Chapter 2 Functions and Graphs

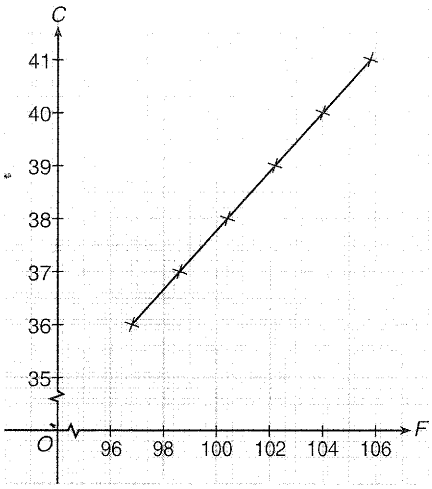
Follow-up

pp.53 – 88

2.1 (a) p.53

F	96.8	98.6	100.4	102.2	104	105.8
C	36	37	38	39	40	41

(b)



(c) $C = \frac{5}{9}(101.3 - 32)$

$= \frac{5}{9}(69.3)$

$= \underline{\underline{38.5}}$

(d) $C = \frac{5}{9}(101.3 - 32)$

$= \frac{5}{9}(69.3)$

$= 38.5$

> 37.3

$\therefore$ Dick is suffering from a fever.

2.2 (a) p.55 $g(-2) = 2(-2)^2 - 4(-2) + 1$

$= 8 + 8 + 1 = \underline{\underline{17}}$

$g(-4) = 2(-4)^2 - 4(-4) + 1$

$= 32 + 16 + 1 = \underline{\underline{49}}$

(b) $\because\ 2g(-2) = 2 \times 17$

$= 34 \neq 49$

$\therefore\ g(-4) \neq 2g(-2)$

2.3 (a) p.56

$f(-2) = -1$

$3 + k(-2)^2 = -1$

$4k = -4$

$k = \underline{\underline{-1}}$

(b) $\because\ f(x) = g(x)$

$\therefore\ 3 - x^2 = 2x$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = \underline{\underline{-3 \text{ or } 1}}$

2.4 (a) p.62 The graph opens upwards.

(b) From the graph of $y = 2x^2 - 8x + k$,

the y-intercept = 11.

$\therefore\ k = \underline{\underline{11}}$

(c) Vertex = $\underline{\underline{(2, 3)}}$

2.5 (a) p.65 For Graph (I): $y = 16 - (x - 1)^2$

Substituting $y = 0$ into the equation, we have

$0 = 16 - (x - 1)^2$

$(x - 1)^2 = 16$

$x - 1 = -4$ or $x - 1 = 4$

$x = -3$ or 5

$\therefore$ The x-intercepts are -3 and 5.

For Graph (II): $y = 16 - 4(x - 1)^2$

Substituting $y = 0$ into the equation, we have

$0 = 16 - 4(x - 1)^2$

$4(x - 1)^2 = 16$

$(x - 1)^2 = 4$

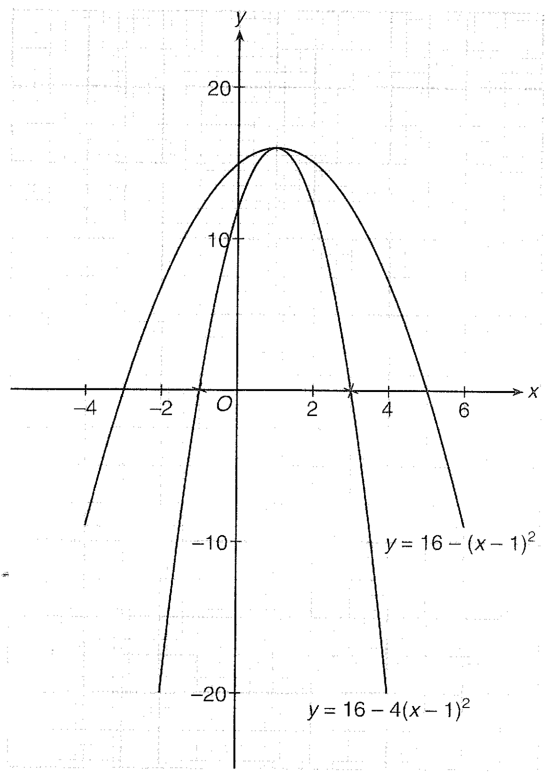
$x - 1 = -2$ or $x - 1 = 2$

$x = -1$ or 3

$\therefore$ The x-intercepts are -1 and 3.

(b) Graph (I) opens wider.

(c)



From the graph, we can see that graph (I) opens wider.

2.6 **(a)** Putting $x = -3$, $y = 5$ into the given equation, p.68
we have

$$5 = -(-3 + 4)^2 + k$$
$$5 = -1 + k$$
$$\therefore \quad k = \underline{\underline{6}}$$

(b) Since the equation of the graph is
$y = -(x + 4)^2 + 6$,
the vertex is $(-4, 6)$.

(c) The axis of symmetry: $x = -4$

(d) Maximum value of y is 6.

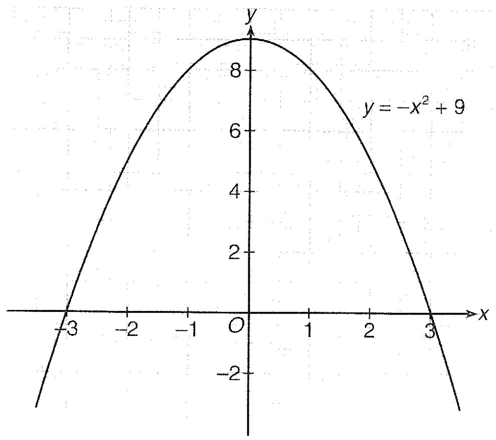
2.7 **(a)** Putting $x = 1$, $y = 8$ into the given equation, p.69
we have

$$8 = -(1 + 3)(1 - k)$$
$$8 = -4 + 4k$$
$$12 = 4k$$
$$\therefore \quad k = \underline{\underline{3}}$$

(b) Since the equation of the graph is
$$y = -(x + 3)(x - 3)$$
$$= -x^2 + 9$$
$\therefore$ The vertex is $(0, 9)$ and the axis of symmetry is $x = 0$.

(c) From the graph,
x-intercepts $= \underline{\underline{-3 \text{ and } 3}}$
y-intercept $= \underline{\underline{9}}$

(d)



2.8 p.71

(a)
$$s = 10t - 5t^2$$
$$= -5(t^2 - 2t)$$
$$= -5(t^2 - 2t + 4 - 4)$$
$$= -5(t - 2)^2 + 20$$
s is maximum when $t = 2$.
$\therefore$ The bullet reach the maximum height after 2 seconds.

(b) When $t = 2$,
$$s = -5(2 - 2)^2 + 20$$
$$= 20$$
$\therefore$ The maximum height is 20 m.

2.9 **(a)** p.71
$$P = -x^2 + 40x + 15$$
$$= -(x^2 - 40x + 20^2 - 20^2) + 15$$
$$= -(x^2 - 40x + 20^2) + 400 + 15$$
$$= -(x - 20)^2 + 415$$
$\therefore$ The maximum profit is \$415.

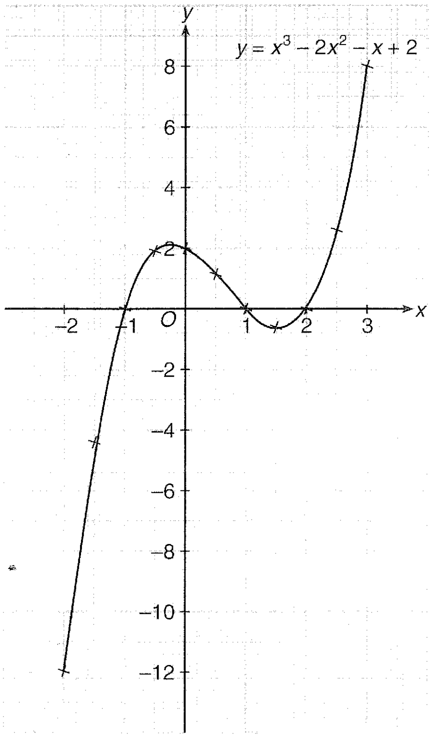
(b) Since P attains its maximum value when $x = 20$,
$\therefore$ 20 cakes should be sold to make the maximum profit.

2.10 p.77

(a)

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3
y	−12	−4.4	0	1.9	2	1.1	0	−0.6	0	2.6	8

(b)



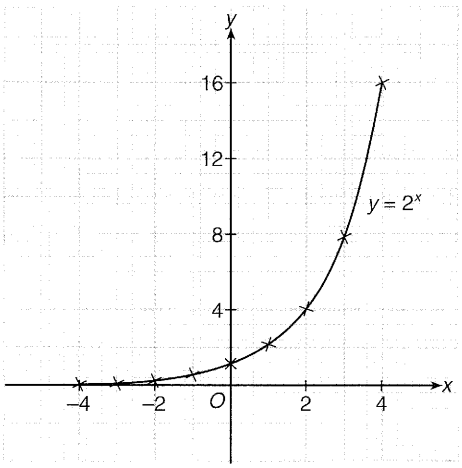
(c) From the graph,
y-intercept = $\underline{\underline{2}}$
Vertex = $\underline{\underline{(-0.2, 2.1)\ \text{and}\ (1.5, -0.6)}}$

2.11 p.78

(a)

x	–4	–3	–2	–1	0	1	2	3	4
y	0.0625	0.125	0.25	0.5	1	2	4	8	16

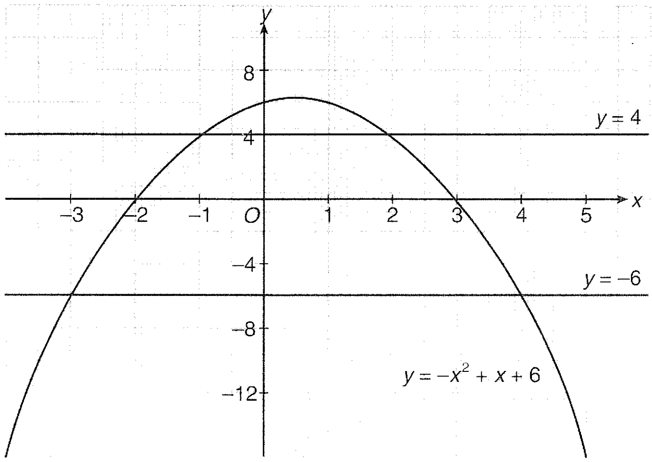
(b)



(c) No, the curve does not have any vertex.

(d) No, y cannot be negative.

2.12 p.81

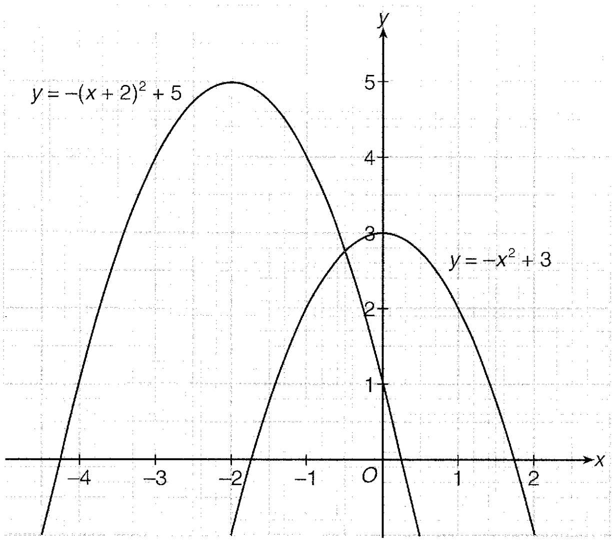


(a) From the graph,
the solution is $-3 \le x \le 4$.

(b) $-x^2 + x + 2 \ge 0$ can be written as
$-x^2 + x + 2 + 4 - 4 \ge 0,$
$-x^2 + x + 6 \ge 4.$
$\therefore$ We should add the line $y = 4$ on the graph.

(c) From the graph,
the solution is $-1 \le x \le 2$.

2.13 (a) p.88



(b) A translation of 2 units to the left and 2 units upwards of the graph of $y = -x^2 + 3$ will give the graph of $y = -(x + 2)^2 + 5$.

Teacher's Example

pp.52 – 88

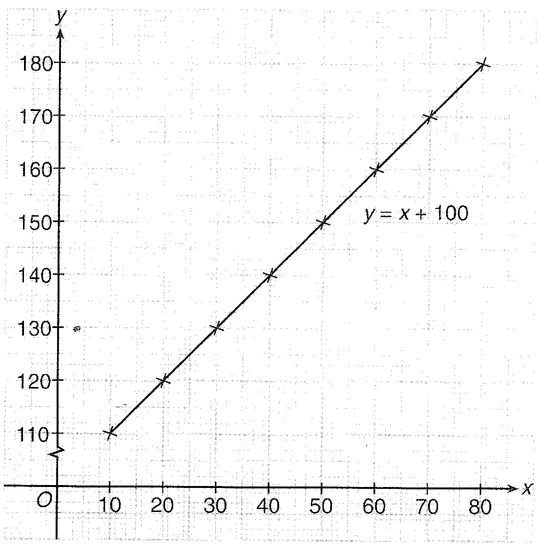
Example 2.1T

p.52

(a)

x	10	20	30	40	50	60	70	80
y	110	120	130	140	150	160	170	180

(b)



(c) $y = 25 + 100$

$= 125$

$\because$ The mobile company has 1000 customers,

$\therefore$ The company earns

$\$ (1000 \times 125)$

$= \underline{\underline{\$125\ 000}}$

Example 2.2T

p.55

(a) $f(-2) = (-2 + 3)(-2 - 4) + 5$

$= 1(-6) + 5$

$= \underline{\underline{-1}}$

$f(0) = (0 + 3)(0 - 4) + 5$

$= 3(-4) + 5$

$= \underline{\underline{-7}}$

$f(2) = (2 + 3)(2 - 4) + 5$

$= 5(-2) + 5$

$= \underline{\underline{-5}}$

(b) Since $f(-2) + f(2) = -1 + (-5)$

$= -6 \neq -7$

$\therefore\ f(-2) + f(2) \neq f(0)$

Example 2.3T

p.55

(a)
$g(2) = 1$

$k - 2(2)^2 = 1$

$k - 8 = 1$

$k = \underline{\underline{9}}$

(b)
$g(x) + 3x = 0$

$9 - 2x^2 + 3x = 0$

$2x^2 - 3x - 9 = 0$

$(2x + 3)(x - 3) = 0$

$x = \underline{\underline{-\frac{3}{2} \text{ or } 3}}$

Example 2.4T

p.62

(a) The graph opens upwards.

(b) The graph passes through the point $(0, -9)$.

$-9 = 2(0)(0 + 3) + k$

$\therefore\ k = \underline{\underline{-9}}$

(c) From the graph,

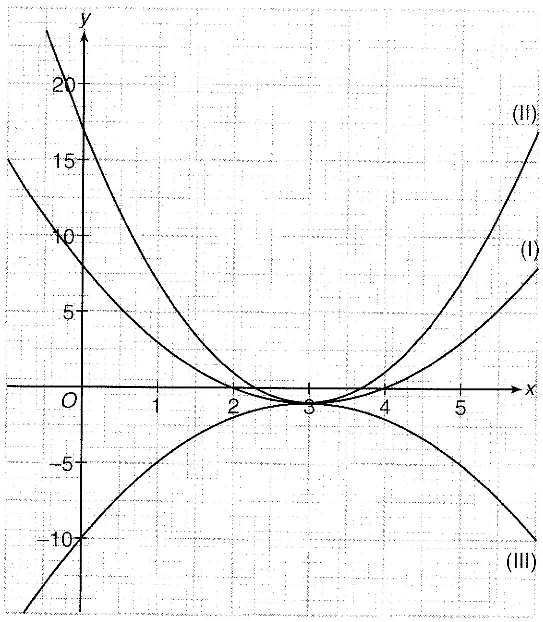
y-intercept = $\underline{\underline{9}}$ and

vertex = $\underline{\underline{(-1.5,\ -13.5)}}$.

Example 2.5T

p.64

(a)



(b) Graphs (I) and (III) are symmetrical about the x-axis.

(c) Graph (II) opens narrower than Graph (I).

Example 2.6T

p.68

(a) Vertex = (−1, 6)

(b) Putting $x = 2$, $y = -12$ into the given equation, we have

$$-12 = a(2+1)^2 + 6$$
$$-18 = 9a$$
$$a = \underline{\underline{-2}}$$

(c) Since $a = -2 < 0$, the graph opens downwards.

∴ It has a maximum value.

Example 2.7T

p.68

(a) Putting $x = -2$, $y = -8$ into the given equation, we have

$$-8 = 2(-2)^2 + b(-2) - 12$$
$$-8 = 8 - 2b - 12$$
$$2b = 4$$
$$b = \underline{\underline{2}}$$

(b) $y = 2x^2 + 2x - 12$

$$= 2\left[x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right] - 12$$
$$= 2\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] - 12$$
$$= 2\left(x + \frac{1}{2}\right)^2 - \frac{25}{2}$$

∴ Vertex = $\underline{\underline{\left(-\frac{1}{2}, -\frac{25}{2}\right)}}$

(c) When $y = 0$,

$$2x^2 + 2x - 12 = 0$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x = -3 \text{ or } 2$$

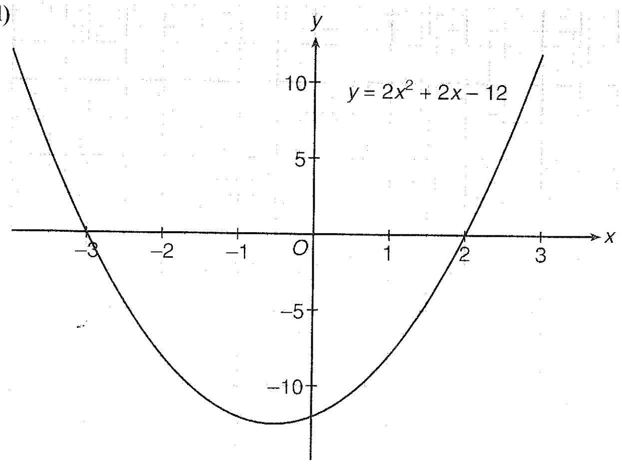
∴ x-intercept = $\underline{\underline{-3 \text{ or } 2}}$

When $x = 0$,

$$y = 2(0)^2 + 2(0) - 12$$
$$= -12$$

∴ y-intercept = $\underline{\underline{12}}$

(d)



Example 2.8T

p.70

(a)
$$s = 45 - \frac{v^2}{20}$$
$$40 = 45 - \frac{v^2}{20}$$
$$v^2 = 100$$
$$v = \underline{\underline{10}} \text{ or } -10 \text{ (rejected)}$$

(b) The maximum velocity is attained when $s = 0$.

$$\therefore \quad 0 = 45 - \frac{v^2}{20}$$
$$v^2 = 900$$
$$v = 30 \text{ or } -30 \text{ (rejected)}$$

∴ The maximum velocity is 30 m/s.

Example 2.9T

p.71

(a) $C = x^2 - 40 + 4100$

$$= x^2 - 40x + \left(\frac{40}{2}\right)^2 - \left(\frac{40}{2}\right)^2 + 4100$$
$$= (x-20)^2 - \left(\frac{40}{2}\right)^2 + 4100$$
$$= (x-20)^2 + 3700$$

∴ The number of bicycle made is 20.

(b) From (a), when 20 bicycles are made, the minimum cost of making bicycles is \$3700.

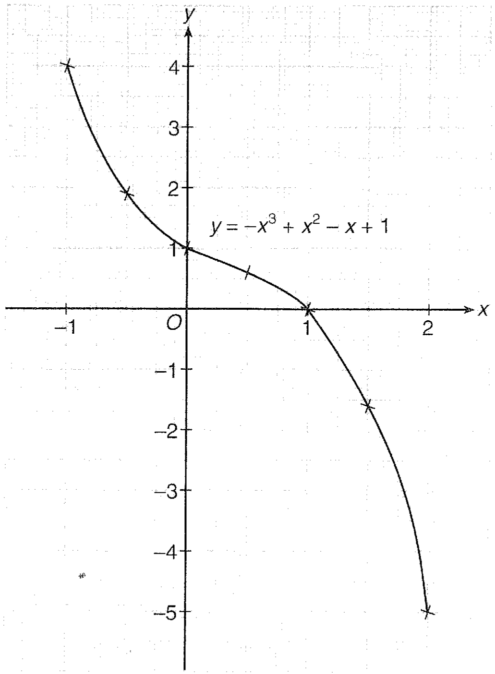
Example 2.10T

p.77

(a)

x	−1	−0.5	0	0.5	1	1.5	2
y	4	1.88	1	0.63	0	−1.63	−5

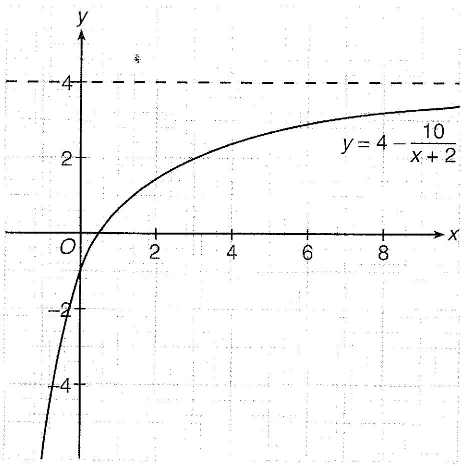
(b)



(c) From the graph,
y-intercept = $\underline{\underline{1}}$
and no vertex.

Example 2.11T p.78

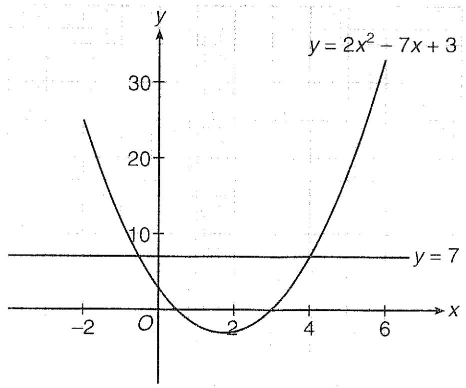
(a)



(b) The curve tends to the line $y = 4$ when x becomes very large.

Example 2.12T p.81

(a)



(b) (i) Draw the line $y = 7$ on the graph, the required solution is $-\frac{1}{2} < x < 4$.

(ii) From the graph, the required solution is $\frac{1}{2} \le x \le 3$.

(iii) $2x^2 - 7x + 1 > -2$ can be written as
$2x^2 - 7x + 3 > 0$,
$\therefore$ The required solutions is $x < \frac{1}{2}$ or $x > 3$.

Example 2.13T p.88

(a) $x^2 - 6x + 10$
$= x^2 - 6x + 3^2 - 3^2 + 10$
$= \underline{\underline{(x - 3)^2 + 1}}$

(b) A translation of 4 units to the left and 3 units downwards of the graph of $y = x^2 - 6x + 10$ will give the graph of $y = (x + 1)^2 - 2$.

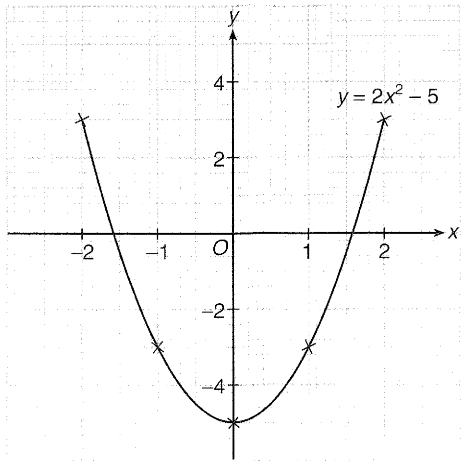
Exercise 2.1 pp.56 – 58

Level 1 p.56

1. (a)

x	−2	−1	0	1	2
y	3	−3	−5	−3	3

(b)



2. **(a)** $f(-3) = 10 + 3(-3)$
$= 10 - 9 = 1$

(b) $f(4) = 10 + 3(4)$
$= 10 + 12 = 22$

(c) $f(-2) = 10 + 3(-2)$
$= 10 - 6 = 4$

(d) $f(2) = 10 + 3(2)$
$= 10 + 6 = 16$

3. **(a)** $g(0) = 0^2 - 4(0) + 1$
$= 0 - 0 + 1 = 1$

(b) $g(1) = 1^2 - 4(1) + 1$
$= 1 - 4 + 1 = -2$

(c) $g(-1) = (-1)^2 - 4(-1) + 1$
$= 1 + 4 + 1 = 6$

(d) $g(-2) = (-2)^2 - 4(-2) + 1$
$= 4 + 8 + 1 = 13$

4. **(a)** $H(2) = (2 + 3)(2 - 2)$
$= 5(0) = 0$

(b) $H(-3) = (-3 + 3)(-3 - 2)$
$= 0(-5) = 0$

(c) $H(1) = (1 + 3)(1 - 2)$
$= 4(-1) = -4$

(d) $H(-2) = (-2 + 3)(-2 - 2)$
$= 1(-4) = -4$

5. **(a)** $G(3) = -\frac{1}{3}(3^2) = -3$

(b) $G(2) = -\frac{1}{3}(2^2) = -\frac{4}{3}$

(c) $G(3) + G(2) = -3 + \left(-\frac{4}{3}\right)$
$= \frac{-9-4}{3} = -\frac{13}{3}$

(b) $G(3) - 3G(2) = -3 - 3\left(-\frac{4}{3}\right)$
$= 1$

6. $f(0) = 3(0) + 2 = 2$
$f(1) = 3(1) + 2 = 5$

(a) $f(0) - f(1) = 2 - 5$
$= -3$

(b) $f(0) \cdot f(1) = 2 \cdot 5$
$= 10$

(c) $\frac{f(0)}{f(1)} = \frac{2}{5}$

(d) $\frac{f(0) + f(1)}{f(1)} = \frac{2+5}{5}$
$= \frac{7}{5}$

7. $h(-1) = 2^{-1} + 1$
$= \frac{1}{2} + 1 = \frac{3}{2}$

$h(-2) = 2^{-2} + 1$
$= \frac{1}{4} + 1 = \frac{5}{4}$

(a) $2h(-1) = 2\left(\frac{3}{2}\right) = 3$

(b) $4h(-2) = 4\left(\frac{5}{4}\right) = 5$

(c) $2h(-1) + 4h(-2) = 3 + 5$
$= 8$

(d) $h(-1) + 2h(-2) = \frac{3}{2} + 2\left(\frac{5}{4}\right)$
$= \frac{3}{2} + \frac{5}{2} = 4$

8. **(a)** $f(2) = 3^2 = \underline{\underline{9}}$

$f(3) = 3^3 = \underline{\underline{27}}$

(b) $f(2+3) = f(5) = 3^5 = 243$

$f(2) + f(3) = 9 + 27$

$= 36 \neq 243$

$\therefore\ f(2+3) \neq f(2) + f(3)$

(c) $f(2 \cdot 3) = f(6) = 3^6 = 729$

$f(2) \cdot f(3) = 9(27)$

$= 243$

$\therefore\ f(2 \cdot 3) \neq f(2) \cdot f(3)$

9. **(a)** $f(-3) = 18,$

$9 - (k-1)(-3) = 18$

$3k - 3 = 9$

$3k = 12$

$\therefore\ k = \underline{\underline{4}}$

(b) $f(x) = -12$

$9 - (4-1)x = -12$

$9 + 12 = 3x$

$21 = 3x$

$\therefore\ x = \underline{\underline{7}}$

10. **(a)** $g(2) = g(-3)$

$2^2 - 2k - 6 = (-3)^2 - (-3)k - 6$

$4 - 2k = 9 + 3k$

$-5k = 5$

$k = \underline{\underline{-1}}$

(b) $g(x) = 0$

$x^2 - (-1)x - 6 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = \underline{\underline{-3 \text{ or } 2}}$

Level 2

p.57

11. **(a)** $f(\sqrt{3}) = (\sqrt{3})^2 + 1$

$= 3 + 1 = \underline{\underline{4}}$

(b) $\sqrt{f(\sqrt{3})} = \sqrt{4}$

$= \underline{\underline{2}}$

(c) $f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2 + 1$

$= \frac{4}{9} + 1 = \underline{\underline{\frac{13}{9}}}$

(d) $\left[f\left(\frac{2}{3}\right)\right]^2 = \left(\frac{13}{9}\right)^2 = \underline{\underline{\frac{169}{81}}}$

12. $f(k-2) - f(k+2)$

$= [(k-2)^2 - 2(k-2)] - [(k+2)^2 - 2(k+2)]$

$= [k^2 - 4k + 4 - 2k + 4] - [k^2 + 4k + 4 - 2k - 4]$

$= k^2 - 6k + 8 - k^2 - 2k$

$= \underline{\underline{-8k + 8}}$

13. **(a)** $f(0) = f(2 \cdot 0)$

$= 4(0^2) - 6(0) + 7$

$= \underline{\underline{7}}$

$f(6) = f(2 \cdot 3)$

$= 4(3^2) - 6(3) + 7$

$= \underline{\underline{25}}$

(b) $f(a) = f\left(2 \cdot \frac{a}{2}\right)$

$= 4\left(\frac{a}{2}\right)^2 - 6\left(\frac{a}{2}\right) + 7$

$= \underline{\underline{a^2 - 3a + 7}}$

14. $g(1) = g(2-1)$

$= 3^2 - 4$

$= 9 - 4 = \underline{\underline{5}}$

15. $f(x+1) = g(3x)$

$(x+1)^2 - 3(x+1) + 1 = 7 + 2(3x)$

$x^2 + 2x + 1 - 3x - 3 + 1 = 7 + 6x$

$x^2 - 7x - 8 = 0$

$(x+1)(x-8) = 0$

$x = \underline{\underline{-1 \text{ or } 8}}$

16. $\because\ h(x) = k(7x + 6)$

$7x^2 - 10x - 12 = -\frac{5}{3}(7x + 6)$

$-3(7x^2 - 10x - 12) = 5(7x + 6)$

$-21x^2 + 30x + 36 = 35x + 30$

$21x^2 + 5x - 6 = 0$

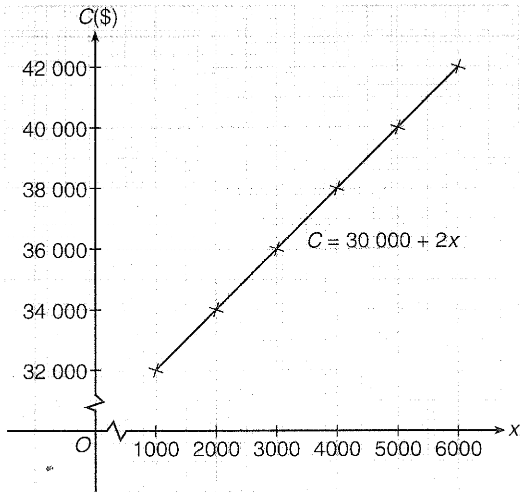
$(7x - 3)(3x + 2) = 0$

$x = \underline{\underline{-\frac{2}{3} \text{ or } \frac{3}{7}}}$

17. (a)

x	1000	2000	3000	4000	5000	6000
C ($)	32 000	34 000	36 000	38 000	40 000	42 000

(b)



(c) $C = 30\ 000 + 2x$

$39\ 000 = 30\ 000 + 2x$

$x = \underline{\underline{4500}}$

18. (a) $E = 500 + 35n + 90n$

$= \underline{\underline{500 + 125n}}$

(b) When $n = 1$,

$E = 500 + 125(1)$

$= \underline{\underline{\$625}}$

When $n = 50$,

$E = 500 + 125(50)$

$= \underline{\underline{\$6750}}$

(c) Expense per scout $= \dfrac{\$6750}{50}$

$= \underline{\underline{\$135}}$

Exercise 2.2

pp.72 – 75

Level 1

p.72

1. $y = (x - 2)^2 + 2$

$= x^2 - 4x + 6$

$\because$ The coefficient of $x^2 = 1 > 0$,

$\therefore$ The graph opens upwards.

2. $y = 4(x + 2)^2 - 3$

$= 4x^2 + 16x + 13$

$\because$ The coefficient of $x^2 = 4 > 0$,

$\therefore$ The graph opens upwards.

3. $y = -(x + 2)^2 + 8$

$= -x^2 - 4x + 4$

$\because$ The coefficient of $x^2 = -1 < 0$,

$\therefore$ The graph opens downwards.

4. $y = 5 - 3(x - 2)^2$

$= 5 - 3x^2 + 6x - 3$

$= -3x^2 + 6x + 2$

$\because$ The coefficient of $x^2 = -3 < 0$,

$\therefore$ The graph opens downwards.

5. $y = (x - 1)^2 + 4$

Vertex = $\underline{\underline{(1, 4)}}$

Substituting $x = 0$ into the given equation,

$y = (0 - 1)^2 + 4$

$= 1 + 4$

$= 5$

$\therefore$ y-intercept = $\underline{\underline{5}}$

6. $y = 2(x + 3)^2 - 6$

Vertex = $\underline{\underline{(-3, -6)}}$

Substituting $x = 0$ into the given equation,

$y = 2(0 + 3)^2 - 6$

$= 2(9) - 6$

$= 12$

$\therefore$ y-intercept = $\underline{\underline{12}}$

7. $y = -(x + 2)^2 - 5$

Vertex = $\underline{\underline{(-2, -5)}}$

Substituting $x = 0$ into the given equation,

$y = -(0 + 2)^2 - 5$

$= -4 - 5$

$= -9$

$\therefore$ y-intercept = $\underline{\underline{-9}}$

8. $y = 8 - 2(x - 1)^2$

$= -2(x - 1)^2 + 8$

Vertex = $\underline{\underline{(1, 8)}}$

Substituting $x = 0$ into the given equation,

$y = -2(0 - 1)^2 + 8$

$= -2 + 8$

$= 6$

$\therefore$ y-intercept = $\underline{\underline{6}}$

9. **(a)** $y = -(x+3)^2 + 7$

Vertex = $(-3,\ 7)$

(b) Axis of symmetry: $x = -3$

10. **(a)** $y = 3(x-2)^2 + 5$

Vertex = $(2,\ 5)$

(b) Axis of symmetry: $x = 2$

11. **(a)** $y = x^2 - 8x + 7$
$= x^2 - 8x + 4^2 - 4^2 + 7$
$= (x-4)^2 - 16 + 7$
$= (x-4)^2 - 9$

(b) Vertex = $(4,\ -9)$

(c) Axis of symmetry: $x = 4$

12. **(a)** $y = -x^2 - 10x + 9$
$= -(x^2 + 10x + 5^2 - 5^2) + 9$
$= -(x+5)^2 + 25 + 9$
$= -(x+5)^2 + 34$

(b) Vertex = $(-5,\ 34)$

(c) Axis of symmetry: $x = -5$

13. $y = x^2 + 6x + 7$
$= x^2 + 6x + 3^2 - 3^2 + 7$
$= (x+3)^2 - 9 + 7$
$= (x+3)^2 - 2$
$\therefore$ The minimum value of y is -2.

14. $y = 2x^2 - 8x + 5$
$= 2(x^2 - 4x) + 5$
$= 2(x^2 - 4x + 2^2 - 2^2) + 5$
$= 2(x-2)^2 - 8 + 5$
$= 2(x-2)^2 - 3$
$\therefore$ The minimum value of y is -3.

15. $y = -x^2 - 8x + 1$
$= -(x^2 + 8x) + 1$
$= -(x^2 + 8x + 4^2 - 4^2) + 1$
$= -(x+4)^2 + 16 + 1$
$= -(x+4)^2 + 17$
$\therefore$ The maximum value of y is 17.

16. $y = -3x^2 + 12x - 4$
$= -3(x^2 - 4x) - 5$
$= -3(x^2 - 4x + 2^2 - 2^2) - 4$
$= -3(x-2)^2 + 12 - 4$
$= -3(x-2)^2 + 8$
$\therefore$ The maximum value of y is 8.

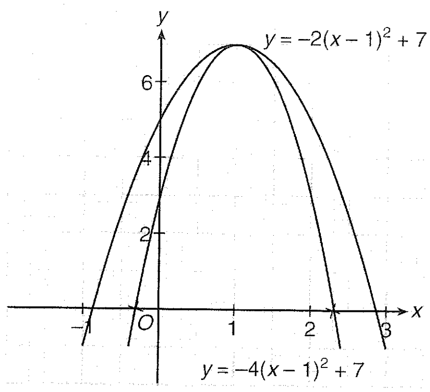
17. $y = (x+1)(x-7) + 3$
$= x^2 - 7x + x - 7 + 3$
$= x^2 - 6x - 4$
$= x^2 - 6x + 3^2 - 3^2 - 4$
$= (x-3)^2 - 13$
$\therefore$ The minimum value of y is -13.

18. $y = x(8-x) + 10$
$= 8x - x^2 + 10$
$= -(x^2 - 8x) + 10$
$= -(x^2 - 8x + 4^2 - 4^2) + 10$
$= -(x-4)^2 + 16 + 10$
$= -(x-4)^2 + 26$
$\therefore$ The maximum value of y is 26.

19. **(a)** Graph (I): Vertex = $(1,\ 7)$

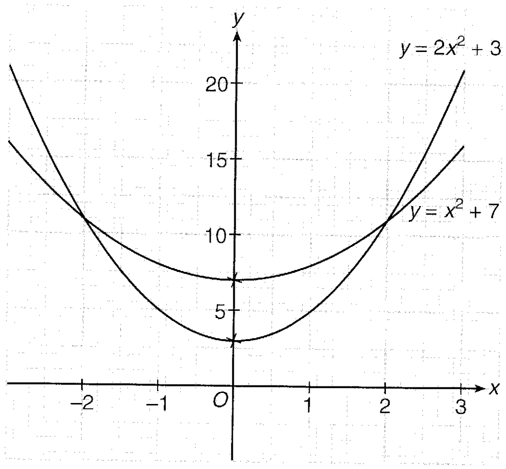
Graph (II): Vertex = $(1,\ 7)$

(b)



Graph (I) opens wider.

20. **(a)**



(b) Graph (II) opens narrower.

Level 2

p.73

21. (a) Vertex = $\underline{\underline{(-3,\ 5)}}$

(b) Substituting $y = 0$ into the given equation,

$$0 = -(x+3)^2 + 5$$
$$x + 3 = \pm\sqrt{5}$$
$$x = -3 \pm \sqrt{5}$$

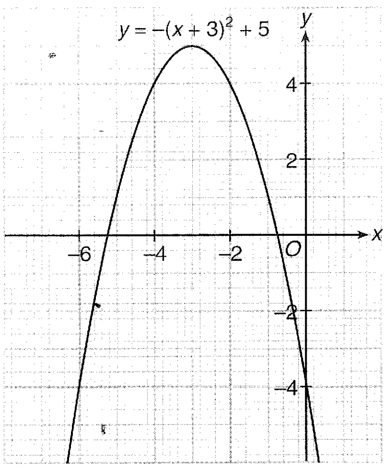
$\therefore$ x-intercepts = $\underline{\underline{-3 \pm \sqrt{5}}}$

Substituting $x = 0$ into the given equation,

$$y = -(0+3)^2 + 5$$
$$= -9 + 5 = -4$$

$\therefore$ y-intercept = $\underline{\underline{-4}}$

(c)



22. (a) Since the graph passes through (2, −9) and (−1, 9), we have

$$\begin{cases} -9 = a(2-2)^2 + k \\ 9 = a(-1-2)^2 + k \end{cases}$$

$$\begin{cases} -9 = k \text{ .... (1)} \\ 9 = 9a + k \text{ .... (2)} \end{cases}$$

From (1), $k = \underline{\underline{-9}}$ .... (3)

Substituting (3) into (2), we have

$$9 = 9a - 9$$
$$18 = 9a$$
$$a = \underline{\underline{2}}$$

(b) The equation of the graph is

$$y = 2(x-2)^2 - 9$$

$\therefore$ Vertex = $\underline{\underline{(2, -9)}}$

(c) Axis of symmetry: $\underline{\underline{x = 2}}$

23. (a) Since the y-intercept is 8, we have

$$c = \underline{\underline{8}}$$

(b) $y = -x^2 + 2x + 8$

$$= -(x^2 - 2x) + 8$$
$$= -(x^2 - 2x + 1 - 1) + 8$$
$$= -(x^2 - 1)^2 + 1 + 8$$
$$= \underline{\underline{-(x-1)^2 + 9}}$$

(c) Vertex = $\underline{\underline{(1,\ 9)}}$

24. (a) Since the y-intercept is −5, we have

$$c = \underline{\underline{-5}}$$

Substituting $x = -1$, $y = 0$ into the given equation, we have

$$0 = (-1)^2 + b(-1) - 5$$
$$b = 1 - 5 = \underline{\underline{-4}}$$

(b) When $y = 0$,

$$x^2 - 4x - 5 = 0$$
$$(x+1)(x-5) = 0$$
$$x = -1 \text{ or } 5$$

$\therefore$ $p = \underline{\underline{5}}$

(c) $y = x^2 - 4x - 5$

$$= x^2 - 4x + 2^2 - 2^2 - 5$$
$$= \underline{\underline{(x-2)^2 - 9}}$$

(d) Vertex = $\underline{\underline{(2, -9)}}$

25. (a) When $y = 0$,

$$-x^2 + 4x + 5 = 0$$
$$x^2 - 4x - 5 = 0$$
$$(x+1)(x-5) = 0$$
$$x = -1 \text{ or } 5$$

Therefore, the coordinates of P and Q are (−1, 0) and (5, 0) respectively.

$\therefore$ $PQ = 5 - (-1) = \underline{\underline{6 \text{ units}}}$

(b) $y = -x^2 + 4x + 5$

$$= -(x^2 - 4x) + 5$$
$$= -(x^2 - 4x + 2^2 - 2^2) + 5$$
$$= -(x-2)^2 + 4 + 5$$
$$= -(x-2)^2 + 9$$

$\therefore$ Vertex = $\underline{\underline{(2,9)}}$

(c) $PQ = 6$ units, $PS = 9$ units

Area of $PQRS = 6 \times 9$

$$= \underline{\underline{54 \text{ sq. units}}}$$

26. (a) Area $= (400 - x)(480 + 2x)$

$$= 192\ 000 - 2x^2 - 480x + 800x$$
$$= \underline{\underline{(-2x^2 + 320x + 192\ 000)\ \text{m}^2}}$$

(b) Area $= -2(x^2 - 160x - 96\ 000)$

$= -2[x - 16x + (80)^2 - (80)^2 - 96\ 000]$

$= -2(x - 80)^2 - 2[-6400 - 96\ 000]$

$= -2(x - 80)^2 + 204\ 800$

$\therefore$ Maximum area is attained when $x = 80$.

$\therefore$ Maximum area is 204 800 m^2.

27. (a) $C = 2x^2 - 84x + 2400$

$C = 2(100)^2 - 84(100) + 2400$

$= \underline{\underline{\$14\ 000}}$

(b) $C = 2x^2 - 84x + 2400$

$= 2(x^2 - 42x + 1200)$

$= 2[x^2 - 42x + (21)^2 - (21)^2 + 1200]$

$= 2(x - 21)^2 + 2[-(21)^2 + 1200]$

$= 2(x - 21)^2 + 1518$

$\therefore$ The minimum cost is \$1518.

(c) From (b), for the minimum cost, the number of watches made everyday is 21.

28. (a) When $h = 1.8$,

$1.8 = 10t - 5t^2$

$5t^2 - 10t + \frac{9}{5} = 0$

$25t^2 - 50t + 9 = 0$

$(5t - 1)(5t - 9) = 0$

$t = \underline{\underline{\frac{1}{5} \text{ or } \frac{9}{5}}}$

(b) The rubber will reach the ground when $h = 0$.

$10t - 5t^2 = 0$

$5t(2 - t) = 0$

$t = 0 \text{ or } 2$

$\therefore$ The rubber will reach the ground after 2 seconds.

(c) $h = 10t - 5t^2$

$= -5(t^2 - 2t)$

$= -5(t^2 - 2t + 1 - 1)$

$= -5(t - 1)^2 + 5$

$\therefore$ The maximum height is 5 m.

29. (a) Profit = Selling price − Cost

$= (150 - x)x - \left(\frac{1}{2}x^2 - 150x + 100\right)$

$= 150x - x^2 - \frac{1}{2}x^2 + 150x - 100$

$= \underline{\underline{-\frac{3}{2}x^2 + 300x - 100}}$ .... (*)

(b) Substituting $x = 80$ into (*), we have

Profit $= -\frac{3}{2}(80)^2 + 300(80) - 100$

$= \underline{\underline{\$14\ 300}}$

(c) Profit $= -\frac{3}{2}x^2 + 300x - 100$

$= -\frac{3}{2}(x^2 - 200x) - 100$

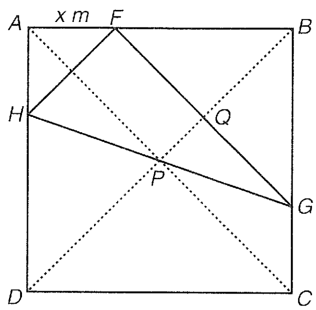
$= -\frac{3}{2}(x^2 - 200x + 100^2 - 100^2) - 100$

$= -\frac{3}{2}(x - 100)^2 + 15\ 000 - 100$

$= -\frac{3}{2}(x - 100)^2 + 14\ 900$

$\therefore$ The maximum profit is \$14 900.

30. (a)



Mark two points P and Q on the figure as shown above.

$\angle DPC = 90°$ *(square properties)*

$\angle PQG = \angle DPC$ *(corr. ∠s, FG // AC)*

$= 90°$

$\angle HFG = \angle PQG$ *(corr. ∠s, HF // DB)*

$= 90°$

$\therefore$ ΔFGH is a right-angled triangle.

(b) $AF = x$, $FB = 12 - x$

$HF = \sqrt{x^2 + x^2}$

$= \sqrt{2x^2}$

$FG = \sqrt{(12 - x)^2 + (12 - x)^2}$

$= \sqrt{2(12 - x)^2}$

Area of $\Delta FGH = \frac{1}{2}(\sqrt{2x^2})(\sqrt{2(12 - x)^2})$

$= \frac{1}{2}\sqrt{4x^2(12 - x)^2}$

$= x(12 - x)$

$= \underline{\underline{-x^2 + 12x}}$

(c) Area $= -x^2 + 12x$

$= -(x^2 - 12x)$

$= -(x^2 - 12x + 6^2 - 6^2)$

$= -(x - 6)^2 + 36$

$\therefore$ The area of ΔFGH attains its maximum value when $x = 6$.

Exercise 2.3

pp.81 – 86

Level 1

p.81

1. y-intercept = 5,
   number of vertex = 0

2. y-intercept = 8,
   number of vertices = 2

3. When $x = 0$,
   $y = -(0 + 1)(0 - 5)$
   $= -(1)(-5) = 5$
   $\therefore$ y-intercept = 5
   When $y = 0$,
   $0 = -(x + 1)(x - 5)$
   $x = -1$ or 5
   $\therefore$ x-intercepts = -1 and 5

4. When $x = 0$,
   $y = \dfrac{12}{0 + 3} = 4$
   $\therefore$ y-intercept = 4
   The graph does not have any x-intercept.

5. (a) From the graph,
   x-intercept = 8 and
   y-intercept = -4.

   (b) The graph does not have any vertex.

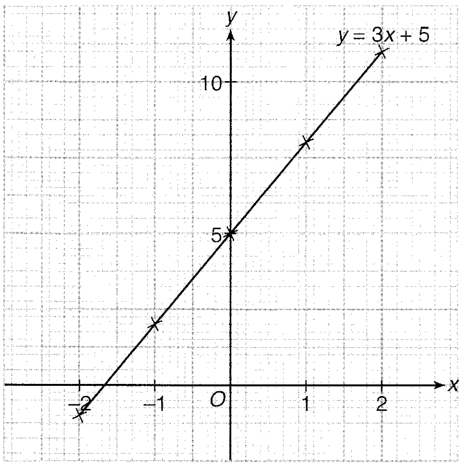
6. (a) From the graph,
   y-intercept = 1

   (b) From the graph, when $y = 6$,
   $x = 0.8$ *(correct to 1 decimal place)*

7. (a)

x	-2	-1	0	1	2
y	-1	2	5	8	11

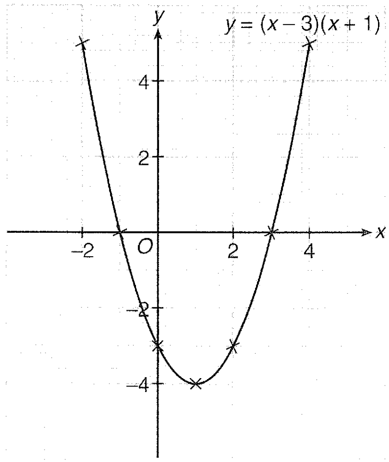
   (b)

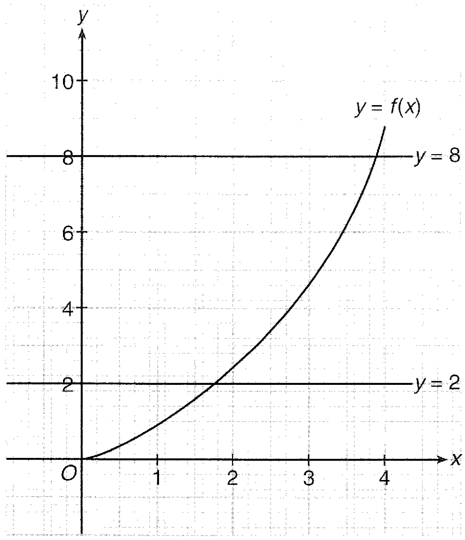


8. (a)

x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

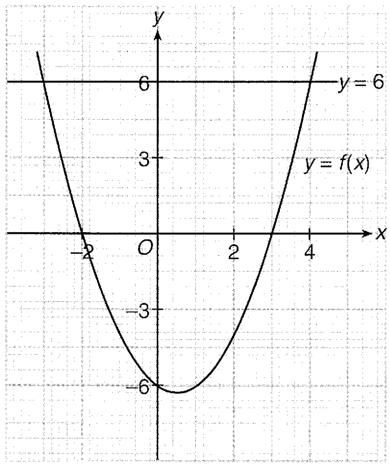
   (b)





9. Draw a line $y = 2$ on the graph, the required solution is $x < 1.8$.

10. Draw a line $y = 8$ on the graph, the required solution is $x \geq 3.9$.



11. From the graph, the required solution is $x < -2$ or $x > 3$.

12. Draw a line $y = 6$ on the graph, the required solution is $-3 \le x \le 4$.

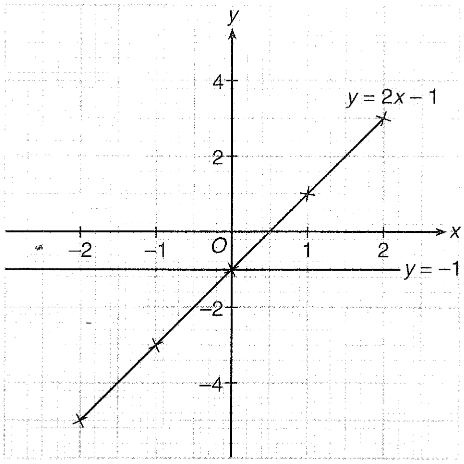
Level 2

p.84

13. (a)

x	-2	-1	0	1	2
y	-5	-3	-1	1	3

(b)



(c) x-intercept = $\underline{\underline{0.5}}$

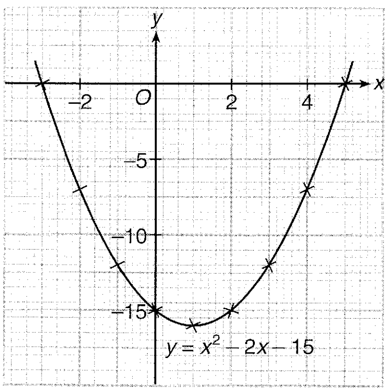
y-intercept = $\underline{\underline{-1}}$

(d) Draw a line $y = -1$ on the graph, the required solution is $x \ge 0$.

14. (a)

x	-3	-2	-1	0	1	2	3	4	5
y	0	-7	-12	-15	-16	-15	-12	-7	0

(b)



(c) (i) x-intercepts = $\underline{\underline{-3 \text{ and } 5}}$

y-intercept = $\underline{\underline{-15}}$

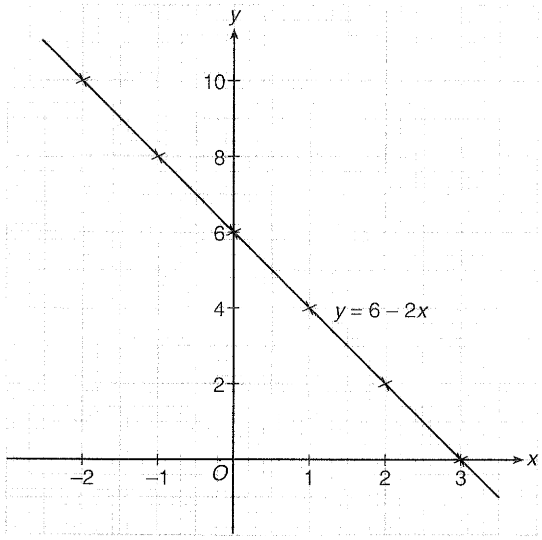
(ii) Vertex = $\underline{\underline{(1, -16)}}$

15. (a) Substituting $x = -2$, $y = 10$ into the function, we have

$$10 = k - 2(-2)$$
$$10 = k + 4$$
$$k = \underline{\underline{6}}$$

x	-2	-1	0	1	2	3
y	10	8	6	4	2	0

(b)



(c) From the above graph,

the area $= \left(\dfrac{3 \times 6}{2}\right) = \underline{\underline{9 \text{ sq. units}}}$

16. (a) Substituting $x = 1$, $y = 6$ into the given equation, we have

$$6 = (1 - 3)^2 + k$$
$$6 = 4 + k$$
$$k = \underline{\underline{2}}$$

(b) y-intercept = $\underline{\underline{11}}$

Vertex = $\underline{\underline{(3,\ 2)}}$

(c) $x^2 - 6x + 7 < 0$ can be written as

$$x^2 - 6x + 3^2 - 3^2 + 7 < 0$$
$$(x - 3)^2 - 9 + 7 < 0$$
$$(x - 3)^2 - 2 < 0$$
$$(x - 3)^2 + 2 - 2 - 2 < 0$$
$$(x - 3)^2 + 2 < 4$$

$\therefore$ Draw a line $y = 4$ on the graph, the required solution is $1.6 < x < 4.4$.

17. (a) From the graph, the y-intercept is 2.

$$\therefore\quad 2 = \frac{4}{0^2 + k}$$
$$k = \frac{4}{2} = \underline{\underline{2}}$$

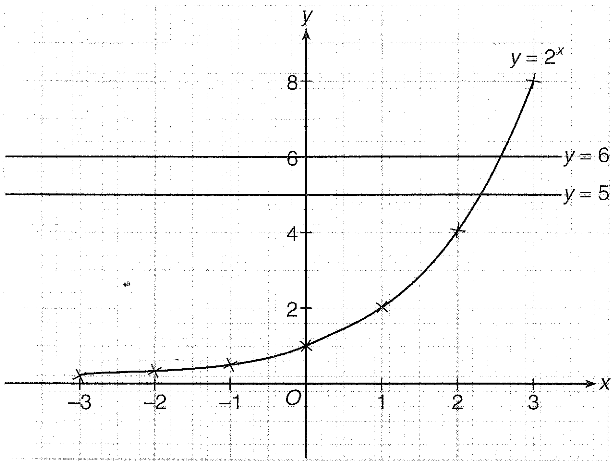
(b) No, the graph does not cut the x-axis.

(c) Draw a line $y = 1$ on the graph, the required solution is $0 \leq x \leq 1.4$.

18. (a)

x	-3	-2	-1	0	1	2	3
y	0.125	0.25	0.5	1	2	4	8

(b)



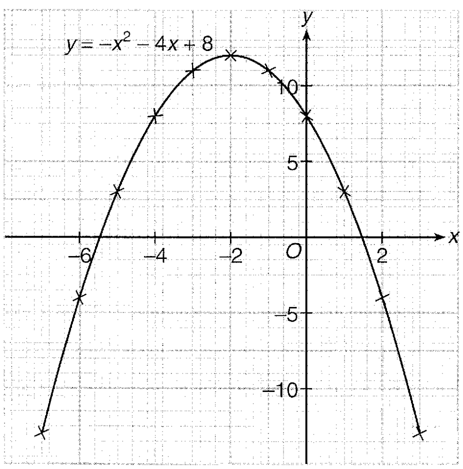
(b) (i) Draw a line $y = 6$ on the graph, the required solution is $x \geq 2.6$.

(ii) Draw a line $y = 5$ on the graph, the required solution is $x \leq 2.3$.

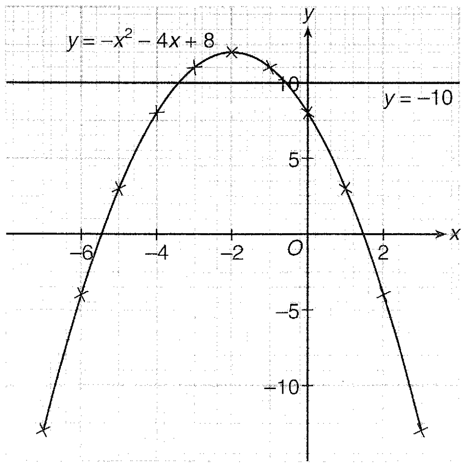
19. (a)

x	-6	-5	-4	-3	-2	-1	0	1	2
y	-4	3	8	11	12	11	8	3	-4

(b)



(c)



(d) (i) Draw a line $y = 0$ on the graph, the required solution is $-5.5 < x < 1.5$.

(ii) $x^2 + 4x - 8 > 10$ can be written as $-x^2 - 4x + 8 < 10$.
Draw a line $y = 10$ on the graph, the required solutions is $x < -3.4$ or $x > -0.6$.

Exercise 2.4

pp.88 – 91

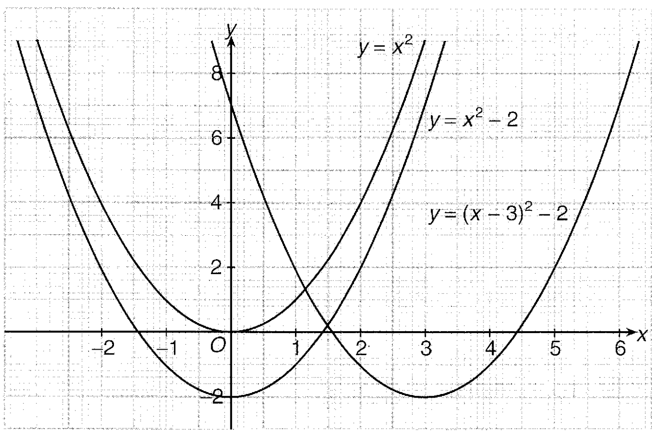
Level 1

p.88

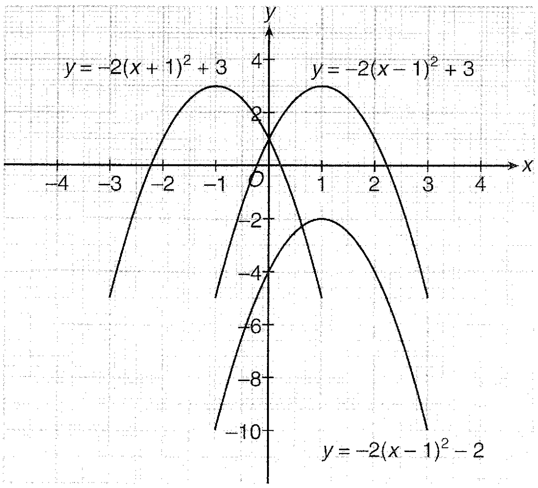
1. (a) Translate the graph of $y = x^2 + 3$
1 unit to the left will obtain the graph of $y = (x + 1)^2 + 3$.

(b) Translate the graph of $y = -(x - 1)^2$
4 units upwards will obtain the graph of $y = -(x - 1)^2 + 2$.

2.



3.



4. **(a)** $y = (x - 7)^2 + 4$

(b) $y = x^2 + 4 + 6$

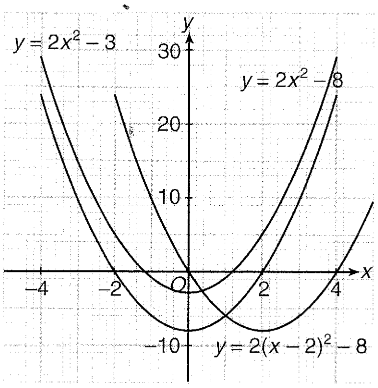
$= \underline{\underline{x^2 + 10}}$

5. **(a)** $y = -(x + 2)^2 + 5$

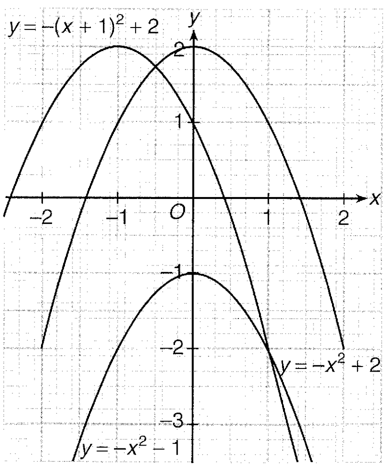
(b) $y = -x^2 + 5 - 4$

$= \underline{\underline{-x^2 + 1}}$

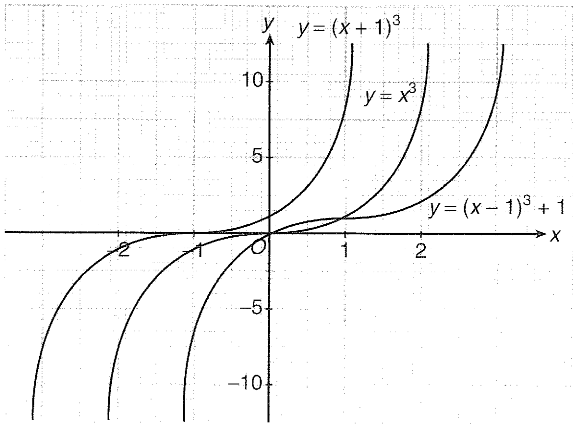
6.



7.



8.



9. $y = (x + 2 - 3)^2 - 3 + 4$

$= (x - 1)^2 + 1$

∴ A translation of 3 units to the right and 4 units upwards of $y = (x + 2)^2 - 3$ will obtain the graph of $y = (x - 1)^2 + 1$.

10. $y = (x - 1)^3 - 1 + 3$

$= (x - 1)^3 + 2$

∴ A translation of 1 unit to the right and 3 units upwards of $y = x^3 - 1$ will obtain the graph of $y = (x - 1)^3 + 2$.

Level 2 p.91

11. **(a)** $-x^2 + 4x + 9 = -(x^2 - 4x) + 9$

$= -(x^2 - 4x + 2^2 - 2^2) + 9$

$= -(x - 2)^2 + 4 + 9$

$= \underline{\underline{-(x - 2)^2 + 13}}$

(b) $y = -x^2 + 4x + 9$

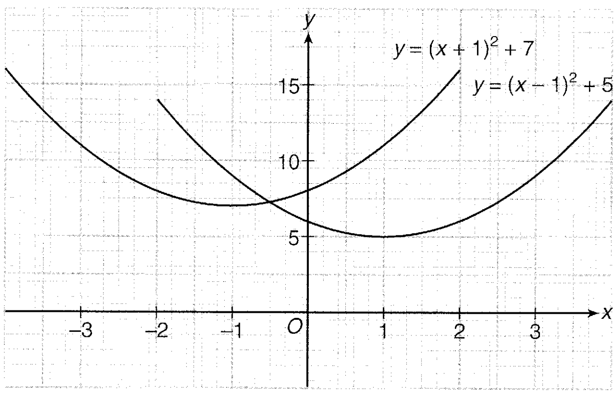
$= -(x - 2)^2 + 13$

If we translate the graph 3 units to the right, then the equation of the image is $y = -(x - 2 - 3)^2 + 13$

$= -(x - 5)^2 + 13$

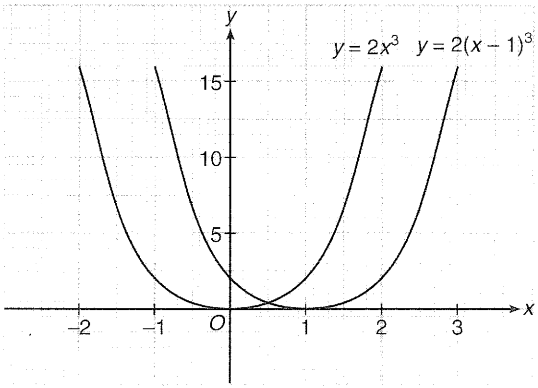
(c) Vertex = $\underline{\underline{(5, 13)}}$

12. **(a)**



(b) A translation of 2 units to the left and 2 units upwards of $y = (x - 1)^2 + 5$ will obtain the graph of $y = (x + 1)^2 + 7$.

13. (a)



(b) A translation of 1 unit to the right of $y = 2x^3$ will obtain the graph of $y = 2(x - 1)^3$.

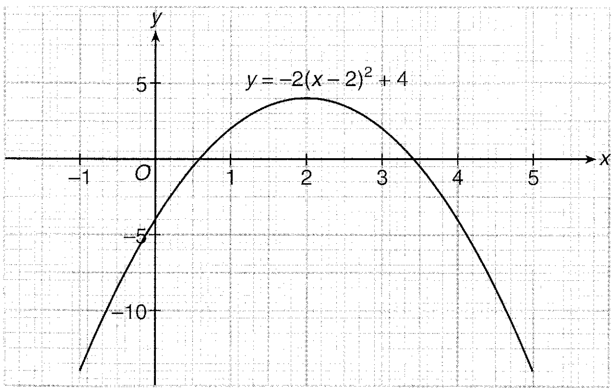
14. (a) If the maximun value of y is 4, then

$$k^2 - k + 2 = 4$$
$$\therefore\ k^2 - k + 2 - 4 = 0$$
$$k^2 - k - 2 = 0$$
$$(k - 2)(k + 1) = 0$$
$$k = \underline{\underline{2}} \text{ or } -1 \text{ (rejected)}$$

(b) When $x = 0$,

$$y = -2(0 - 2)^2 + (2^2 - 2 + 2)$$
$$= -2(4)^2 + 4$$
$$= \underline{\underline{-4}}$$

(c)



(d) $y = -2(x - 2 - 2)^2 + 4$

$$= \underline{\underline{-2(x - 4)^2 + 4}}$$

15. (a)

$$2x^2 - 5x + 0.75$$
$$= 2x^2 - 5x + \frac{3}{4}$$
$$= 2\left(x^2 - \frac{5}{2}x + \frac{3}{8}\right)$$
$$= 2\left[x^2 - \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 + \frac{3}{8}\right]$$
$$= 2\left(x - \frac{5}{4}\right)^2 + 2\left[-\left(\frac{5}{4}\right)^2 + \frac{3}{8}\right]$$
$$= \underline{\underline{2\left(x - \frac{5}{4}\right)^2 - \frac{19}{8}}}$$

(b) Translate the graph of $y = 2x^2 - 5x + 0.75$ 8 units to the left, we have

$$2\left(x - \frac{5}{4} + 8\right)^2 - \frac{19}{8}$$
$$= \underline{\underline{2\left(x + \frac{27}{4}\right)^2 - \frac{19}{8}}}$$

(c) From (b), translate the graph of $y = 2\left(x + \frac{27}{4}\right)^2 - \frac{19}{8}$ 1 unit upwards, we have

$$2\left(x + \frac{27}{4}\right)^2 - \frac{19}{8} + 1$$
$$= \underline{\underline{2\left(x + \frac{27}{4}\right)^2 - \frac{11}{8}}}$$

(d) From part (c),

$$\text{Vertex} = \underline{\underline{\left(-\frac{27}{4}, \frac{11}{8}\right)}}$$

16. (a)

$$-4x^2 + 8x + 12$$
$$= -4(x^2 - 2x - 3)$$
$$= -4[x^2 - 2x + (1)^2 - (1)^2 - 3]$$
$$= -4(x - 1)^2 - 4[-(1)^2 - 3]$$
$$= \underline{\underline{-4(x - 1)^2 + 16}}$$

(b) Translate the graph m units to the right, the equation becomes $-4(x - 1 - m)^2 + 16$.

$\because$ The graph passes through the point (2, 0),

$$\therefore\ -4(2 - 1 - m)^2 + 16 = 0$$
$$(1 - m)^2 = 4$$
$$1 - m = \pm 2$$
$$m = \underline{\underline{-1 \text{ or } 3}}$$

(c) For $m = -1$,

$$y = -4[x - 1 - (-1)]^2 + 16$$
$$= \underline{\underline{-4x^2 + 16}}$$

For $m = 3$,

$$y = -4[x - 1 - (3)]^2 + 16$$
$$= \underline{\underline{-4(x - 4)^2 + 16}}$$

Revision Exercise 2

pp.96 – 107

Level 1

p.96

1. **(a)** n is the independent variable.

(b) From the graph, $C = 900$ when $n = 0$.

$\therefore \; 900 = k + 80(0)$

$k = \underline{\underline{900}}$

(c) When $n = 30$,

$C = 900 + 80(30)$

$= \underline{\underline{3300}}$

2. **(a)** $f(5) = \sqrt{2(5) - 1}$

$= \sqrt{9} = \underline{\underline{3}}$

$f(3) = \sqrt{2(3) - 1}$

$= \underline{\underline{\sqrt{5}}}$

(b) Since $\sqrt{5}$ is an irrational number, $f(3)$ is an irrational number.

3. **(a)** $g(4) = \dfrac{6}{4 - 2} = \underline{\underline{3}}$

(b) $g(3) = \dfrac{6}{3 - 2} = 6$

$g(-4) = \dfrac{6}{-4 - 2} = -1$

$\therefore \; g(3) - g(-4) = 6 - (-1)$

$= \underline{\underline{7}}$

4. **(a)** $f(-2) = (-2)^2 - 4(-2) + 2$

$= 4 + 8 + 2 = \underline{\underline{14}}$

(b) $f(2) = 2^2 - 4(2) + 2$

$= -2$

$\therefore \; 2f(-2) + 3f(2) = 2(14) + 3(-2)$

$= 28 - 6 = \underline{\underline{22}}$

5. **(a)** $h(a - 1) - h(a + 1)$

$= [(a - 1)^2 - 3(a - 1) - 6] - [(a + 1)^2 - 3(a + 1) - 6]$

$= [a^2 - 2a + 1 - 3a + 3 - 6] - [a^2 + 2a + 1 - 3a - 3 - 6]$

$= (a^2 - 5a - 2) - (a^2 - a - 8)$

$= a^2 - 5a - 2 - a^2 + a + 8$

$= \underline{\underline{-4a + 6}}$

(b) $h(a + 1) = 4$

$a^2 - a - 8 = 4$

$a^2 - a - 12 = 0$

$(a + 3)(a - 4) = 0$

$a = \underline{\underline{-3 \text{ or } 4}}$

6. **(a)** $g(0) = 4 \cdot 6^0 - 2 \cdot 3^0$

$= 4 - 2 = 2$

$g(1) = 4 \cdot 6^1 - 2 \cdot 3^1$

$= 24 - 6 = 18$

$\therefore \; g(0) + g(1) = 2 + 18$

$= \underline{\underline{20}}$

(b) $g(-1) = 4 \cdot 6^{-1} - 2 \cdot 3^{-1}$

$= \dfrac{4}{6} - \dfrac{2}{3} = 0$

$g(1) - g(-1) = 18 - 0$

$= \underline{\underline{18}}$

7. $f(k + 1) = k$

$(k + 1)^2 - 2(k + 1) - 5 = k$

$k^2 + 2k + 1 - 2k - 2 - 5 - k = 0$

$k^2 - k - 6 = 0$

$(k + 2)(k - 3) = 0$

$k = \underline{\underline{-2 \text{ or } 3}}$

8. **(a)** $g(0) = g(2 \cdot 0)$

$= 8(0)^2 - 6(0) + 1$

$= \underline{\underline{1}}$

(b) $g(4) = g(2 \cdot 2)$

$= 8(2)^2 - 6(2) + 1$

$= 32 - 12 + 1 = \underline{\underline{21}}$

(c) $g(-2) = g(2 \cdot -1)$

$= 8(-1)^2 - 6(-1) + 1$

$= 8 + 6 + 1 = \underline{\underline{15}}$

(d) $g(1) = g(2 \cdot \frac{1}{2})$

$= 8\left(\dfrac{1}{2}\right)^2 - 6\left(\dfrac{1}{2}\right) + 1$

$= 2 - 3 + 1 = \underline{\underline{0}}$

9. **(a)** Axis of symmetry: $\underline{\underline{x = 2}}$

(b) y is minimum when $x = 2$.

$\therefore$ Minimum value is 7.

(c) Vertex = $\underline{\underline{(2,\ 7)}}$

10. **(a)** y-intercept = $\underline{\underline{-21}}$

(b) When $x = 0, y = -21$.

$\therefore\ -21 = -(0-5)^2 + k$

$-21 = -25 + k$

$k = \underline{\underline{4}}$

(c) Vertex = $\underline{\underline{(5,\ 4)}}$

11. **(a)** The graph opens upwards.

(b) When $x = 0$, $y = 0^2 - 4(0) + 7$

$= 7$

$\therefore$ y-intercept = $\underline{\underline{7}}$

(c)
$$\begin{aligned} y &= x^2 - 4x + 7 \\ &= x^2 - 4x + 2^2 - 2^2 + 7 \\ &= (x-2)^2 - 4 + 7 \\ &= (x-2)^2 + 3 \end{aligned}$$

$\therefore$ Vertex = $\underline{\underline{(2,\ 3)}}$

Axis of symmetry: $\underline{\underline{x = 2}}$

12.
$$\begin{aligned} y &= -x^2 - 4x + 5 \\ &= -(x^2 + 4x) + 5 \\ &= -(x^2 + 4x + 2^2 - 2^2) + 5 \\ &= -(x+2)^2 + 4 + 5 \\ &= -(x+2)^2 + 9 \end{aligned}$$

$\therefore$ The maximum value of y is 9.

13.
$$\begin{aligned} y &= 2x^2 - 12x - 1 \\ &= 2(x^2 - 6x) - 1 \\ &= 2(x^2 - 6x + 3^2 - 3^2) - 1 \\ &= 2(x-3)^2 - 18 - 1 \\ &= 2(x-3)^2 - 19 \end{aligned}$$

$\therefore$ The minimum value of y is -19.

14.
$$\begin{aligned} y &= x(2+x) - 11 \\ &= x^2 + 2x - 11 \\ &= x^2 + 2x + 1 - 1 - 11 \\ &= (x+1)^2 - 12 \end{aligned}$$

$\therefore$ The minimum value of y is -12.

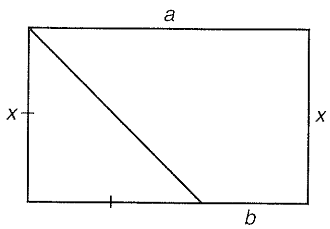
15.
$$\begin{aligned} y &= (x+3)(9-x) \\ &= 9x - x^2 + 27 - 3x \\ &= -x^2 + 6x + 27 \\ &= -(x^2 - 6x) + 27 \\ &= -(x^2 - 6x + 3^2 - 3^2) + 27 \\ &= -(x-3)^2 + 9 + 27 \\ &= -(x-3)^2 + 36 \end{aligned}$$

$\therefore$ The maximum value of y is 36.

16. **(a)**
$$\begin{aligned} A &= \left(\frac{x}{4}\right)^2 + \left(\frac{24-x}{4}\right)^2 \\ &= \frac{x^2 + 576 - 48x + x^2}{16} \\ &= \frac{2x^2 - 48x + 576}{16} \\ &= \frac{2(x^2 - 24x + 12^2 - 12^2) + 576}{16} \\ &= \frac{2(x-12)^2 - 288 + 576}{16} \\ &= \underline{\underline{\frac{1}{8}(x-12)^2 + 18}} \end{aligned}$$

(b) From (a), the minimum value of A is 18 sq. units.

17. **(a)**



$a = \dfrac{12 - 2x}{2} = 6 - x$

$b = 6 - x - x = 6 - 2x$

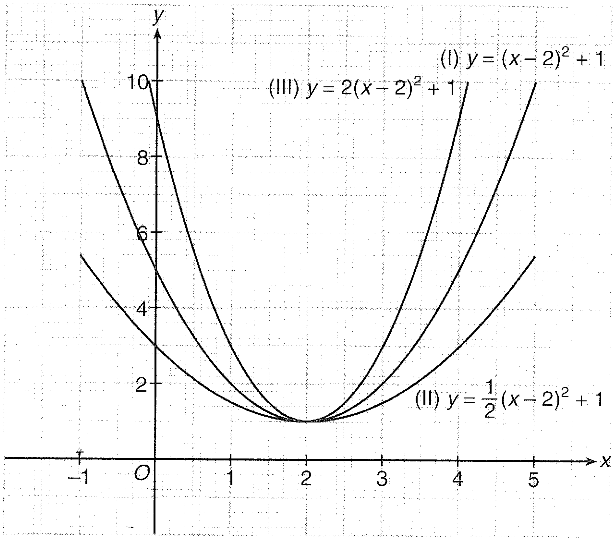
$$\begin{aligned} \therefore\ \text{Area } A &= (a+b)\frac{x}{2} \\ &= (6 - x + 6 - 2x)\frac{x}{2} \\ &= \underline{\underline{\frac{(12-3x)x}{2}}} \end{aligned}$$

(b)
$$\begin{aligned} A &= \frac{(12-3x)x}{2} \\ &= \frac{12x - 3x^2}{2} \\ &= \frac{-3(x^2 - 4x)}{2} \\ &= \frac{-3(x^2 - 4x + 2^2 - 2^2)}{2} \\ &= \frac{-3(x-2)^2 + 12}{2} \\ &= -\frac{3}{2}(x-2)^2 + 6 \end{aligned}$$

$\therefore$ A is maximum when $x = 2$.

(c) The maximum value of A is 6.

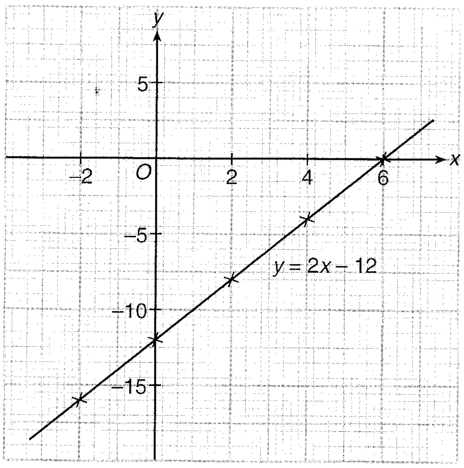
18. (a)



(b) Graph (II) opens the widest.

19. (a)

x	−2	0	2	4	6
y	−16	−12	−8	−4	0

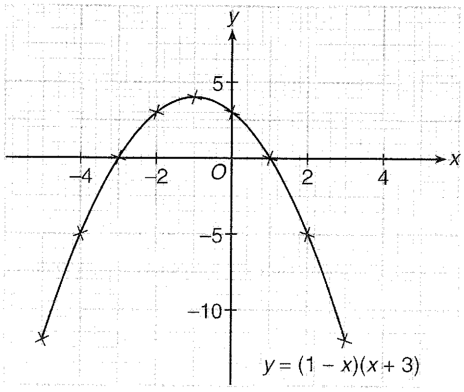


(b) x-intercept = $\underline{\underline{6}}$

y-intercept = $\underline{\underline{-12}}$

20. (a)

x	−5	−4	−3	−2	−1	0	1	2	3
y	−12	−5	0	3	4	3	0	−5	−12

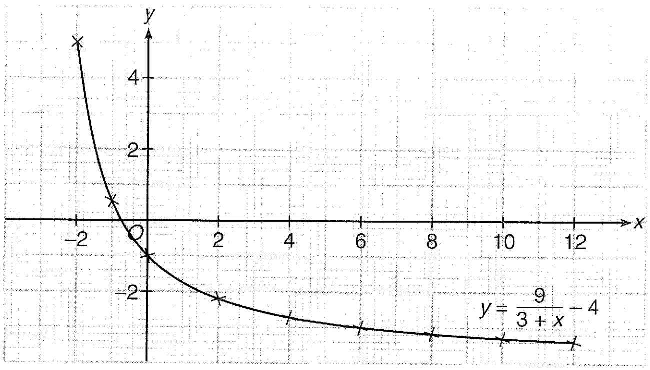


(b) Vertex = $\underline{\underline{(-1,\ 4)}}$

Axis of symmetry: $\underline{\underline{x = -1}}$

21 (a)

x	−2	−1	0	2	4	6	8	10	12
y	5	0.5	−1	−2.2	−2.7	−3	−3.2	−3.3	−3.4



(b) y-intercept = $\underline{\underline{-1}}$

(c) If $x \geq -2$, then the range of the values of y is $-4 < y \leq 5$.

22. (a) x-intercept = $\underline{\underline{-0.8}}$

y-intercept = $\underline{\underline{1.9}}$

(b) From the graph,

(i) the required solution is

$x < -0.8$;

(ii) the required solution is

$x \geq -0.8$.

23. (a) From the graph, the required solution is

$-0.8 \leq x \leq 2.7$.

(b)
$$x^2 - 2x - 5 > 0$$
$$x^2 - 2x - 2 - 3 > 0$$
$$x^2 - 2x - 2 > 3$$

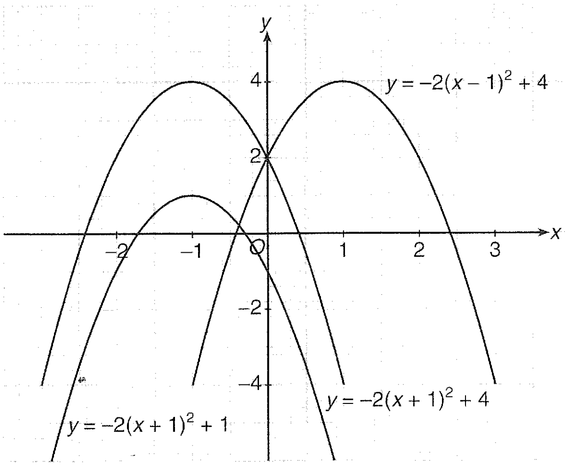
Draw a line $y = 3$ on the graph, the required solution is

$x < 1.4$ or $x > 3.4$.

24. (a) $y = (x - 3 - 4)^2 + 2$
$= (x - 7)^2 + 2$

(b) $y = (x - 3)^2 + 2 + 3$
$= (x - 3)^2 + 5$

25.



Level 2

p.100

26. (a) $f(a) = f(b)$

$$3a^2 - 9a + 4 = 3b^2 - 9b + 4$$
$$3a^2 - 3b^2 - 9(a - b) = 0$$
$$3(a - b)(a + b) - 9(a - b) = 0$$
$$3(a - b)[(a + b) - 3] = 0$$
$\because \ a \neq b$
$\therefore \ (a + b) - 3 = 0$
$a + b = \underline{\underline{3}}$

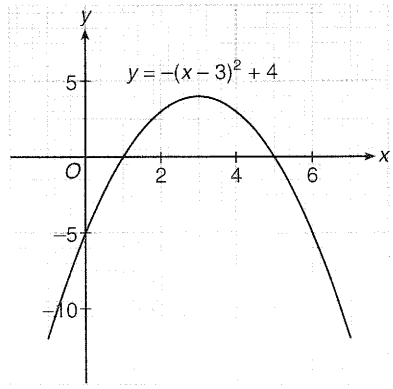
(b) $f(a + b) = f(3)$
$= 3(3^2) - 9(3) + 4$
$= \underline{\underline{4}}$

27. (a) $y = -x^2 + 6x + c$
$= -(x^2 - 6x) + c$
$= -(x^2 - 6x + 3^2 + 3^2) + c$
$= \underline{\underline{-(x - 3)^2 + 9 + c}}$

(b) The maximum value of y is 4.
$\therefore \ 9 + c = 4$
$c = \underline{\underline{-5}}$

(c) y-intercept $= \underline{\underline{-5}}$

(d)



28. (a) Since the graph touches the x-axis at only one point, $\Delta = 0$.
$$[-(8 + k)]^2 - 4(-3)(k - 1) = 0$$
$$k^2 + 16k + 64 + 12k - 12 = 0$$
$$k^2 + 28k + 52 = 0$$
$$(k + 26)(k + 2) = 0$$
$$k = \underline{\underline{-26 \text{ or } -2}}$$

(b) If k takes the smallest value, then the equation of the graph becomes $y = -3x^2 + 18x - 27$.
When $y = 0$,
$$-3x^2 + 18x - 27 = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x - 3)^2 = 0$$
$$x = 3$$
$\therefore \ T = \underline{\underline{(3, 0)}}$

29. (a) Since the y-intercept $= -6$,
$c = \underline{\underline{-6}}$

(b) $y = x^2 - 4x - 6$
$= x^2 - 4x + 2^2 - 2^2 - 6$
$= (x - 2)^2 - 10$
$\therefore \ V = \underline{\underline{(2, -10)}}$
When $y = -6$,
$$x^2 - 4x - 6 = -6$$
$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$
$$x = 0 \text{ or } 4$$
$\therefore \ Q = \underline{\underline{(4, -6)}}$

(c) Area of $OPQR = 4 \times 6$
$= 24$ sq. units

Area of $\Delta OVR = \frac{1}{2}(4)(10)$
$= 20$ sq. units

$\therefore$ The difference between the areas is $24 - 20 = 4$ sq. units.

30. (a) $A = \frac{(120 - 3x)}{2}x$

$= \frac{1}{2}(120x - 3x^2)$

(b) $A = \frac{1}{2}(120x - 3x^2)$

$= \frac{1}{2}(-3)(x^2 - 40x)$

$= -\frac{3}{2}(x^2 - 40x + 20^2 - 20^2)$

$= -\frac{3}{2}(x - 20)^2 + \frac{3}{2}(400)$

$= -\frac{3}{2}(x - 20)^2 + 600$

∴ The maximum value of A is 600.

(c) Number of cow can be kept $= \frac{600}{4}$

$= 150$

31. (a) Substituting $x = 2, y = 3$ into the equation of graph (I),

$3 = -(2 - 1)^2 + k$

$3 = -1 + k$

∴ $k = 4$

The equation of graph (I) is

$y = -(x - 1)^2 + 4.$

∴ Vertex = (1, 4)

(b) A translation of 1 unit to the right and 2 units upwards of graph (I) will obtain graph (II).

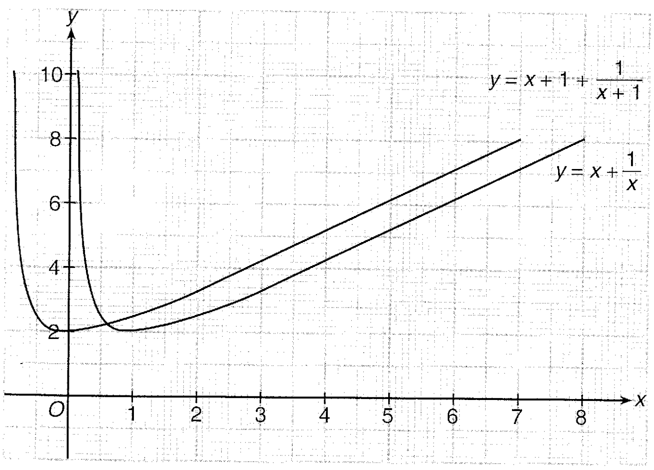
(c) $y = -(x - 1 - 1)^2 + 4 + 2$

$= -(x - 2)^2 + 6$

32. (a) When $x = 0$, y is undefined.

(b)

x	0.1	1	2	3	4	5	6	7	8
y	10.1	2	2.5	3.3	4.25	5.2	6.2	7.1	8.1



(c) Vertex = (1, 2)

33. (a) $y = -2x^2 + 8x + 2$

$= -2(x^2 - 4x) + 2$

$= -2(x^2 - 4x + 2^2 - 2^2) + 2$

$= -2(x - 2)^2 + 8 + 2$

$= -2(x - 2)^2 + 10$

(b) Maximum height = 10 m

(c) When $x = 3$,

$y = -2(3)^2 + 8(3) + 2$

$= -18 + 24 + 2$

$= 8$

∴ The ball will fall into the basket.

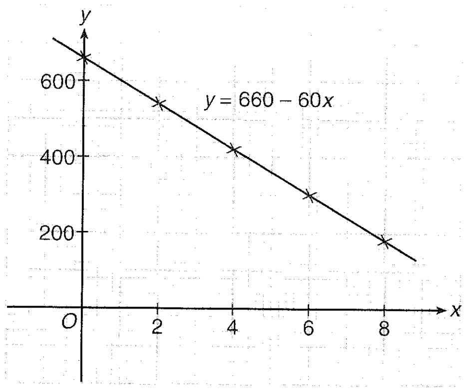
34. (a) Total distance travelled for x hours $= 60x$

∴ $y = 660 - 60x$

(b)

x	0	2	4	6	8
y	660	540	420	300	180

(c)



(d) If they arrive at Fuzhou, then $y = 0$.

∴ $0 = 660 - 60x$

$x = \frac{660}{60} = 11$

They will arrive at Fuzhou after 11 hours, that is, they will arrive at 7:00 p.m.

35. (a) $f(2) = 8$

$f(1 + 1) = 8$

$1^2 + 4(1) + k = 8$

$k = 3$

(b) $f(-1) = f(-2 + 1)$

$= (-2)^2 + 4(-2) + 3$

$= 4 - 8 + 3$

$= -1$

(c) $f(x) = f(x - 1 + 1)$

$= (x - 1)^2 + 4(x - 1) + 3$

$= x^2 - 2x + 1 + 4x - 4 + 3$

$= x^2 + 2x$

(d)
$$f(x) = 3$$
$$x^2 + 2x = 3$$
$$x^2 + 2x - 3 = 0$$
$$(x + 3)(x - 1) = 0$$
$$x = \underline{\underline{-3 \text{ or } 1}}$$

36 – 40 (HKCEE Questions)

Extended Question

p.104

41. (a)
$$y = -\frac{x^2}{200} + x$$
$$= -\frac{1}{200}(x^2 - 200x)$$
$$= -\frac{1}{200}(x^2 - 200x + 100^2 - 100^2)$$
$$= -\frac{1}{200}(x - 100)^2 + \frac{10\ 000}{200}$$
$$= \underline{\underline{-\frac{1}{200}(x - 100)^2 + 50}}$$

(b) The maximum height = $\underline{\underline{50 \text{ m}}}$

(c) When $y = 0$,
$$-\frac{x^2}{200} + x = 0$$
$$-x^2 + 200x = 0$$
$$-x(x - 200) = 0$$
$$x = 0 \text{ or } 200$$
∴ The target is 200 units from the muzzle.

(d) Yes, the equation will be different from the original one.

Multiple-choice Questions

pp.105 – 107

1. A

2. B
$$f(-1) = (-1)^{2004} - (-1 + 1)^{2005} - (-1 + 2)^{2006}$$
$$= 1 - 0 - 1$$
$$= \underline{\underline{0}}$$

3. C
$$f(2) = 8$$
$$(2 - 2)(2 + 3)\, g(2) + a(2) + b = 8$$
$$2a + b = 8 \quad \text{.............................} (1)$$
$$f(-3) = -2$$
$$(-3 - 2)(-3 + 3)\, g(-3) + a(-3) + b = -2$$
$$-3a + b = -2 \quad \text{.................} (2)$$

(1) – (2): $5a = 10$
$$a = \underline{\underline{2}}$$
Substituting $a = 2$ into (1), we have
$$2(2) + b = 8$$
$$b = \underline{\underline{4}}$$

4. B
Quadratic equations are in the form
$ax^2 + bx + c$.

5. D
For a quadratic function $y = ax^2 + bx + c$, it has a maximum value if $a < 0$.
Consider the function in D,
$$y = 12 - (x + 1)(x - 3)$$
$$= 12 - (x^2 - 3x + x - 3)$$
$$= -x^2 + 2x + 15$$
∵ $a = -1 < 0$
∴ It has a maximum value.

6. C

7. A
Since $a < 0$, the graph opens downwards,
∴ C is false.
Since $L < 0$, the y-intercept is negative,
∴ B is false.
Since the line of symmetry is $x = -\frac{b}{2a} > 0$,
(∵ $b > 0$ and $a < 0$),
∴ D is false.
Therefore, A is the correct answer.

8. A
∵ y has a maximum value.
∴ $k < 0$
$$k^2 - k - 2 = 4$$
$$k^2 - k - 6 = 0$$
$$(k + 2)(k - 3) = 0$$
$$k = -2 \text{ or } 3 \text{ (rejected)}$$

9. B
Consider the quadratic graph of $y = -3(x + 4)^2 + 9$.

(I) Vertex = $(-4, 9)$;

(II) ∴ $a = -3 < 0$
∴ It opens downwards.

(III) The maximum value of y is 9.
Therefore, only II is true.

10. C

11. B

When $x = 0$, $y = \frac{1}{0+10}$

$= \frac{1}{10}$

$\therefore$ The y-intercept of the graph is $\frac{1}{10}$.

12. A

13. A

14. B

15. D

16. B

$y = -(x + 3)^2 - 7$

$= -(x + 3 + 4)^2 - 7 + 1$

$= \underline{\underline{-(x + 7)^2 - 6}}$

17. C

18. A

$f(2x) = 4x^2 - 8x + 9$

$= (2x)^2 - 4(2x) + 9$

$\therefore$ $f(x) = x^2 - 4x + 9$

$f(x + 1) = (x + 1)^2 - 4(x + 1) + 9$

$= x^2 + 2x + 1 - 4x - 4 + 9$

$= \underline{\underline{x^2 - 2x + 6}}$