= \cos\alpha$

(c) $\sin[180k° + (-1)^k\alpha]$

If k is even, let $k = 2m$.

$\sin[180(2m)° + (-1)^{2m}\alpha] = \sin(360m° + \alpha)$
$= \sin\alpha$

If k is odd, let $k = 2m + 1$,

$\sin[180(2m+1)° + (-1)^{2m+1}\alpha]$

$= \sin[(360m° + 180°) + (-1)\alpha]$

$= \sin[360m° + (180° - \alpha)]$

$= \sin(180° - \alpha)$

$= \sin\alpha$

Combining the above results,

$\sin[180k° + (-1)^k\alpha] = \sin\alpha$

9. $\because$ The equation has equal roots.

$\therefore$ $D = 0$

$4 - 4(2\sin\phi)(-\cos\phi) = 0$

$1 + 2\sin\phi\cos\phi = 0$

$\sin^2\phi + 2\sin\phi\cos\phi + \cos^2\phi = 0$

$(\sin\phi + \cos\phi) = 0$

$\sin\phi = -\cos\phi$

$\tan\phi = -1$

$\therefore$ $\phi = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

10. $\frac{\sin^2\theta}{1 + 2\cos^2\theta} = \frac{3}{19}$

$19\sin^2\theta = 3 + 6\cos^2\theta$

$19\sin^2\theta = 3\sin^2\theta + 3\cos^2\theta + 6\cos^2\theta$

$16\sin^2\theta = 9\cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} = \frac{9}{16}$

$\tan^2\theta = \frac{9}{16}$

$\tan\theta = \frac{3}{4}$ $(\because \pi < \theta < \frac{3\pi}{2}, \therefore \tan\theta > 0)$

$\therefore$ $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$

$\frac{\sin\theta}{1 + 2\cos\theta} = \frac{-\frac{3}{5}}{1 + 2(-\frac{4}{5})} = 1$

11. $\because$ The equation has equal roots.

$\therefore$ $D = 0$

$(4\cos\theta)^2 - 4(2)(3\sin\theta) = 0$

$16\cos^2\theta - 24\sin\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$

$2(1 - \sin^2\theta) - 3\sin\theta = 0$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(\sin\theta + 2) = -1$

$2\sin\theta - 1 = 0$ or $\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -2$ (rejected)

$\theta = \frac{5\pi}{6}$ or $\frac{\pi}{3}$ (rejected)

12. (a)

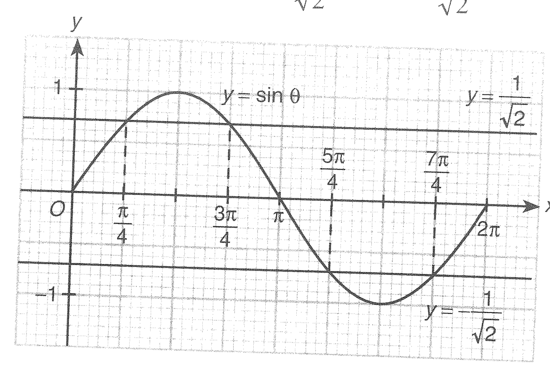
$2\sin^2\theta \le 1$

$\sin^2\theta \le \frac{1}{2}$

$(\sin^2\theta - \frac{1}{2}) \le 0$

$(\sin\theta + \frac{1}{\sqrt{2}})(\sin\theta - \frac{1}{\sqrt{2}}) \le 0$

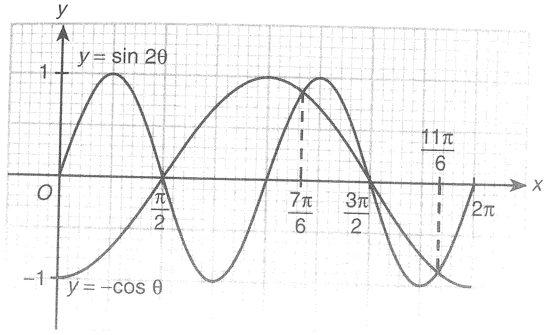
$-\frac{1}{\sqrt{2}} \le \sin\theta \le \frac{1}{\sqrt{2}}$



$\therefore$ $0 \le \theta \le \frac{\pi}{4}, \frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}, \frac{7\pi}{4} \le \theta \le 2\pi$

(b) $\sin 2\theta + \cos\theta < 0$

$\sin 2\theta < -\cos\theta$



The graph of $y = -\cos\theta$ lies above the graph $y = \sin 2\theta$ when

$\frac{\pi}{2} < \theta < \frac{7\pi}{6}$, $\frac{3\pi}{2} < \theta < \frac{11\pi}{6}$

13. $\sin\theta$ and $\sec\theta$ are roots of $2x^2 + kx + 1 = 0$

Sum of the roots $= \sin\theta + \sec\theta = -\frac{k}{2}$ ....(1)

Product of the roots $= \sin\theta\sec\theta = \frac{1}{2}$ ....(2)

From (2),

$\tan\theta = \frac{1}{2}$

$\sin\theta = \frac{1}{\sqrt{5}}$, $\sec = \frac{\sqrt{5}}{2}$ ($\because\ 0 < \theta < \frac{\pi}{2}$)

$\therefore$ From (1),

$\frac{1}{\sqrt{5}} + \frac{\sqrt{5}}{2} = -\frac{k}{2}$

$\frac{2 + (\sqrt{5})^2}{2\sqrt{5}} = -\frac{k}{2}$

$k = (-2)\frac{2+5}{2\sqrt{5}} = -\frac{7}{\sqrt{5}} = -\frac{7\sqrt{5}}{5}$

14. (a) $\sin\theta$ and $\cos\theta$ are roots of $3x^2 - 4x + k = 0$

Sum of the roots $= \sin\theta + \cos\theta = \frac{4}{3}$ ......(1)

Product of the roots $= \sin\theta\cos\theta = \frac{k}{3}$ ....(2)

From (1),

$(\sin\theta + \cos\theta)^2 = \frac{16}{9}$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \frac{16}{9}$

$1 + 2(\frac{k}{3}) = \frac{16}{9}$

$\frac{2k}{3} = \frac{7}{9}$

$k = \frac{7}{6}$

(b) $\frac{\sin\theta}{1 - \cot\theta} + \frac{\cos\theta}{1 - \tan\theta}$

$= \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} - \frac{\cos^2\theta}{\sin\theta - \cos\theta}$

$= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta - \cos\theta}$

$= \sin\theta + \cos\theta$

$= \frac{4}{3}$

15. (a) $\angle OAB = \angle OBA$

$= \frac{1}{2}(180° - 36°)$ (base $\angle$s, isos Δ)

$= 72°$

$\angle BCA = 72°$ (base $\angle$s, isos Δ)

$\angle ABC + 2(72°) = 180°$ ($\angle$sum of Δ)

$\angle ABC = 36° = \angle BOA$

$\Delta OAB \sim \Delta BAC$ (A.A.A.)

$\therefore\ \frac{AB}{OA} = \frac{AC}{AB}$

$(AB)^2 = (OA)(AC)$

(b) $\angle OBC = \angle BCA - \angle BOC = 72° - 36° = 36°$

$\therefore$ ΔOCB is an isosceles triangle.

$OC = BC = AB$

(c) $AB = x$

$x^2 = (1)(AC)$ (by **(a)**)

$\therefore\ AC = x^2$

$OC + CA = OA$

$x + x^2 = 1$

$x^2 = 1 - x$

(d) $x^2 + x - 1 = 0$ (by **(c)**)

$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

Since $x > 0$, $x = \frac{-1 + \sqrt{5}}{2}$

$AB = \frac{\sqrt{5} - 1}{2}$ cm

$AB = 2(1)\sin 18°$ cm

$2\sin 18° = \frac{\sqrt{5} - 1}{2}$

$\sin 18° = \frac{\sqrt{5} - 1}{4}$

Enrichment 5 (p.138)

1. (a) $\because\ \begin{cases} \alpha^2 - 4\alpha\sin\theta - 2 = 0 \text{................(1)} \\ \alpha^2 - 4\alpha\cos\theta + 2 = 0 \text{...............(2)} \end{cases}$

(1) − (2),

$4\alpha\cos\theta - 4\alpha\sin\theta - 4 = 0$

$\alpha(\cos\theta - \sin\theta) = 1$

$\alpha = \frac{1}{\cos\theta - \sin\theta}$

(b) Substitute $\alpha = \frac{1}{\cos\theta - \sin\theta}$ into (1).

$(\frac{1}{\cos\theta - \sin\theta})^2 - 4(\frac{1}{\cos\theta - \sin\theta})\sin\theta - 2 = 0$

$1 - 4\sin\theta(\cos\theta - \sin\theta) - 2(\cos\theta - \sin\theta)^2 = 0$

$1 - 4\sin\theta\cos\theta + 4\sin^2\theta$

$- 2(\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta) = 0$

$1 + 4\sin^2\theta - 2(1) = 0$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

2. (a) $y = \sin^6 x + 2\sin^2 x\cos^2 x + \cos^6 x$

$= (\sin^2 x)^3 + (\cos^2 x)^3 + 2\sin^2 x\cos^2 x$

$= (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x\cos^2 x$

$+ \cos^4 x) + 2\sin^2 x\cos^2 x$

$= (\sin^4 x + \cos^4 x) + \sin^2 x\cos^2 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$+ \sin^2 x\cos^2 x$

$= 1 - \sin^2 x\cos^2 x$

(b) (i) Let $a = \sin^2 x$.

$y = 1 - a(1 - a)$

$= 1 - a + a^2$

$= a^2 - a + (\frac{1}{2})^2 - (\frac{1}{2})^2 + 1$

$= \frac{3}{4} + (a - \frac{1}{2})^2$

(ii) y is maximum when $(a - \frac{1}{2})^2$ is maximum.

$(a - \frac{1}{2})^2$ is maximum when a is maximum.

$\because\ 0 \le a = \sin^2 x \le 1$

$\therefore$ The maximum value of y

$= \frac{3}{4} + (1 - \frac{1}{2})^2$

$= 1$

y is minimum when $(a - \frac{1}{2})^2$ is minimu

$(a - \frac{1}{2})^2$ is minimum when $(a - \frac{1}{2})^2 =$

$\therefore$ The minimum value of y

$= \frac{3}{4} + (0)$

$= \frac{3}{4}$

3. $\because\ \sin\theta - \cos\theta = m$

$\therefore\ (\sin\theta - \cos\theta)^2 = m^2$

$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = m^2$

$1 - 2\sin\theta\cos\theta = m^2$

$\sin\theta\cos\theta = \frac{1 - m^2}{2}$

(a) $\sin^3\theta - \cos^3\theta$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)$

$= (m)(1 + \frac{1 - m^2}{2})$

$= m(\frac{2 + 1 - m^2}{2})$

$= \frac{1}{2}m(3 - m^2)$

(b) Product of the roots

$= (\sin^3\theta)(-\cos^3\theta)$

$= -(\sin\theta\cos\theta)^3$

$= -(\frac{1 - m^2}{2})^3$

$= -\frac{1}{8}(1 - 3m^2 + 3m^4 - m^6)$

Sum of the roots

$= \sin^3\theta + (-\cos^3\theta)$

$= \sin^3\theta + (-\cos^3\theta)$

$= \sin^3\theta - \cos^3\theta$

$= \frac{1}{2}m(3 - m^2)$ (by **(a)**)

$\because$ A quadratic equation with roots $\sin^3\theta$ and $-\cos^3\theta$ is

$x^2 - \frac{1}{2}m(3 - m^2)x$

$-\frac{1}{8}(1 - 3m^2 + 3m^4 - m^6) = 0$

$8x^2 - 4m(3 - m^2)x$

$-(1 - 3m^2 + 3m^4 - m^6) = 0$

4. (a) (i) $y = \dfrac{\sec^2 x - \tan x}{\sec^2 x + \tan x} = \dfrac{1+\tan^2 x - \tan x}{1+\tan^2 x + \tan x}$

Let $t = \tan x$, $y = \dfrac{1+t^2-t}{1+t^2+t} = \dfrac{t^2-t+1}{t^2+t+1}$

(ii) $t^2+t+1 = t^2+t+(\frac{1}{2})^2-(\frac{1}{2})^2+1$

$= (t+\frac{1}{2})^2+\frac{3}{4}$

$(t+\frac{1}{2})^2 \geq 0$

$(t+\frac{1}{2})^2+\frac{3}{4} \geq 0+\frac{3}{4}$

$t^2+t+1 \geq \frac{3}{4} > 0$ for all t

(b) $y = \dfrac{t^2-t+1}{t^2+t+1}$

$y(t^2+t+1) = t^2-t+1$

$(y-1)t^2+(y+1)t+(y-1) = 0$

Since t is real,

$\therefore \quad D \geq 0$

$\therefore \quad (y+1)^2 - 4(y-1)^2 \geq 0$

$(y^2+2y+1)-4(y^2-2y+1) \geq 0$

$y^2+2y+1-4y^2+8y-4 \geq 0$

$-3y^2+10y-3 \geq 0$

$3y^2-10y+3 \leq 0$

$(y-3)(3y-1) \leq 0$

$\begin{cases} y-3 \leq 0 \\ 3y-1 \geq 0 \end{cases}$ or $\begin{cases} y-3 \geq 0 \\ 3y-1 \leq 0 \end{cases}$

$\frac{1}{3} \leq y \leq 3$ or no solution

$\therefore \quad \frac{1}{3} \leq y \leq 3$

i.e. $\dfrac{1}{3} \leq \dfrac{\sec^2 x - \tan x}{\sec^2 x + \tan x} \leq 3$ for all $x \neq k\pi$.

Classwork 1 (p.102)

1. (a) $69.3° = 69.3 \times \dfrac{\pi}{180}$

$= \underline{\underline{1.21}}$ (corr. to 2 d.p.)

(b) $138.7° = 138.7 \times \dfrac{\pi}{180}$

$= \underline{\underline{2.42}}$ (corr. to 2 d.p.)

2. (a) $60° = 60 \times \dfrac{\pi}{180}$

$= \underline{\underline{\dfrac{\pi}{3}}}$

(b) $150° = 150 \times \dfrac{\pi}{180}$

$= \underline{\underline{\dfrac{5\pi}{6}}}$

3. (a) $4.69^c = 4.69 \times \dfrac{180°}{\pi}$

$= \underline{\underline{269°}}$ (corr. to 3 sig. fig.)

(b) $\dfrac{6\pi}{5}$ rad. $= \dfrac{6\pi}{5} \times \dfrac{180°}{\pi}$

$= \underline{\underline{216°}}$

Classwork 2 (p.103)

1 revolution = 2π radians

1 minute = 60 seconds

(a) 60 rev./min. $= 60 \cdot \dfrac{2\pi}{60}$ rad./s

$= \underline{\underline{2\pi \text{ rad./s}}}$

(b) Angular speed $= \dfrac{\text{angle rotated}}{\text{time}}$

$= \dfrac{15\pi}{\text{time}}$

The time required $= 15\pi \cdot \dfrac{1}{2\pi}$ s

$= \underline{\underline{7.5 \text{ s}}}$

Classwork 3 (p.104)

1.

Degree Measure	10°	60°	80°	150°	225°	330°	390°	645°
Radian Measure	$\frac{\pi}{18}$	$\frac{\pi}{3}$	$\frac{4\pi}{9}$	$\frac{5\pi}{6}$	$\frac{5\pi}{4}$	$\frac{11\pi}{6}$	$\frac{13\pi}{6}$	$\frac{43\pi}{12}$

2.

θ(c)	r (cm)	s (cm)	A (cm²)
$\frac{\pi}{3}$	2	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$
$\frac{5\pi}{6}$	3	$\frac{5\pi}{2}$	$\frac{15\pi}{4}$
$\frac{5\pi}{18}$	$\frac{5}{2}$	$\frac{25\pi}{36}$	$\frac{125\pi}{144}$
$\frac{5\pi}{3}$	$\frac{4}{5}$	$\frac{4\pi}{3}$	$\frac{8\pi}{15}$
2π	4	8π	16π

Classwork 4 (p.105)

(a) $\cos 30° = \dfrac{BC}{20 \text{ cm}}$

$BC = 20\cos 30°$ cm

$= \underline{\underline{17.3 \text{ cm}}}$ (corr. to 3 sig. fig.)

(b) Area of $\Delta OAB = \dfrac{1}{2}(BC)(OA)$

$= \dfrac{1}{2}(20\cos 30°)(20)$ cm²

$= 200\cos 30°$ cm²

$\angle BOC = 180° - 90° - 30° = 60°$

reflex $\angle BOC = 360° - 60° = 300°$

Area of major sector OBA

$= \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2} \times (20)^2(300 \times \dfrac{\pi}{180})$ cm²

$= \dfrac{1\,000\pi}{3}$ cm²

$\therefore$ The area of the major segment cut off by AB

$= (\dfrac{1\,000\pi}{3} + 200\cos 30°)$ cm²

$= \underline{\underline{1\,220 \text{ cm}^2}}$ (corr. to 3 sig. fig.)

Classwork 5 (p.112)

θ	Quadrant of Angle θ	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
85°	I	+	+	+	+	+	+
$\frac{3\pi}{4}$	II	+	−	−	+	−	−
$\frac{13\pi}{9}$	III	−	−	+	−	−	+
300°	IV	−	+	−	−	+	−
$\frac{7\pi}{6}$	III	−	−	+	−	−	+
$\frac{15\pi}{8}$	IV	−	+	−	−	+	−

Classwork 6 (p.113)

1. (a) $\sin 150° = \sin(180° - 30°) = \sin 30° = \underline{\underline{\dfrac{1}{2}}}$

(b) $\tan 300° = \tan(360° - 60°) = -\tan 60° = \underline{\underline{-\sqrt{3}}}$

(c) $\cos(\dfrac{5}{4}\pi) = \cos(\pi + \dfrac{\pi}{4}) = -\cos\dfrac{\pi}{4} = \underline{\underline{-\dfrac{\sqrt{2}}{2}}}$

2. (a) $\sin(-150°) = -\sin 150°$

$= -\sin(180° - 30°)$

$= -\sin 30°$

$= \underline{\underline{-\dfrac{1}{2}}}$

(b) $\sec(-60°) = \sec 60° = \underline{\underline{2}}$

(c) $\cot(-\dfrac{\pi}{4}) = -\cot\dfrac{\pi}{4} = \underline{\underline{-1}}$

Classwork 7 (p.114)

1. $\sin(90° + \theta) - \cos\theta = \cos\theta - \cos\theta = \underline{\underline{0}}$

2. $\cos^2(180° - \theta) + \cos^2(270° - \theta)$

$= (-\cos\theta)^2 + (-\sin\theta)^2$

$= \cos^2\theta + \sin^2\theta$

$= \underline{\underline{1}}$

3. $\cos(90° - \theta)\tan(90° + \theta) = (\sin\theta)(-\cot\theta)$

$= \sin\theta(-\dfrac{\cos\theta}{\sin\theta})$

$= \underline{\underline{-\cos\theta}}$

4. $1 + \tan^2(270° + \theta) = 1 + (-\cot\theta)^2$

$= 1 + \cot^2\theta$

$= \underline{\underline{\csc^2\theta}}$

5. $\dfrac{\cot(180° + \theta)\sin(-\theta)}{\sec(360° - \theta)\cos^2(180° - \theta)} = \dfrac{(\cot\theta)(-\sin\theta)}{(\sec\theta)(-\cos\theta)^2}$

$= \dfrac{(\cot\theta)(-\sin\theta)}{(\cos\theta)}$

$= (\cot\theta)(-\tan\theta)$

$= \underline{\underline{-1}}$

Classwork 8 (p.115)

1. $\tan\theta\csc\theta = \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\sin\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

2. $\sin(90° + \theta)\sec(90° - \theta) = (\cos\theta)(\csc\theta)$

$= \cos\theta\dfrac{1}{\sin\theta}$

$= \cot\theta$

3. $\dfrac{\sec^2\theta - 1}{\tan^2\theta + 1} = \dfrac{\tan^2\theta}{\sec^2\theta} = \dfrac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta = \sin^2\theta$

Classwork 9 (p.119)

1. $2\cos\theta + \sqrt{3} = 0$

$\cos\theta = -\dfrac{\sqrt{3}}{2}$

$\theta = \underline{\underline{\dfrac{5\pi}{6}, \dfrac{7\pi}{6}}}$

2. $\sin\theta = 4\cos\theta$

$\tan\theta = 4$

$\theta = \underline{\underline{1.33, 4.47}}$ (corr. to 2 d.p.)

3. $\sec^2\theta - \tan^2\theta + \tan\theta = 0$

$(1 + \tan^2\theta) - \tan^2\theta + \tan\theta = 0$

$\tan\theta = -1$

$\theta = \underline{\underline{\dfrac{3\pi}{4}, \dfrac{7\pi}{4}}}$

4. $\sin^2\theta + 2\cos\theta - 2 = 0$

$(1 - \cos^2\theta) + 2\cos\theta - 2 = 0$

$\cos^2\theta - 2\cos\theta + 1 = 0$

$(\cos\theta - 1)^2 = 0$

$\cos\theta = 1$

$\theta = \underline{\underline{0}}$

5. $(1+\sqrt{3})\sin^2\theta + (1+\sqrt{3})\sin\theta\cos\theta + 2\cos^2\theta = 1$

$\sin^2\theta + \sqrt{3}\sin^2\theta + (1+\sqrt{3})\sin\theta\cos + 2\cos^2\theta = 1$

$\sqrt{3}\sin^2\theta + (1+\sqrt{3})\sin\theta\cos + \cos^2\theta = 0$

$(\sqrt{3}\sin\theta + \cos\theta)(\sin\theta + \cos\theta) = 0$

$\sqrt{3}\sin\theta + \cos\theta = 0$ or $\sin\theta + \cos\theta = 0$

$\tan\theta = -\dfrac{1}{\sqrt{3}}$ or $\tan\theta = -1$

$\theta = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$ or $\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

$\therefore \theta = \underline{\underline{\dfrac{3\pi}{4}, \dfrac{5\pi}{6}, \dfrac{7\pi}{4}, \dfrac{11\pi}{6}}}$

6. $\sin\theta + \sqrt{2}\cos\theta = 1$

$\sin\theta = 1 - \sqrt{2}\cos\theta$

$\sin^2\theta = 1 - 2\sqrt{2}\cos\theta + 2\cos^2\theta$

$1 - \cos^2\theta = 1 - 2\sqrt{2}\cos\theta + 2\cos^2\theta$

$3\cos^2\theta - 2\sqrt{2}\cos\theta = 0$

$\cos\theta(3\cos\theta - 2\sqrt{2}) = 0$

$\cos\theta = 0$ or $3\cos\theta - 2\sqrt{2} = 0$

$\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ or $\theta = 0.34, 5.94$ (corr. to 2 d.p.)

Check:

(i) When $\theta = \dfrac{\pi}{2}$,

L.H.S. $= 1 + \sqrt{2}(0) = 1 =$ R.H.S.

$\therefore \dfrac{\pi}{2}$ is a solution.

(ii) When $\theta = \dfrac{3\pi}{2}$,

L.H.S. $= -1 + \sqrt{2}(0) = -1 \neq$ R.H.S

$\therefore \dfrac{3\pi}{2}$ is not a solution.

(iii) When $\theta = 0.34$,

L.H.S. $= \sin 0.34 + \sqrt{2}\cos 0.34 = 1.67 \neq$ R.H.S

$\therefore$ 0.34 is not a solution.

(iv) When $\theta = 5.94$,

L.H.S. $= \sin 5.94 + \sqrt{2}\cos 5.94 = 1 =$ R.H.S.

$\therefore$ 5.94 is a solution.

$\therefore \theta = \underline{\underline{\dfrac{\pi}{2}, 5.94}}$

Classwork 10 (p.126)

1. (a) $\sin 90° = \underline{\underline{1}}$

(b) $\tan 570° = \tan(360° + 210°)$

$= \tan 210°$

$= \tan 30°$

$= \underline{\underline{\dfrac{\sqrt{3}}{3}}}$

(c) $\cos\dfrac{14\pi}{3} = \cos[2(2\pi) + \dfrac{2\pi}{3}]$

$= \cos\dfrac{2\pi}{3}$

$= -\cos\dfrac{\pi}{3}$

$= \underline{\underline{-\dfrac{1}{2}}}$

(d) $\sec\dfrac{23\pi}{3} = \sec[3(2\pi) + \dfrac{5\pi}{3}]$

$= \sec\dfrac{5\pi}{3}$

$= \sec(2\pi - \dfrac{\pi}{3})$

$= \sec\dfrac{\pi}{3}$

$= \underline{\underline{2}}$

2. (a) $y = \dfrac{\sqrt{3}}{2}\sin x$

$\because -1 \le \sin x \le 1$ for all values of x.

When $\sin x = 1$, $\dfrac{\sqrt{3}}{2}\sin x$ is maximum.

$\therefore$ The maximum value is $\underline{\underline{\dfrac{\sqrt{3}}{2}}}$.

When $\sin x = -1$, $\dfrac{\sqrt{3}}{2}\sin x$ is minimum.

$\therefore$ The minimum value is $\underline{\underline{-\dfrac{\sqrt{3}}{2}}}$.

(b) $y = -\dfrac{1}{4}\cos\dfrac{x}{2}$

$\because -1 \le \cos\dfrac{x}{2} \le 1$ for all values of x.

When $\cos\dfrac{x}{2} = -1$, $-\dfrac{1}{4}\cos\dfrac{x}{2}$ is maximum.

$\therefore$ The maximum value is $\underline{\underline{\dfrac{1}{4}}}$.

When $\cos\dfrac{x}{2} = 1$, $-\dfrac{1}{4}\cos\dfrac{x}{2}$ is minimum.

$\therefore$ The minimum value is $\underline{\underline{-\dfrac{1}{4}}}$.

(c) $y = 2\cos^2(\pi + \dfrac{x}{2})$

$\because 0 \le \cos^2(\pi + \dfrac{x}{2}) \le 1$ for all values of x.

When $\cos^2(\pi + \dfrac{x}{2}) = 1$, $2\cos^2(\pi + \dfrac{x}{2})$ is maximum.

$\therefore$ The maximum value is $\underline{\underline{2}}$.

When $\cos^2(\pi + \dfrac{x}{2}) = 0$, $2\cos^2(\pi + \dfrac{x}{2})$ is minimum.

$\therefore$ The minimum value is $\underline{\underline{0}}$.

(d) $y = 4\sin^2(\dfrac{x}{2}) - 3$

$\because 0 \le \sin^2(\dfrac{x}{2}) \le 1$ for all values of x.

When $\sin^2(\dfrac{x}{2}) = 1$, $4\sin^2(\dfrac{x}{2}) - 3$ is maximum.

$\therefore$ The maximum value is $\underline{\underline{1}}$.

When $\sin^2(\dfrac{x}{2}) = 0$, $4\sin^2(\dfrac{x}{2}) - 3$ is minimum.

$\therefore$ The minimum value is $\underline{\underline{-3}}$.

Classwork 11 (p.132)

1. (a) $y = 2\cos x$

period $= \underline{\underline{2\pi}}$

(b) $y = \sin\dfrac{x}{2}$

period $= \underline{\underline{4\pi}}$

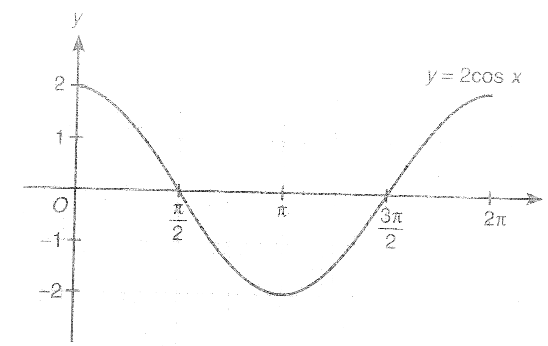
(c) $y = \tan 4x$

period $= \underline{\underline{\dfrac{\pi}{4}}}$

(d) $y = \sec\dfrac{2x}{3}$

period $= \underline{\underline{3\pi}}$

2. (a)



(b)