CHAPTER 2

Exercise 2A (p.49)

1. $7(5-3x) \ge 5(7-2x)$
$35-21x \ge 35-10x$
$0 \ge 11x$
$\underline{\underline{x \le 0}}$

2. $4-2(2-x) \le 4(x-5)-6$
$4-4+2x \le 4x-20-6$
$26 \le 2x$
$\underline{\underline{x \ge 13}}$

3. $5-x \ge 4(x-3)-2(x-1)$
$5-x \ge 4x-12-2x+2$
$15 \ge 3x$
$\underline{\underline{x \le 5}}$

4. $9x-5(3x-8) \le 4$
$9x-15x+40 \le 4$
$36 \le 6x$
$\underline{\underline{x \ge 6}}$

5. $18-5(x+1) > 3(x-1)$
$18-5x-5 > 3x-3$
$16 > 8x$
$\underline{\underline{x < 2}}$

6. $4(x-1)+8+5x < 3(x-2)$
$4x-4+8+5x < 3x-6$
$6x < -10$
$\underline{\underline{x < -\frac{5}{3}}}$

7. $\frac{x-1}{3}+\frac{5}{12} > \frac{x}{12}+\frac{2x+10}{15}$
$20x-20+25 > 5x+8x+40$
$7x > 35$
$\underline{\underline{x > 5}}$

8. $6+\frac{x+1}{2}+\frac{x+2}{3} < \frac{3+x}{4}$
$72+6x+6+4x+8 < 9+3x$
$7x < -77$
$\underline{\underline{x < -11}}$

9. $\frac{1}{12}(3x-2)-\frac{1}{45}(x-3) > 4$
$45x-30-4x+12 > 720$
$41x > 738$
$\underline{\underline{x > 18}}$

10. $\frac{x+24}{21}-\frac{x+5}{9} < \frac{x+6}{30}$
$\frac{x+24}{7}-\frac{x+5}{3} < \frac{x+6}{10}$
$30x+720-70x-350 < 21x+126$
$244 < 61x$
$\underline{\underline{x > 4}}$

11. $x(x+7) \ge 3-x(1-x)$ or $(x-14)-6(2x+5) \ge 10(x+4)$
$x^2+7x \ge 3-x+x^2$ or $x-14-12x-30 \ge 10x+40$
$8x \ge 3$ or $-84 \ge 21x$
$x \ge \frac{3}{8}$ or $x \le -4$
$\underline{\underline{x \le -4 \text{ or } x \ge \frac{3}{8}}}$

12. $\frac{7x}{10}-1\frac{4}{5} > \frac{x}{5}-\frac{7}{10}$ or $\frac{x-3}{3}+\frac{5}{4} < \frac{x}{12}+\frac{2x+9}{15}$
$7x-18 > 2x-7$ or $20x-60+75 < 5x+8x+36$
$5x > 11$ or $7x < 21$
$x > \frac{11}{5}$ or $x < 3$
$\therefore$ The solution is $\underline{\underline{\text{all real numbers}}}$.

13. $\frac{2x-3}{3}-\frac{x-5}{2} > \frac{2}{5}-\frac{x+1}{3}$ or $\frac{3x+1}{4} \le \frac{2}{3}$
$10(2x-3)-15(x-5) > 12-10(x+1)$ or $9x+3 \le 8$
$15x > -43$ or $9x \le 5$
$x > -\frac{43}{15}$ or $x \le \frac{5}{9}$
$\therefore$ The solution is $\underline{\underline{\text{all real numbers}}}$.

14. $6(14x-3) > 1+13(7x-2)$ and $9x \ge 4+5(3x-8)$
$84x-18 > 1+91x-26$ and $9x \ge 4+15x-40$
$7 > 7x$ and $36 \ge 6x$
$x < 1$ and $x \le 6$
$\underline{\underline{x < 1}}$

15. $2(2x+3)+\frac{x}{5} > 7$ and $\frac{x+3}{4} > \frac{x+4}{3}-\frac{1}{4}$
$20x+30+x > 35$ and $3x+9 > 4x+16-3$
$21x > 5$ and $-4 > x$
$x > \frac{5}{21}$ and $x < -4$
There is $\underline{\underline{\text{no solution}}}$.

16. $\frac{x+1}{2}+\frac{2x-1}{4} < \frac{1}{5}$
$10(x+1)+5(2x-1) < 4$
$20x < -1$
$x < -\frac{1}{20}$
and
$\frac{4x-3}{3} > 2-\frac{x-3}{4}$
$4(4x-3) > 24-3(x-3)$
$19x > 45$
$x > \frac{45}{19}$
There is $\underline{\underline{\text{no solution}}}$.

Exercise 2B (p.57)

1. $x^2+3x-10 > 0$
$(x+5)(x-2) > 0$
$\begin{cases} x+5>0 \\ x-2>0 \end{cases}$ or $\begin{cases} x+5<0 \\ x+5<0 \end{cases}$
$\underline{\underline{x > 2 \text{ or } x < -5}}$

2. $x^2+x-12 < 0$
$(x+4)(x-3) < 0$
$\begin{cases} x+4<0 \\ x-3>0 \end{cases}$ or $\begin{cases} x+4>0 \\ x-3<0 \end{cases}$
no solution or $-4 < x < 3$
$\underline{\underline{-4 < x < 3}}$

3. $5x^2-13x+6 \ge 0$
$(5x-3)(x-2) \ge 0$
$\begin{cases} 5x-3 \ge 0 \\ x-2 \ge 0 \end{cases}$ or $\begin{cases} 5x-3 \le 0 \\ x-2 \le 0 \end{cases}$
$\underline{\underline{x \ge 2 \text{ or } x \le \frac{3}{5}}}$

4. $4x^2-23x+30 \le 0$
$(4x-15)(x-2) \le 0$
$\begin{cases} 4x-15 \le 0 \\ x-2 \ge 0 \end{cases}$ or $\begin{cases} 4x-15 \ge 0 \\ x-2 \le 0 \end{cases}$
$2 \le x \le \frac{15}{4}$ or no solution
$\underline{\underline{2 \le x \le \frac{15}{4}}}$

5. $2x^2+3x+4 > 0$
$x^2+\frac{3}{2}x+2 > 0$
$x^2+\frac{3}{2}x+\frac{9}{16}+2-\frac{9}{16} > 0$
$(x+\frac{3}{4})^2+\frac{23}{16} > 0$
The solution is $\underline{\underline{\text{all real numbers}}}$.

6. $4x^2-7x+6 < 0$
$x^2-\frac{7}{4}x+\frac{3}{2} < 0$
$x^2-\frac{7}{4}x+\frac{49}{64}+\frac{3}{2}-\frac{49}{64} < 0$
$(x-\frac{7}{8})^2+\frac{47}{64} < 0$
Since $(x-\frac{7}{8})^2 \ge 0$ for all real values x,
there is $\underline{\underline{\text{no solution}}}$.

7. $(4x+7)^2 \ge 0$
There does not exist any real value x such that $(4x+7)^2 < 0$.
The solution is $\underline{\underline{\text{all real numbers}}}$.

8. $(5x-1)(10x+7) \le 5(3x+1)$
$50x^2+25x-7 \le 15x+5$
$50x^2+10x-12 \le 0$
$25x^2+5x-6 \le 0$
$(5x+3)(5x-2) \le 0$
$\begin{cases} 5x+3 \le 0 \\ 5x-2 \ge 0 \end{cases}$ or $\begin{cases} 5x+3 \ge 0 \\ 5x-2 \le 0 \end{cases}$
no solution or $-\frac{3}{5} \le x \le \frac{2}{5}$
$\underline{\underline{-\frac{3}{5} \le x \le \frac{2}{5}}}$

9. $(2x+1)^2 > 3(2x+1)$
$(2x+1)^2-3(2x+1) > 0$
$(2x+1)(2x+1-3) > 0$
$(2x+1)(x-1) > 0$

$\begin{cases} 2x+1>0 \\ x-1>0 \end{cases}$ or $\begin{cases} 2x+1<0 \\ x-1<0 \end{cases}$

$\underline{\underline{x>1 \text{ or } x<-\frac{1}{2}}}$

10. $(3x-4)(2x-3) \le -2(3x-4)$

$(3x-4)(2x-3)+2(3x-4) \le 0$

$(3x-4)(2x-3+2) \le 0$

$(3x-4)(2x-1) \le 0$

$\begin{cases} 3x-4 \le 0 \\ 2x-1 \ge 0 \end{cases}$ or $\begin{cases} 3x-4 \ge 0 \\ 2x-1 \le 0 \end{cases}$

$\frac{1}{2} \le x \le \frac{4}{3}$ or no solution

$\underline{\underline{\frac{1}{2} \le x \le \frac{4}{3}}}$

11. (a) $3x^2+kx+12=0$

If the equation has real roots, $D \ge 0$.

$k^2-4(3)(12) \ge 0$

$k^2-144 \ge 0$

$(k+12)(k-12) \ge 0$

$\begin{cases} k+12 \ge 0 \\ k-12 \ge 0 \end{cases}$ or $\begin{cases} k+12 \le 0 \\ k-12 \le 0 \end{cases}$

$\underline{\underline{k \ge 12 \text{ or } k \le -12}}$

(b) $x^2+kx+3x-k=0$

$x^2+(k+3)x-k=0$

If the equation has real roots, $D \ge 0$.

$(k+3)^2-4(-k) \ge 0$

$k^2+6k+9+4k \ge 0$

$k^2+10k+9 \ge 0$

$(k+1)(k+9) \ge 0$

$\begin{cases} k+1 \ge 0 \\ k+9 \ge 0 \end{cases}$ or $\begin{cases} k+1 \le 0 \\ k+9 \le 0 \end{cases}$

$\underline{\underline{k \ge -1 \text{ or } k \le -9}}$

12. (a) $kx^2-8x+2=0$

If the equation does not have real roots, $D<0$.

$(-8)^2-4k(2) <0$

$64<8k$

$\underline{\underline{k>8}}$

(b) $5x^2-4kx+3k+5=0$

If the equation does not have real roots, $D<0$.

$(-4k)^2-4(5)(3k+5) <0$

$16k^2-60k-100<0$

$4k^2-15k-25<0$

$(4k+5)(k-5)<0$

$\begin{cases} 4k+5>0 \\ k-5<0 \end{cases}$ or $\begin{cases} 4k+5<0 \\ k-5>0 \end{cases}$

$-\frac{5}{4}<k<5$ or no solution

$\underline{\underline{-\frac{5}{4}<k<5}}$

13. $x^2-6x-1+\mu(2x+1)=0$

$x^2+(2\mu-6)x+(\mu-1)=0$

If the equation has real roots, $D \ge 0$.

$(2\mu-6)^2-4(\mu-1) \ge 0$

$\mu^2-7\mu+10 \ge 0$

$(\mu-2)(\mu-5) \ge 0$

$\begin{cases} \mu-2 \ge 0 \\ \mu-5 \ge 0 \end{cases}$ or $\begin{cases} \mu-2 \le 0 \\ \mu-5 \le 0 \end{cases}$

$\underline{\underline{\mu \ge 5 \text{ or } \mu \le 2}}$

14. $s^2x^2-(s+2)x+1=0$

If the equation has real roots, $D \ge 0$.

$[-(s+2)]^2-4s^2(1) \ge 0$

$s^2+4s+4-4s^2 \ge 0$

$3s^2-4s-4 \le 0$

$(3s+2)(s-2) \le 0$

$\begin{cases} 3s+2 \le 0 \\ s-2 \ge 0 \end{cases}$ or $\begin{cases} 3s+2 \ge 0 \\ s-2 \le 0 \end{cases}$

no solution or $-\frac{2}{3} \le s \le 2$

$\underline{\underline{-\frac{2}{3} \le s \le 2}}$

15. $x^2-4x+k>0$ for all real values of x.

$\therefore \quad D<0$

$(-4)^2-4k<0$

$16<4k$

$\underline{\underline{k>4}}$

16. $-3x^2+6x-k \le 0$ for all real values of x.

$3x^2-6x+k \ge 0$ for all real values of x.

$\therefore \quad D \le 0$

$(-6)^2-4(3)(k) \le 0$

$36 \le 12k$

$\underline{\underline{k \ge 3}}$

17. $4k^2x^2+2(k+3)x+9>0$ for all real values of x.

$\therefore \quad D<0$

$[2(k+3)]^2-4(4k^2)(9)<0$

$k^2+6k+9-36k^2<0$

$35k^2-6k-9>0$

$(7k+3)(5k-3)>0$

$\begin{cases} 7k+3>0 \\ 5k-3>0 \end{cases}$ or $\begin{cases} 7k+3<0 \\ 5k-3<0 \end{cases}$

$\underline{\underline{k>\frac{3}{5} \text{ or } k<-\frac{3}{7}}}$

18. $x^2-(k-4)x+k^2-5k+4 \ge 0$ for all real values of x

$\therefore \quad D \le 0$

$[-(k-4)]^2-4(k^2-5k+4) \le 0$

$k^2-8k+16-4k^2+20k-16 \le 0$

$3k^2-12k \ge 0$

$k^2-4k \ge 0$

$k(k-4) \ge 0$

$\begin{cases} k \ge 0 \\ k-4 \ge 0 \end{cases}$ or $\begin{cases} k \le 0 \\ k-4 \le 0 \end{cases}$

$\underline{\underline{k \ge 4 \text{ or } k \le 0}}$

19. $2x^2-2(k-3)x+(k+1)>0$ for all real values of x.

$\therefore \quad D<0$

$[-2(k-3)]^2-4(2)(k+1)<0$

$k^2-6k+9-2k-2<0$

$k^2-8k+7<0$

$(k-1)(k-7)<0$

$\begin{cases} k-1<0 \\ k-7>0 \end{cases}$ or $\begin{cases} k-1>0 \\ k-7<0 \end{cases}$

no solution or $1<k<7$

$\underline{\underline{1<k<7}}$

20. $y=\frac{x}{x^2+4}$

(a) $x^2y+4y=x$

$x^2y-x+4y=0$

(b) x has real roots, then $D \ge 0$

$(-1)^2-4y(4y) \ge 0$

$1-16y^2 \ge 0$

$16y^2-1 \le 0$

$(4y+1)(4y-1) \le 0$

$\begin{cases} 4y+1 \le 0 \\ 4y-1 \ge 0 \end{cases}$ or $\begin{cases} 4y+1 \ge 0 \\ 4y-1 \le 0 \end{cases}$

no solution or $-\frac{1}{4} \le y \le \frac{1}{4}$

$\therefore \quad -\frac{1}{4} \le y \le \frac{1}{4}$

21. (a) $y=\frac{x+1}{x^2+2x+2}$

$x^2y+2xy+2y=x+1$

$x^2y+(2y-1)x+(2y-1)=0$

(b) As x is real, $D \ge 0$

$(2y-1)^2-4y(2y-1) \ge 0$

$(2y-1)(2y+1) \le 0$

$\begin{cases} 2y-1 \ge 0 \\ 2y+1 \le 0 \end{cases}$ or $\begin{cases} 2y-1 \le 0 \\ 2y+1 \ge 0 \end{cases}$

no solution or $-\frac{1}{2} \le y \le \frac{1}{2}$

$-\frac{1}{2} \le y \le \frac{1}{2}$

22. $\frac{3x-1}{x(x+1)}=k$

$3x-1=kx^2+kx$

$kx^2+(k-3)x+1=0$

It has no or just one real root. $D \le 0$

$(k-3)^2-4k \le 0$

$k^2-6k+9-4k \le 0$

$k^2-10k+9 \le 0$

$(k-1)(k-9) \le 0$

$\begin{cases} k-1 \le 0 \\ k-9 \ge 0 \end{cases}$ or $\begin{cases} k-1 \ge 0 \\ k-9 \le 0 \end{cases}$

no solution or $1 \le k \le 9$

$1 \le k \le 9$

Maximum value of k is $\underline{\underline{9}}$.

Minimum value of k is $\underline{\underline{1}}$.

23. $f(x)=(k-2)x^2+(2k-1)x+(k-5)$

(a) $f(x)=0$ has no real root.

$D<0$, $(2k-1)^2-4(k-2)(k-5)<0$

$24k<39$

$\underline{\underline{k<\frac{13}{8}}}$

(b) $f(x)<-3$

$$(k-2)x^2+(2k-1)x+(k-2)<0$$

$$\begin{cases}k-2<0\\D<0\end{cases}$$

$$\begin{aligned}D&=(2k-1)^2-4(k-2)^2\\&=3(4k-5)<0\end{aligned}$$

$$\begin{cases}k-2<0\\4k-5<0\end{cases}$$

$$\underline{\underline{k<\frac{5}{4}}}$$

24. $$\begin{aligned}f(x)&=x^2+4x+2+m(2x+1)\\&=x^2+2(m+2)x+(m+2)\end{aligned}$$

(a) $f(x)=0$ has no real root.

$$D<0,\ 4(m+2)^2-4(m+2)<0$$
$$(m+2)(m+1)<0$$

$$\begin{cases}m+2>0\\m+1<0\end{cases}\quad\text{or}\quad\begin{matrix}m+2<0\\m+1>0\end{matrix}$$

$-2<m<-1$ or no solution

$\underline{\underline{-2<m<-1}}$

(b) $f(x)>-2$

$$x^2+2(m+2)x+(m+4)>0$$

$$D<0,\ 4(m+2)^2-4(m+4)<0$$
$$m(m+3)<0$$

$$\begin{cases}m>0\\m+3<0\end{cases}\quad\text{or}\quad\begin{matrix}m<0\\m+3>0\end{matrix}$$

no solution or $-3<m<0$

$\underline{\underline{-3<m<0}}$

25 – 27. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

Revision Exercise 2 (p.59)

1. $$\begin{aligned}2(2x-3)-5(x+1)&>3(2x-5)\\4x-6-5x-5&>6x-15\\4&>7x\\x&<\frac{4}{7}\end{aligned}$$

2. $$\begin{aligned}(2x+1)(x-4)&<x(2x+5)\\2x^2-7x-4&<2x^2+5x\\-4&<12x\\x&>-\frac{1}{3}\end{aligned}$$

3. $$\begin{aligned}(x-3)(2x+1)&<2(2x+1)\\(x-3)(2x+1)-2(2x+1)&<0\\(2x+1)(x-5)&<0\end{aligned}$$

$$\begin{cases}2x+1>0\\x-5<0\end{cases}\quad\text{or}\quad\begin{matrix}2x+1<0\\x-5>0\end{matrix}$$

$-\frac{1}{2}<x<5$ or no solution

$\underline{\underline{-\frac{1}{2}<x<5}}$

4. $$\begin{aligned}(2x-3)(x-2)&>-1\\2x^2-7x+6+1&>0\\2x^2-7x+7&>0\\x^2-\frac{7}{2}x+\frac{49}{16}+\frac{7}{16}&>0\\(x-\frac{7}{4})^2+\frac{7}{16}&>0\end{aligned}$$

The solution is $\underline{\underline{\text{all real numbers}}}$.

5. $x(x+7)\geq 3-x(1-x)$ and $(x-14)-6(2x+5)\geq 10(x+4)$

$x^2+7x\geq 3-x+x^2$ and $x-14-12x-30\geq 10x+40$

$8x\geq 3$ and $-84\geq 21x$

$x\geq\frac{3}{8}$ and $x\leq -4$

There is $\underline{\underline{\text{no solution}}}$.

6. $3(14x-3)>2+13(3x-2)$ or $9x\geq 4+5(3x-8)$

$42x-9>2+39x-26$ or $9x\geq 4+15x-40$

$3x>-15$ or $36\geq 6x$

$x>-5$ or $x\leq 6$

The solution is $\underline{\underline{\text{all real numbers}}}$.

7. $3(k+1)x-x^2+2k-3=0$

$-x^2+3(k+1)x+2k-3=0$

If the equation has real roots, $D\geq 0$.

$$\begin{aligned}\therefore\quad [3(k+1)]^2-4(-1)(2k-3)&\geq 0\\9k^2+18k+9+4(2k-3)&\geq 0\\9k^2+18k+9+8k-12&\geq 0\\9k^2+26k-3&\geq 0\end{aligned}$$

$$\begin{cases}k+3\geq 0\\9k-1\geq 0\end{cases}\quad\text{or}\quad\begin{matrix}k+3\leq 0\\9k-1\leq 0\end{matrix}$$

$\underline{\underline{k\geq\frac{1}{9}\text{ or }k\leq -3}}$

8. $x^2+(3m-1)x+2m\geq -10$

$x^2+(3m-1)x+2m+10\geq 0$

Consider the discriminant of the equation $x^2+(3m-1)x+2m+10=0$.

$$\begin{aligned}D&=(3m-1)^2-4(2m+10)\\&=9m^2-6m+1-8m-40\\&=9m^2-14m-39\\&=(m-3)(9m+13)\end{aligned}$$

If $x^2+(3m-1)x+2m+10\geq 0$ for all real values of x, then $D\leq 0$.

$(m-3)(9m+13)\leq 0$

$$\begin{cases}m-3\leq 0\\9m+13\geq 0\end{cases}\quad\text{or}\quad\begin{matrix}m-3\geq 0\\9m+13\leq 0\end{matrix}$$

$-\frac{13}{9}\leq m\leq 3$ or no solution

$\underline{\underline{-\frac{13}{9}\leq m\leq 3}}$

9. $(m-1)x^2-3x+m>1$

$(m-1)x^2-3x+m-1>0$

For any real values of x, $m-1>0$, $m>1$.

Consider the discriminant of the equation $(m-1)x^2-3x+m-1=0$.

$$\begin{aligned}D&=(-3)^2-4(m-1)(m-1)\\&=9-4m^2+8m-4\\&=-4m^2+8m+5\\&=(-2m+5)(2m+1)\end{aligned}$$

If $(m-1)x^2-3x+m-1>0$ for any real values of x, then $D<0$.

$(-2m+5)(2m+1)<0$

$$\begin{cases}-2m+5>0\\2m+1<0\end{cases}\quad\text{or}\quad\begin{matrix}-2m+5<0\\2m+1>0\end{matrix}$$

$m<-\frac{1}{2}$ (rejected) or $m>\frac{5}{2}$

$\underline{\underline{m>\frac{5}{2}}}$

10. If the equation has two unequal roots, its discriminant is greater than zero.

$8x^2-(k+1)x+k=5$

$8x^2-(k+1)x+(k-5)=0$

$$\begin{aligned}D=[-(k+1)]^2-4(8)(k-5)&>0\\k^2+2k+1-32k+160&>0\\k^2-30k+161&>0\\(k-23)(k-7)&>0\end{aligned}$$

$$\begin{cases}k-23>0\\k-7>0\end{cases}\quad\text{or}\quad\begin{matrix}k-23<0\\k-7<0\end{matrix}$$

$k>23$ or $k<7$

If the equation has positive roots,

sum of the roots $=\frac{k+1}{8}>0$ and

product of the roots $=\frac{k-5}{8}>0$

$$\begin{cases}k>-1\\k>5\end{cases}\quad\therefore\quad k>5$$

$\therefore\quad\underline{\underline{5<k<7\text{ or }k>23}}$

11. $y=\frac{x^2-2x+3}{x^2+2x+3}$

$x^2y+2xy+3y=x^2-2x+3$

$(y-1)x^2+(2y+2)x+3y-3=0$

As x is real, $D\geq 0$.

$$\begin{aligned}(2y+2)^2-4(y-1)(3y-3)&\geq 0\\4y^2+8y+4-12y^2+24y-12&\geq 0\\8y^2-32y+8&\leq 0\\y^2-4y+1&\leq 0\\(y-2+\sqrt{3})(y-2-\sqrt{3})&\leq 0\end{aligned}$$

$$\begin{cases}y-2+\sqrt{3}\leq 0\\y-2-\sqrt{3}\geq 0\end{cases}\quad\text{or}\quad\begin{matrix}y-2+\sqrt{3}\geq 0\\y-2-\sqrt{3}\leq 0\end{matrix}$$

no solution or $2-\sqrt{3}\leq y\leq 2+\sqrt{3}$

$\underline{\underline{2-\sqrt{3}\leq y\leq 2+\sqrt{3}}}$

12. $y=\frac{2x+1}{x^2+2}$

$yx^2+2y=2x+1$

$yx^2-2x+2y-1=0$

As x is real, $D\geq 0$

$$\begin{aligned}(-2)^2-4y(2y-1)&\geq 0\\4-8y^2+4y&\geq 0\\2y-y-1&\leq 0\\(y-1)(2y+1)&\leq 0\end{aligned}$$

$$\begin{cases}y-1\leq 0\\2y+1\geq 0\end{cases}\quad\text{or}\quad\begin{matrix}y-1\geq 0\\2y+1\leq 0\end{matrix}$$

$-\frac{1}{2}\leq y\leq 1$ or no solution

$\underline{\underline{-\frac{1}{2}\leq y\leq 1}}$

$\because \quad x^2 \geq 0,\ x^2+2>0$

$\dfrac{|2x+1|}{x^2+2} \geq 0$

$\therefore \quad 0 \leq \dfrac{|2x+1|}{x^2+2} \leq 1$

13. (a) $(x-1)^2 = k^2-k+2$

$x^2-2x-(k^2-k+1)=0$

$D = 4+4(k^2-k+1)$

$= 4k^2-4k+8$

$= (2k-1)^2+7>0$

for all real values of k.

$\therefore$ α and β are real and distinct.

(b) As $\alpha+\beta=2$, $\alpha\beta=-k^2+k-1$

$(\alpha-\beta)^2 = (\alpha+\beta)^2-4\alpha\beta$

$= 4-4(-k^2+k-1)$

$= 4k^2-4k+8$

$= (2k-1)^2+7$

It attains minimum when $k=\dfrac{1}{2}$ and the minimum value is 7.

$\therefore$ Minimum value of $|\alpha-\beta| = \sqrt{7}$

14. (a) $x^2-(m-1)x+3(m-5)=0$

$D=(m-1)^2-4(3)(m-5)$

$= m^2-14m+61$

$= (m-7)^2+12>0$

for all real values of m.

$\therefore$ α and β are real and distinct.

(b) As $\alpha+\beta=m-1$, $\alpha\beta=3(m-5)$

$(3-\alpha)(3-\beta) = 9-3(\alpha+\beta)+\alpha\beta$

$= 9-3(m-1)+3(m-5)$

$= -3<0$

15. (a) $x^2+(k+1)x+k(k-a)=0$

$\alpha+\beta = -(k+1)$

$\alpha\beta = k(k-a)$

(b) $(\alpha+\beta)^2 = [-(k+1)]^2$

$\alpha^2+2\alpha\beta+\beta^2 = k^2+2k+1$

$\alpha^2+\beta^2 = k^2+2k+1-2\alpha\beta$

$= k^2+2k+1-2k(k-a)$

$= k^2+2k+1-2k^2+2ka$

$= -k^2+2(a+1)k+1$

(c) $\alpha^2+\beta^2 = -k^2+2(a+1)k+1 \leq 5$

$-k^2+2(a+1)k-4 \leq 0$

for all values of k, $D \leq 0$

$[2(a+1)]^2-4(-1)(-4) \leq 0$

$4(a^2+2a+1)-16 \leq 0$

$a^2+2a-3 \leq 0$

$(a+3)(a-1) \leq 0$

$\begin{cases} a+3 \leq 0 \\ a-1 \geq 0 \end{cases}$ or $\begin{cases} a+3 \geq 0 \\ a-1 \leq 0 \end{cases}$

no solution or $-3 \leq a \leq 1$

$-3 \leq a \leq 1$

16. $f(x) = k(x+2)^2-x(x-6k)+5k$

$= (k-1)x^2+10kx+9k$

(a) $f(x)=0$ has equal roots, $D=0$

$100k^2-4(k-1)(9k)=0$

$25k^2-9k^2+9k=0$

$16k^2+9k=0$

$k(16k+9)=0$

$k=0,\ -\dfrac{9}{16}$

(b) $f(x)=5x$

$(k-1)x^2+5(2k-1)x+9k=0$

Roots are of opposite sign, (i.e. sum of the roots = 0)

$2k-1=0$

$k=\dfrac{1}{2}$

(c) $f(x) \leq 0$ for all real values of x.

$\begin{cases} k-1<0 \\ D \leq 0 \end{cases}$

By **(a)**,

$D = k(16k+9) \leq 0$

$-\dfrac{9}{16} \leq k \leq 0$

$\therefore \begin{cases} k<1 \\ -\dfrac{9}{16} \leq k \leq 0 \end{cases}$

$-\dfrac{9}{16} \leq k \leq 0$

17. (a) $x^2-(m+\dfrac{3}{m})x+2=0$

$\alpha+\beta = m+\dfrac{3}{m}$, $\alpha\beta=2$

Since for all real numbers a, b $(a-b)^2 \geq 0$.

$(m-\dfrac{3}{m})^2 \geq 0$

$m^2-6+\dfrac{9}{m^2} \geq 0$

$m^2+\dfrac{9}{m^2} \geq 6$

$m^2+6+\dfrac{9}{m^2} \geq 12$

$(m+\dfrac{3}{m})^2 \geq 12$

(b) Consider the equation

$x^2-(m+\dfrac{3}{m})x+2=0$.

$D=(m+\dfrac{3}{m})^2-4(2)$

$\geq 12-8$

$=4>0$

It has distinct real roots.

(c) $\alpha+\beta < 2\alpha\beta$

$m+\dfrac{3}{m} < 2(2)$

$m^2-4m+3<0$

$(m-1)(m-3)<0$

$\therefore \begin{cases} m-1<0 \\ m-3>0 \end{cases}$ or $\begin{cases} m-1>0 \\ m-3<0 \end{cases}$

$\therefore$ no solution or $1<m<3$

$1<m<3$

18 – 19. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

Enrichment 2 (p.62)

1 – 3. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

Classwork 1 (p.44)

1. If $ac>bc$,

let $a=-2$, $b=-1$. $c=-1$

$ac=(-2)(-1)=2$

$bc=(-1)(-1)=1$

however $a<b$

$\therefore$ The statement is false.

2. If $\dfrac{a}{b}<1$,

let $a=1$, $b=-3$.

$\dfrac{a}{b}=\dfrac{1}{-3}<1$

however $a>b$

$\therefore$ The statement is false.

3. If $a<1$,

$\because$ $a>0$, $\dfrac{a}{a}<\dfrac{1}{a}$

$0<a<1<\dfrac{1}{a}$

i.e. $a<\dfrac{1}{a}$

$\therefore$ The statement is true.

4. If $a^2>b^2$,

let $a=-3$, $b=2$

$a^2=(-3)^2=9$

$b^2=(2)^2=4$

however, $a<b$

$\therefore$ The statement is false.

5. For any real values of a, b,

$(a-b)^2 \geq 0$

$a^2-2ab+b^2 \geq 0$

$a^2+b^2 \geq 2ab$

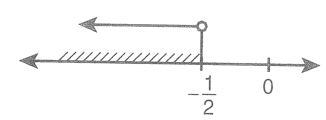
$\therefore$ The statement is true.

Classwork 2 (p.45)

1. $2-x>x+3$

$-1>2x$

$x<-\dfrac{1}{2}$

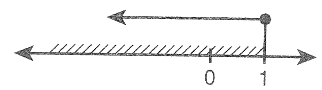


2. $3(x-3)-8 \geq 4(2x-5)-2$

$3x-9-8 \geq 8x-20-2$

$5 \geq 5x$

$x \leq 1$

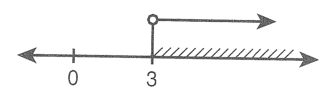


3. $x-4>\dfrac{7-3x}{2}$

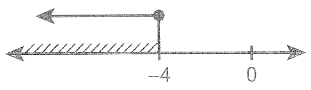
$2x-8>7-3x$

$5x>15$

$x>3$

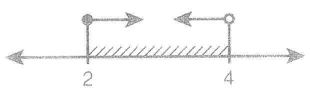


4. $\frac{3(4+x)}{8} \le \frac{-(x+2)}{2} - 1$
$3(4+x) \le -4(x+2) - 8$
$12 + 3x \le -4x - 8 - 8$
$7x \le -28$
$\underline{\underline{x \le -4}}$

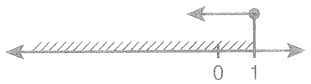


Classwork 3 (p.46)

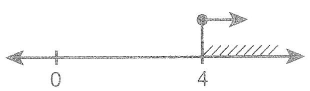
1. $x \ge 2$ and $x < 4$



2. $x < -3$ or $x \le 1$



3. $x > 1$ and $x \ge 4$

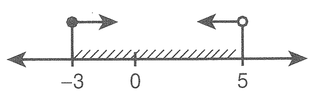


4. $x < -2$ or $x \ge -5$

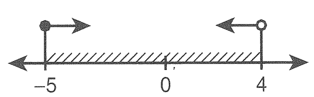
5. $x \ge 0$ and $x < -1$

Classwork 4 (p.48)

1. $5(x+1) \ge 2(x-2)$ and $9(x-3) < 5(x+1) - 12$
$5x + 5 \ge 2x - 4$ and $9x - 27 < 5x + 5 - 12$
$3x \ge -9$ and $4x < 20$
$x \ge -3$ and $x < 5$
$\therefore$ The solution is $\underline{\underline{-3 \le x < 5}}$.

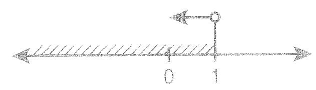


2. $-3(x+5) \le 7(x+5) < 75 - 3x$
$-3x - 15 \le 7x + 35 < 75 - 3x$
$-15 \le 10x + 35 < 75$
$-50 \le 10x < 40$
$\therefore$ The solution is $\underline{\underline{-5 \le x < 4}}$.



3. $\frac{x-2}{3} > \frac{2-x}{2}$ and $\frac{2x+3}{2} < \frac{x+4}{3} - \frac{1}{6}$
$2x - 4 > 6 - 3x$ and $3(2x+3) < 2(x+4) - 1$
$5x > 10$ and $6x + 9 < 2x + 8 - 1$
$x > 2$ and $x < -\frac{1}{2}$
$\therefore$ There is $\underline{\underline{\text{no solution}}}$.

4. $7 - 3x > 10$ or $6 - 5x > 2 - x$
$-3x > 3$ or $-4x > -4$
$x < -1$ or $x < 1$
$\therefore$ The solution is $\underline{\underline{x < 1}}$.



5. $\frac{5+x}{9} > 3$ or $4 - \frac{x-3}{2} > 5$
$5 + x > 27$ or $8 - x + 3 > 10$
$x > 22$ or $x < 1$
$\therefore$ The solution is $\underline{\underline{x < 1 \text{ or } x > 22}}$.

6. $9 - 2(x+1) > -5$ or $8(x+1) + x > 5 - (2 - 4x)$
$9 - 2x - 2 > -5$ or $8x + 8 + x > 5 - 2 + 4x$
$12 - 2x > 0$ or $5x > -5$
$x < 6$ or $x > -1$
$\therefore$ The solution is $\underline{\underline{\text{all real numbers}}}$.

Classwork 5 (p.52)

1. $x^2 - x - 6 < 0$
$(x+2)(x-3) < 0$
$\begin{cases} x+2<0 \\ x-3>0 \end{cases}$ or $\begin{cases} x+2>0 \\ x-3<0 \end{cases}$
no solution or $-2 < x < 3$
$\underline{\underline{-2 < x < 3}}$

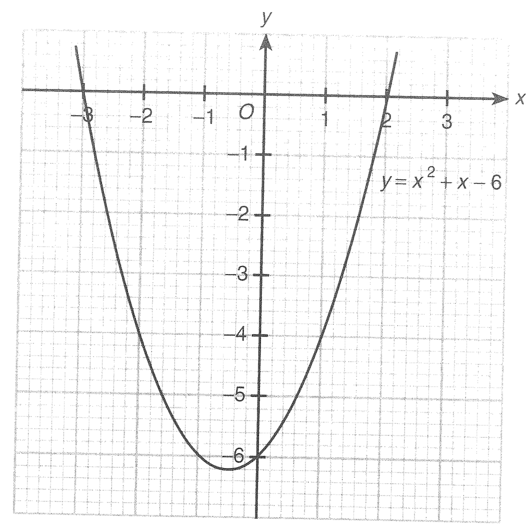
2. $-2x^2 - x + 10 < 0$
$(2x+5)(-x+2) < 0$
$\therefore \begin{cases} 2x+5<0 \\ -x+2>0 \end{cases}$ or $\begin{cases} 2x+5>0 \\ -x+2<0 \end{cases}$
$\therefore \underline{\underline{x < -\frac{5}{2} \text{ or } x > 2}}$

3. $2x^2 + 4x + 11 > 0$
$2(x^2 + 2x + \frac{11}{2}) > 0$
$x^2 + 2x + 1 - 1 + \frac{11}{2} > 0$
$(x+1)^2 + \frac{9}{2} > 0$
For all real values of x,
$(x+1)^2 \ge 0$
$(x+1)^2 + \frac{9}{2} > 0$
$\therefore$ The solution is $\underline{\underline{\text{all real numbers}}}$.

4. $4x(1-x) > 2$
$4x - 4x^2 > 2$
$4x^2 - 4x + 2 < 0$
$x^2 - x + \frac{1}{2} < 0$
$x^2 - x + (\frac{1}{2})^2 - (\frac{1}{2})^2 + \frac{1}{2} < 0$
$(x - \frac{1}{2})^2 < -\frac{1}{4}$
Since $(x - \frac{1}{2})^2 \ge 0$ for all real values of x.
$\therefore$ No real number satisfies the inequality.
$\therefore$ There is $\underline{\underline{\text{no solution}}}$.

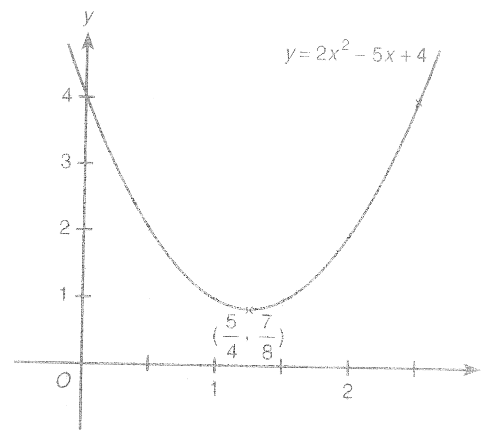
Classwork 6 (p.57)

1. (a) Consider $y = x^2 + x - 6$
$= (x-2)(x+3)$
$\therefore$ The graph of $y = x^2 + x - 6$ is as follows.



$\therefore$ The solution is $\underline{\underline{x > 2 \text{ or } x < -3}}$.

(b) Consider $y = 2x^2 - 5x + 4$
$= 2(x^2 - \frac{5}{2}x + 2)$
$= 2[x^2 - \frac{5}{2}x + (\frac{5}{4})^2 - (\frac{5}{4})^2 + 2]$
$= 2(x - \frac{5}{4})^2 + \frac{7}{8}$
$\therefore$ The graph of $y = 2x^2 - 5x + 4$ is a[s] follows.



Note that $y = 2x^2 - 5x + 4 > 0$ for all rea[l] values of x.
$\therefore$ The solution is $\underline{\underline{\text{all real numbers}}}$.

2. If the equation has no real roots, $D < 0$.
$(-2k)^2 - 4(3k) < 0$
$4k^2 - 12k < 0$
$k(k-3) < 0$
$\begin{cases} k<0 \\ k-3>0 \end{cases}$ or $\begin{cases} k>0 \\ k-3<0 \end{cases}$
no solution or $0 < k < 3$
$\underline{\underline{0 < k < 3}}$

3. For the expression $x^2 + 3kx + 1$, the coefficient of x^2 is 1, which is greater than 0.
Consider the discriminant of the equation $x^2 + 3kx + 1 = 0$.
$D = (3k)^2 - 4(1)$
$= 9k^2 - 4$
If $x^2 + 3kx + 1 > 0$ for any real value of x, then $D < 0$.

$$9k^2 - 4 < 0$$
$$(3k-2)(3k+2) < 0$$
$$\begin{cases} 3k-2>0 \\ 3k+2<0 \end{cases} \quad \text{or} \quad \begin{matrix} 3k-2<0 \\ 3k+2>0 \end{matrix}$$
$$\text{no solution} \quad \text{or} \quad -\frac{2}{3} < k < \frac{2}{3}$$
$$\underline{\underline{-\frac{2}{3} < k < \frac{2}{3}}}$$

CHAPTER 3

Exercise 3A (p.71)

1. Let $P(n)$ be the proposition

"$1+4+7+\cdots+(3n-2) = \frac{n}{2}(3n-1)$".

When $n=1$, L.H.S. $=1$

R.H.S. $= \frac{1}{2}[3(1)-1] = 1$

$\therefore$ $P(1)$ is true.

Assume $P(k)$ is true for any positive integer k.

i.e. $1+4+7+\cdots+(3k-2) = \frac{k}{2}(3k-1)$

Then $1+4+7+\cdots+(3k-2)+[3(k+1)-2]$

$$= \frac{k}{2}(3k-1)+(3k+1)$$
$$= \frac{3k^2}{2} - \frac{k}{2} + 3k + 1$$
$$= \frac{3k^2 - k + 6k + 2}{2}$$
$$= \frac{3k^2 + 5k + 2}{2}$$
$$= \frac{(k+1)(3k+2)}{2}$$
$$= \frac{k+1}{2}[3(k+1)-1]$$

Thus assuming $P(k)$ is true for any positive integer k, $P(k+1)$ is also true. By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

2. Let $P(n)$ be the proposition

"$2^3+4^3+6^3+\cdots+(2n)^3 = 2n^2(n+1)^2$".

When $n=1$, L.H.S. $= 2^3 = 8$

R.H.S. $= 2(1)^2(1+1)^2 = 8$

$\therefore$ $P(1)$ is true.

Assume $P(k)$ is true for any positive integer k.

i.e. $2^3+4^3+6^3+\cdots+(2k)^3 = 2k^2(k+1)^2$

Then $2^3+4^3+\cdots+(2k)^3+[2(k+1)]^3$

$$= 2k^2(k+1)^2 + 8(k+1)^3$$
$$= 2(k+1)^2(k^2+4k+4)$$
$$= 2(k+1)^2(k+2)^2$$
$$= 2(k+1)^2[(k+1)+1]^2$$

Thus assuming $P(k)$ is true for any positive integer k, $P(k+1)$ is also true. By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

3. Let $P(n)$ be the proposition

"$2^2+4^2+6^2+\cdots+(2n)^2 = \frac{2}{3}n(n+1)(2n+1)$"

When $n=1$, L.H.S. $= 2^2 = 4$

R.H.S. $= \frac{2}{3}(1)(1+1)(2+1) = 4$

$\therefore$ $P(1)$ is true.

Assume $P(k)$ is true for any positive integer k.

i.e. $2^2+4^2+6^2+\cdots+(2k)^2 = \frac{2}{3}k(k+1)(2k+1)$

Then $2^2+4^2+6^2+\cdots+(2k)^2+[2(k+1)]^2$

$$= \frac{2}{3}k(k+1)(2k+1)+4(k+1)^2$$
$$= \frac{2}{3}(k+1)[k(2k+1)+6(k+1)]$$
$$= \frac{2}{3}(k+1)(2k^2+k+6k+6)$$
$$= \frac{2}{3}(k+1)(2k^2+7k+6)$$
$$= \frac{2}{3}(k+1)(k+2)(2k+3)$$
$$= \frac{2}{3}(k+1)[(k+1)+1][2(k+1)+1]$$

Thus assuming $P(k)$ is true for any positive integer k, $P(k+1)$ is also true. By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

4. Let $P(n)$ be the proposition

"$a+(a+d)+(a+2d)+\cdots+[a+(n-1)d]$

$= \frac{n}{2}[2a+(n-1)d]$".

When $n=1$, L.H.S. $= a$

R.H.S. $= \frac{1}{2}[2a+(1-1)d] = a$

$\therefore$ $P(1)$ is true.

Assume $P(k)$ is true for any positive integer k.

i.e. $a+(a+d)+(a+2d)+\cdots+[a+(k-1)d]$

$= \frac{k}{2}[2a+(k-1)d]$

Then

$a+(a+d)+\cdots+[a+(k-1)d]+\{a+[(k+1)-1]d\}$

$$= \frac{k}{2}(2a+(k-1)d]+(a+kd)$$
$$= \frac{1}{2}[2ka+k(k-1)d+2a+2kd]$$
$$= \frac{1}{2}[(k+1)2a+k^2d-kd+2kd]$$
$$= \frac{1}{2}[(k+1)2a+k(k+1)d]$$
$$= \frac{1}{2}(k+1)(2a+kd)$$
$$= \frac{k+1}{2}\{2a+[(k+1)-1]d\}$$