CHAPTER 8

Exercise 8A (p.186)

1. $\cos\theta = -1$ for $0° \le \theta \le 180°$

$\theta = \underline{\underline{180°}}$

2. $6\tan\theta = \frac{-\sqrt{3}}{2}$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\tan\theta = \frac{-\sqrt{3}}{12}$

$\theta = \underline{\underline{-0.14}}$ (corr. to 2 d.p.)

3. $\sin\theta = 0.28$

$\theta = \underline{\underline{n\pi + (-1)^n 0.28}}$

where n is any integer.

4. $3\tan\theta = -\sqrt{3}$

$\tan\theta = \frac{-\sqrt{3}}{3}$

$\theta = \underline{\underline{n\pi - \frac{\pi}{6}}}$

where n is any integer.

5. $\sin 3\theta = \frac{1}{\sqrt{2}}$

$3\theta = n\pi + (-1)^n \frac{\pi}{4}$

$\theta = \underline{\underline{\frac{n\pi}{3} + (-1)^n \frac{\pi}{12}}}$

where n is any integer.

6. $\tan(2\theta - \frac{\pi}{4}) = \sqrt{3}$

$2\theta - \frac{\pi}{4} = n\pi + \frac{\pi}{3}$

$2\theta = n\pi + \frac{7\pi}{12}$

$\theta = \underline{\underline{\frac{n\pi}{2} + \frac{7\pi}{24}}}$

where n is any integer.

7. $2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = -1$

$\theta = \underline{\underline{2n\pi \pm \frac{2\pi}{3}}}$ or $\theta = \underline{\underline{2n\pi \pm \pi}}$

where n is any integer.

8. $3\cos\theta = 2\sin^2\theta - 3$

$3\cos\theta = 2 - 2\cos^2\theta - 3$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = -1$

$\theta = \underline{\underline{2n\pi \pm \frac{2\pi}{3}}}$ or $\theta = \underline{\underline{2n\pi \pm \pi}}$

where n is any integer.

9. $\sqrt{3}\sec^2\theta = 4\tan\theta$

$\sqrt{3}(1 + \tan^2\theta) = 4\tan\theta$

$\sqrt{3} + \sqrt{3}\tan^2\theta = 4\tan\theta$

$\sqrt{3}\tan^2\theta - 4\tan + \sqrt{3} = 0$

$(\tan\theta - \sqrt{3})(\sqrt{3}\tan\theta - 1) = 0$

$\tan\theta = \sqrt{3}$ or $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \underline{\underline{n\pi + \frac{\pi}{3}}}$ or $\theta = \underline{\underline{n\pi + \frac{\pi}{6}}}$

where n is any integer.

10. $6\tan\theta = 5\csc\theta$

$6\frac{\sin\theta}{\cos\theta} = \frac{5}{\sin\theta}$

$6\sin^2\theta = 5\cos\theta$

$6 - 6\cos^2\theta = 5\cos\theta$

$6\cos^2\theta + 5\cos\theta - 6 = 0$

$(3\cos\theta - 2)(2\cos\theta + 3) = 0$

$\cos\theta = \frac{2}{3}$ or $\cos\theta = -\frac{3}{2}$ (rejected)

$\theta = \underline{\underline{2n\pi \pm 0.84}}$ (corr. to 2 d.p.)

where n is any integer.

11. $\sin 5\theta = \cos 4\theta$

$\cos 4\theta = \cos(\frac{\pi}{2} - 5\theta)$

$4\theta = 2n\pi \pm (\frac{\pi}{2} - 5\theta)$

$4\theta = 2n\pi + (\frac{\pi}{2} - 5\theta)$

$9\theta = 2n\pi + \frac{\pi}{2}$

$\theta = \frac{2}{9}n\pi + \frac{\pi}{18}$ where n is any integer.

or

$4\theta = 2n\pi - (\frac{\pi}{2} - 5\theta)$

$-\theta = 2n\pi - \frac{\pi}{2}$

$\theta = -2n\pi + \frac{\pi}{2}$

Since the values of $\theta = \frac{2}{9}n\pi + \frac{\pi}{18}$ include those of $-2n\pi + \frac{\pi}{2}$, the general solution of the equation is $\theta = \underline{\underline{\frac{2}{9}n\pi + \frac{\pi}{18}}}$, where n is any integer.

12. $\tan 4\theta = \cot 3\theta$

$\tan 4\theta = \tan(\frac{\pi}{2} - 3\theta)$

$4\theta = n\pi + \frac{\pi}{2} - 3\theta$

$7\theta = n\pi + \frac{\pi}{2}$

$\theta = \underline{\underline{\frac{1}{7}n\pi + \frac{\pi}{14}}}$

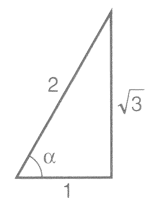
where n is any integer.

13. Let α be an acute angle such that $\tan\alpha = \sqrt{3}$. Then

$2\cos\alpha = 1$

$2\sin\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$



The given equation becomes

$(2\cos\alpha)\cos\theta - (2\sin\alpha)\sin\theta = 1$

$2(\cos\theta\cos\alpha - \sin\theta\sin\alpha) = 1$

$\cos(\theta + \alpha) = \frac{1}{2}$

$\theta + \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3}$

$\theta = \underline{\underline{2n\pi}}$ or $\theta = \underline{\underline{2n\pi - \frac{2\pi}{3}}}$

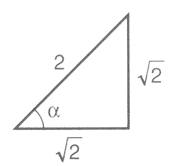
where n is any integer.

14. Let α be an acute angle such that $\tan\alpha = 1$. Then

$2\cos\alpha = \sqrt{2}$

$2\sin\alpha = \sqrt{2}$

$\alpha = \frac{\pi}{4}$



The given equation becomes

$2(\sin\theta + \cos\theta) = \sqrt{6}$

$\sqrt{2}\sin\theta + \sqrt{2}\cos\theta = \sqrt{3}$

$(2\sin\alpha)\sin\theta + (2\cos\alpha)\cos\theta = \sqrt{3}$

$2(\cos\theta\cos\alpha + \sin\theta\sin\alpha) = \sqrt{3}$

$\cos(\theta - \alpha) = \frac{\sqrt{3}}{2}$

$\theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{6}$

$\theta = \underline{\underline{2n\pi + \frac{\pi}{12}}}$ or $\theta = \underline{\underline{2n\pi + \frac{5\pi}{12}}}$

where n is any integer.

15. $\sin 4\theta + \sin 2\theta = 0$

$2\sin 3\theta\cos\theta = 0$

$\sin 3\theta = 0$ or $\cos\theta = 0$

$3\theta = n\pi$ or $\theta = 2n\pi \pm \frac{\pi}{2}$

$\theta = \underline{\underline{\frac{n\pi}{3}}}$ or $\theta = \underline{\underline{2n\pi \pm \frac{\pi}{2}}}$, where n is any integer.

16. $\tan\theta + 3\cot\theta = 5\sec\theta$

$\frac{\sin\theta}{\cos\theta} + \frac{3\cos\theta}{\sin\theta} = \frac{5}{\cos\theta}$

$\sin^2\theta + 3\cos^2\theta = 5\sin\theta$

$\sin^2\theta + 3 - 3\sin^2\theta = 5\sin\theta$

$2\sin^2\theta + 5\sin\theta - 3 = 0$

$(2\sin\theta - 1)(\sin\theta + 3) = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -3$ (rejected)

$\theta = \underline{\underline{n\pi + (-1)^n \frac{\pi}{6}}}$

where n is any integer.

17. $\sin^2\theta + 1 = 3\cos\theta(\sin\theta + \cos\theta)$

$2\sin^2\theta + \cos^2\theta = 3\sin\theta\cos\theta + 3\cos^2\theta$

$2\tan^2\theta + 1 = 3\tan\theta + 3$

$2\tan^2\theta - 3\tan\theta - 2 = 0$

$(2\tan\theta + 1)(\tan\theta - 2) = 0$

$\tan\theta = -\frac{1}{2}$ or $\tan\theta = 2$

$\theta = \underline{\underline{n\pi - 0.46}}$ (corr. to 2 d.p.) or

$\theta = \underline{\underline{n\pi + 1.11}}$ (corr. to 2 d.p.)

where n is any integer.

18. $\cos 2\theta = 3\cos\theta + 4$

$2\cos^2\theta - 1 = 3\cos\theta + 4$

$2\cos^2\theta - 3\cos\theta - 5 = 0$

$(2\cos\theta - 5)(\cos\theta + 1) = 0$

$\cos\theta = \frac{5}{2}$ (rejected) or $\cos\theta = -1$

$\theta = \underline{\underline{2n\pi \pm \pi}}$

where n is any integer.

19. $\tan 3\theta + \sec 3\theta = 1$

$\frac{\sin 3\theta}{\cos 3\theta} + \frac{1}{\cos 3\theta} = 1$

$\sin 3\theta + 1 = \cos 3\theta$ and $\cos 3\theta \neq 0$

$\cos 3\theta - \sin 3\theta = 1$

$\sqrt{2}\cos(3\theta + \frac{\pi}{4}) = 1$

$\cos(3\theta + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$3\theta + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$3\theta = 2n\pi$ or $3\theta = 2n\pi - \frac{\pi}{2}$

$\theta = \frac{2}{3}n\pi$ $\quad\theta = \frac{2}{3}n\pi - \frac{\pi}{6}$

The condition $\cos 3\theta \neq 0$ means

$3\theta \neq 2n\pi \pm \frac{\pi}{2}$

$\theta \neq \frac{2}{3}n\pi \pm \frac{\pi}{6}$

$\therefore\ \theta = \frac{2}{3}n\pi$, where n is any integer.

20. $\sin 2\theta = \cos 2\theta - \sin^2\theta + 1$

$2\sin\theta\cos\theta = 2\cos^2\theta - \sin^2\theta$

$2\tan\theta = 2 - \tan^2\theta$

$\tan^2\theta + 2\tan\theta - 2 = 0$

$(\tan\theta + 1)^2 = 3$

$\tan\theta + 1 = \pm\sqrt{3}$

$\tan\theta = -1 + \sqrt{3}$ or $\tan\theta = -1 - \sqrt{3}$

$\theta = n\pi + 0.63$ (corr. to 2 d.p.) or

$\theta = n\pi - 1.22$ (corr. to 2 d.p.)

where n is any integer.

21. $\sqrt{2}(\cos\theta + \sin\theta) = \cos 2\theta + \sqrt{3}\sin 2\theta$

$\sqrt{2}[\sqrt{2}(\sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta)]$

$= 2\sin\frac{\pi}{6}\cos 2\theta + 2\cos\frac{\pi}{6}\sin 2\theta)]$

$\therefore\ \sin(\frac{\pi}{4} + \theta) = \sin(\frac{\pi}{6} + 2\theta)$

$\therefore\ \frac{\pi}{6} + 2\theta = k\pi + (-1)^k(\frac{\pi}{4} + \theta)$

where k is any integer.

When $k = 2n$,

$\frac{\pi}{6} + 2\theta = 2n\pi + \frac{\pi}{4} + \theta$

$\theta = 2n\pi + \frac{\pi}{12}$

where n is any integer.

When $k = 2n+1$,

$\frac{\pi}{6} + 2\theta = (2n+1)\pi - \frac{\pi}{4} - \theta$

$\theta = \frac{2n+1}{3}\pi - \frac{5\pi}{36}$

$\therefore\ \theta = 2n\pi + \frac{\pi}{12}$ or $\frac{2n+1}{3}\pi - \frac{5\pi}{36}$

where n is any integer.

22. $\sin(\theta - \frac{\pi}{4}) + \sqrt{3}\sin(\theta + \frac{\pi}{4}) = 2$

$\sin(\theta - \frac{\pi}{4}) + \sqrt{3}\sin(\frac{\pi}{2} - \frac{\pi}{4} + \theta) = 2$

$\therefore\ \frac{1}{2}\sin(\theta - \frac{\pi}{4}) + \frac{\sqrt{3}}{2}\cos(\theta - \frac{\pi}{4}) = 1$

Let $r = (\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 = 1$,

$r\cos\alpha = \frac{1}{2}$, $r\sin\alpha = \frac{\sqrt{3}}{2}$, $\therefore\ \alpha = \frac{\pi}{3}$

$\therefore\ \sin(\theta - \frac{\pi}{4})\cos\frac{\pi}{3} + \cos(\theta - \frac{\pi}{4})\sin\frac{\pi}{3} = 1$

$\sin(\theta - \frac{\pi}{4} + \frac{\pi}{3}) = 1$

$\therefore\ \theta = 2n\pi + \frac{5\pi}{12}$ where n is any integer.

23. $\sin\theta + \sin 2\theta + \sin 3\theta = 0$

$(\sin 3\theta + \sin\theta) + \sin 2\theta = 0$

$2\sin 2\theta\cos\theta + \sin 2\theta = 0$

$\sin 2\theta(2\cos\theta + 1) = 0$

$2\cos\theta + 1 = 0$ or $\sin 2\theta = 0$

$\cos\theta = -\frac{1}{2}$ or $2\theta = n\pi$

$\theta = 2n\pi \pm \frac{2\pi}{3}$ or $\theta = \frac{n\pi}{2}$, where n is any integer.

24. $\sin 11\theta\sin 4\theta + \sin 5\theta\sin 2\theta = 0$

$\frac{1}{2}(\cos 7\theta - \cos 15\theta) + \frac{1}{2}(\cos 3\theta - \cos 7\theta) = 0$

$\frac{1}{2}(\cos 3\theta - \cos 15\theta) = 0$

$\sin 9\theta\sin 6\theta = 0$

$\sin 9\theta = 0$ or $\sin 6\theta = 0$

$9\theta = n\pi$ or $6\theta = n\pi$

$\theta = \frac{1}{9}n\pi$ or $\theta = \frac{1}{6}n\pi$, where n is any integer.

25. $4\cos\frac{\theta}{2}\cos\frac{3\theta}{2} = 1$

$4 \cdot \frac{1}{2}(\cos 2\theta + \cos\theta) = 1$

$\cos 2\theta + \cos\theta = \frac{1}{2}$

$2\cos^2\theta - 1 + \cos\theta - \frac{1}{2} = 0$

$4\cos^2\theta + 2\cos\theta - 3 = 0$

$\cos\theta = 0.651\,39$ or -1.151 (rejected)

$\therefore\ \theta = 2n\pi \pm 0.86$ (corr. to 2 d.p.)

where n is any integer.

26. $\sin\theta\sin 7\theta = \sin 3\theta\sin 5\theta$

$\frac{\cos 6\theta - \cos 8\theta}{2} = \frac{\cos 2\theta - \cos 8\theta}{2}$

$\therefore\ \cos 6\theta = \cos 2\theta$

$6\theta = 2n\pi \pm 2\theta$ for any integer n.

$\theta = \frac{n\pi}{2}$ or $\theta = \frac{n\pi}{4}$

Since the values of $\theta = \frac{n\pi}{4}$ include those of $\frac{n\pi}{2}$, the general solution of the equation is $\theta = \frac{n\pi}{4}$, where n is any integer.

27. $\cot\theta + \cot(\theta + \frac{\pi}{4}) = 3$

$\frac{1}{\tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta} = 3$

$1 + \tan\theta + \tan\theta(1 - \tan\theta) = 3\tan\theta(1 + \tan\theta)$

$4\tan^2\theta + \tan\theta - 1 = 0$

$\tan\theta = 0.390\,4$ or $\tan\theta = -0.640\,4$

$\theta = n\pi + 0.37$ or $\theta = n\pi - 0.57$

(corr. to 2 d.p.) (corr to 2 d.p.)

where n is any integer.

28. $\sin^3\theta - \cos^3\theta = \sin\theta - \cos\theta$

$(\sin\theta - \cos\theta)(\sin^2\theta + \sin\cos\theta + \cos^2\theta)$

$= \sin\theta - \cos\theta$

$(\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta) = \sin\theta - \cos\theta$

$(\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta) - (\sin\theta - \cos\theta) = 0$

$(\sin\theta - \cos\theta)\sin\theta\cos\theta = 0$

$\sin\theta - \cos\theta = 0$ or $\sin\theta\cos\theta = 0$

$\tan\theta = 1$ or $\sin 2\theta = 0$

$\theta = n\pi + \frac{\pi}{4}$ or $2\theta = n\pi$

$\theta = \frac{n}{2}\pi$

where n is any integer.

29. (a) $\sin^4 x + \cos^4 x$

$= \sin^4 x + 2\sin^2 x\cos^2 x$

$+ \cos^4 x - 2\sin^2 x\cos^2 x$

$= (\sin^2 x + \cos^2 x)^2 - \frac{1}{2}\sin^2 2x$

$= 1 - \frac{1}{2}\sin^2 2x$

(b) $4(\sin^4 x + \cos^4 x) - \sin 2x - 3 = 0$

$4(1 - \frac{1}{2}\sin^2 2x) - \sin 2x - 3 = 0$ (by **(a)**)

$2\sin^2 2x + \sin 2x - 1 = 0$

$(\sin 2x + 1)(2\sin 2x - 1) = 0$

$\therefore\ \sin 2x = -1$ or $\sin 2x = \frac{1}{2}$

$2x = n\pi - (-1)^n\frac{\pi}{2}$ or $2x = n\pi + (-1)^n\frac{\pi}{6}$

$\therefore\ x = \frac{n\pi}{2} - (-1)^n\frac{\pi}{4}$ or $\frac{n\pi}{2} + (-1)^n\frac{\pi}{12}$

where n is any integer.

For $x = \frac{n\pi}{2} - (-1)^n\frac{\pi}{4}$,

let $n = 2m$

$x = \frac{2m\pi}{2} - (-1)^{2m}\frac{\pi}{4}$

$= m\pi - \frac{\pi}{4}$

let $n = 2m + 1$

$x = \frac{(2m+1)\pi}{2} - (-1)^{2m+1}\frac{\pi}{4}$

$= \frac{(2m+1)\pi}{2} + \frac{\pi}{4}$

$= \frac{(2m+1)\pi}{2} - \frac{\pi}{4} + \frac{\pi}{2}$

$= (m+1)\pi - \frac{\pi}{4}$

$\therefore\ x = n\pi - \frac{\pi}{4}$

$\therefore\ x = n\pi - \frac{\pi}{4}$ or $x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{12}$

where n is any integer.

30. (a) $\frac{1}{2}[(\sin x + \cos x)^2 - 1]$

$= \frac{1}{2}(\sin^2 x + 2\sin x\cos x + \cos^2 x - 1)$

$= \frac{1}{2}(2\sin x\cos x)$

$= \sin x\cos x$

(b) Let $t = \sin x + \cos x$,

$\therefore$ The given equation becomes

$t + \frac{1}{2}(t^2 - 1) = 1$

$t^2 + 2t - 3 = 0$

$t = 1$ or $t = -3$

As $|\sin x| \le 1$ and $|\cos x| \le 1$,

$\therefore$ $t = \sin x + \cos x = -3$ is rejected.

$\therefore$ $\sin x + \cos x = 1$

$\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore$ $\sin(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$x + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}$

$\therefore$ $x = n\pi + [(-1)^n - 1]\frac{\pi}{4}$, where n is any integer.

31. (a) $\tan\theta\tan(\theta+\alpha) = k$

$\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} = k$

$\sin\theta(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$
$= k\cos\theta(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

$\sin^2\theta\cos\alpha + \sin\theta\cos\theta\sin\alpha$
$= k\cos^2\theta\cos\alpha - k\sin\theta\cos\theta\sin\alpha$

$(k+1)\sin\theta\cos\theta\sin\alpha$
$= (k\cos^2\theta - \sin^2\theta)\cos\alpha$

$\frac{k+1}{2}\sin 2\theta\sin\alpha$
$= [k(\frac{\cos 2\theta + 1}{2}) - \frac{1 - \cos 2\theta}{2}]\cos\alpha$

$(k+1)\sin 2\theta\sin\alpha$
$= (k\cos 2\theta + k - 1 + \cos 2\theta)\cos\alpha$
$= [(k+1)\cos 2\theta + (k-1)]\cos\alpha$

$\therefore$ $(1-k)\cos\alpha$
$= (k+1)(\cos 2\theta\cos\alpha - \sin 2\theta\sin\alpha)$
$= (k+1)\cos(2\theta+\alpha)$

(b) Substitute $\alpha = \frac{\pi}{3}$, $k = 2$ into the result of **(a)**,

$\therefore$ $3\cos(2\theta + \frac{\pi}{3}) = -\cos\frac{\pi}{3}$

$\cos(2\theta + \frac{\pi}{3}) = -\frac{1}{6}$

$2\theta + \frac{\pi}{3} = 2n\pi \pm 1.738$

$\theta = \frac{6n-1}{6}\pi \pm 0.87$ (corr. to 2 d.p.)

where n is any integer.

Revision Exercise 8 (p.188)

1. $\sec 5\theta + \csc 2\theta = 0$

$\frac{1}{\cos 5\theta} + \frac{1}{\sin 2\theta} = 0$

$\therefore$ $-\sin 2\theta = \cos 5\theta$

$\therefore$ $\cos 5\theta = \sin(-2\theta) = \cos(\frac{\pi}{2} + 2\theta)$

$\frac{\pi}{2} + 2\theta = 2n\pi \pm 5\theta$, where n is any integer.

$\theta = \frac{-2n\pi}{3} + \frac{\pi}{6}$ or $\theta = \frac{2n\pi}{7} - \frac{\pi}{14}$

$\therefore$ $\theta = \frac{2n\pi}{3} + \frac{\pi}{6}$ or $\theta = \frac{2n\pi}{7} - \frac{\pi}{14}$

2. $3 + \sin 2x - \sin x = 6\cos x$
$3 + 2\sin x\cos x - \sin x = 6\cos x$
$\sin x(2\cos x - 1) + 3(1 - 2\cos x) = 0$
$(2\cos x - 1)(\sin x - 3) = 0$

$\cos x = \frac{1}{2}$ or $\sin x = 3$ (rejected)

$x = 2n\pi \pm \frac{\pi}{3}$, where n is any integer.

3. $3\tan(x - \frac{\pi}{12}) = \tan(x + \frac{\pi}{12})$

$3\cdot\frac{\sin(x-\frac{\pi}{12})}{\cos(x-\frac{\pi}{12})} = \frac{\sin(x+\frac{\pi}{12})}{\cos(x+\frac{\pi}{12})}$

$3\sin(x - \frac{\pi}{12})\cos(x + \frac{\pi}{12}) = \sin(x + \frac{\pi}{12})\cos(x - \frac{\pi}{12})$

$\frac{3}{2}(\sin 2x - \sin\frac{\pi}{6}) = \frac{1}{2}(\sin 2x + \sin\frac{\pi}{6})$

$\sin 2x = 1$

$2x = 2n\pi + \frac{\pi}{2}$

$x = n\pi + \frac{\pi}{4}$, where n is any integer.

4. (a) $2\sin x\cos(p - x) = 2\cdot\frac{1}{2}[\sin p + \sin(2x - p)]$
$= \sin(2x - p) + \sin p$

(b) $\sin x + 2\sin x\cos(p - x) - \sin p = 0$

$\therefore$ $\sin x + \sin(2x - p) + \sin p - \sin p = 0$

$\therefore$ $\sin x + \sin(2x - p) = 0$
$\sin(2x - p) = \sin(-x)$

$\therefore$ $2x - p = n\pi + (-1)^n(-x)$
$[2 + (-1)^n]x = p + n\pi$

$\therefore$ $x = \frac{p + n\pi}{2 + (-1)^n}$

where n is any integer.

5. $\sin x + \sin 2x + \sin 3x + \sin 4x + \sin 5x = 0$
$\sin x + \sin 5x + \sin 2x + \sin 4x + \sin 3x = 0$
$2\sin 3x\cos 2x + 2\sin 3x\cos x + \sin 3x = 0$
$\sin 3x(2\cos 2x + 2\cos x + 1) = 0$

$\sin 3x = 0$ or $2\cos 2x + 2\cos x + 1 = 0$

$3x = n\pi$ or $4\cos^2 x - 2 + 2\cos x + 1 = 0$

$x = \frac{n\pi}{3}$ or $4\cos^2 x + 2\cos x - 1 = 0$

$\cos x = 0.309$ or $\cos x = -0.809$

$x = 2n\pi \pm \frac{2\pi}{5}$ or $x = 2n\pi \pm \frac{4\pi}{5}$

where n is any integer.

6. No solution is provided for the H.K.C.E.E. question because of the copyright reasons.

7. (a) $\sin^2 x + \sin^2 2x + \sin^2 3x$

$= \frac{1 - \cos 2x}{2} + \frac{1 - \cos 4x}{2} + \frac{1 - \cos 6x}{2}$

$= \frac{3}{2} - \frac{1}{2}(\cos 2x + \cos 4x + \cos 6x)$

$= \frac{3}{2} - \frac{1}{2}(2\cos 2x\cos 4x + \cos 4x)$

$= \frac{3}{2} - \frac{1}{2}\cos 4x(2\cos 2x + 1)$

(b) $\sin^2 x + \sin^2 2x + \sin^2 3x = \frac{3}{2}$

$\sin^2 x + \sin^2 2x + \sin^2 3x - \frac{3}{2} = 0$

$-\frac{1}{2}\cos 4x(2\cos 2x + 1) = 0$

$\cos 4x(2\cos 2x + 1) = 0$

$\therefore$ $\cos 4x = 0$ or $\cos 2x = -\frac{1}{2}$

$4x = 2n\pi \pm \frac{\pi}{2}$ or $2x = 2n\pi \pm \frac{2\pi}{3}$

$x = \frac{n\pi}{2} \pm \frac{\pi}{8}$ or $x = n\pi \pm \frac{\pi}{3}$

for any integer n.

8. (a) $\sin(\theta + \alpha) = k\cos(\theta - \alpha)$

$\sin\theta\cos\alpha + \cos\theta\sin\alpha$
$= k(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$\sin\theta(\cos\alpha - k\sin\alpha)$
$= \cos\theta(k\cos\alpha - \sin\alpha)$

$\sin\theta(1 - k\tan\alpha) = \cos\theta(k - \tan\alpha)$

$\tan\theta = \frac{k - \tan\alpha}{1 - k\tan\alpha}$

(b) $k = \frac{2}{3}\sqrt{3}$, $\alpha = \frac{\pi}{3}$, $\therefore$ $\tan\alpha = \sqrt{3}$

$\tan\theta = \frac{\frac{2}{3}\sqrt{3} - \sqrt{3}}{1 - \frac{2}{3}\sqrt{3}\cdot\sqrt{3}} = \frac{-\frac{1}{3}\sqrt{3}}{-1} = \frac{1}{\sqrt{3}}$

$\theta = n\pi + \frac{\pi}{6}$, where n is any integer.

9. (a) Let $c = \cos 2x$,

$\therefore$ $\sin^2 x = \frac{1 - \cos 2x}{2} = \frac{1 - c}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2} = \frac{1 + c}{2}$

$\therefore$ (*) becomes

$(\frac{1-c}{2})^4 + (\frac{1+c}{2})^4 = \frac{17}{16}c^2$

(b) $(\frac{1-c}{2})^4 + (\frac{1+c}{2})^4 = \frac{17}{16}c^2$

$(1-c)^4 + (1+c)^4 = 17c^2$

$2c^4 - 5c^2 + 2 = 0$

$(c^2 - 2)(2c^2 - 1) = 0$

$\therefore$ $c^2 = 2$ or $c^2 = \frac{1}{2}$

As $c = \cos 2x$, $\therefore$ $c^2 = 2$ is rejected.

$\therefore$ $\cos 2x = \pm\frac{\sqrt{2}}{2}$

$2x = 2n\pi \pm \frac{\pi}{4}$ or $2x = (2n+1)\pi \pm \frac{\pi}{4}$

$\therefore$ $x = n\pi \pm \frac{\pi}{8}$ or $x = (\frac{2n+1}{2})\pi \pm \frac{\pi}{8}$

where n is any integer.

10. (a) Let $P(n)$ be the proposition

" $\sin\theta - \sin 3\theta + \sin 5\theta + \cdots$
$+ (-1)^{n+1}\sin(2n-1)\theta = \frac{(-1)^{n+1}\sin 2n\theta}{2\cos\theta}$ ".

When $n = 1$,

L.H.S. $= \sin\theta$

R.H.S. $= \frac{(-1)^2\sin 2\theta}{2\cos\theta} = \frac{2\sin\theta\cos\theta}{2\cos\theta} = \sin\theta$

$\therefore$ $P(1)$ is true.

Assume $P(k)$ is true for any positive integer k.

i.e. $\sin\theta - \sin 3\theta + \sin 5\theta + \cdots$
$+ (-1)^{k+1}\sin(2k-1)\theta$
$= \frac{(-1)^{k+1}\sin 2k\theta}{2\cos\theta}$

Then

$\sin\theta-\sin 3\theta+\sin 5\theta+\cdots+(-1)^{k+1}\sin(2k-1)\theta$
$+(-1)^{k+1+1}\sin[2(k+1)-1]\theta$
$=\dfrac{(-1)^{k+1}\sin 2k\theta}{2\cos\theta}+(-1)^{k+2}\sin(2k+1)\theta$
$=[(-1)^{k+1}\sin 2k\theta$
$+2(-1)^{k+2}\sin(2k+1)\theta\cos\theta]\div 2\cos\theta$
$=\{(-1)^{k+1}\sin 2k\theta+2(-1)^{k+2}\dfrac{1}{2}[\sin 2k\theta$
$+\sin 2(k+1)\theta]\}\div 2\cos\theta$
$=[-(-1)^{k}\sin 2k\theta+(-1)^{k}\sin 2k\theta$
$+(-1)^{k+2}\sin 2(k+1)\theta]\div 2\cos\theta$
$=\dfrac{(-1)^{(k+1)+1}\sin 2(k+1)\theta}{2\cos\theta}$

Thus assuming $P(k)$ is true for any positive integer k, $P(k+1)$ is also true. By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

(b) $\sin\theta\cos\theta-\sin 3\theta\cos\theta+\sin 5\theta\cos\theta$
$+\cdots+\sin 9\theta\cos\theta=0$
$\cos\theta(\sin\theta-\sin 3\theta+\sin 5\theta+\cdots+\sin 9\theta)=0$
$\cos\theta=0$ (rejected) or
$\sin\theta-\sin 3\theta+\sin 5\theta+\cdots+\sin 9\theta=0$
$\dfrac{(-1)^{5+1}\sin 2(5)\theta}{2\cos\theta}=0$
$\sin 10\theta=0$ for $\cos\theta\neq 0$
$10\theta=n\pi$
$\theta=\dfrac{n\pi}{10}$
where n is any integer.

11. (a) Sum of the roots $=\tan\theta+\cot\theta$
$=\dfrac{4\sqrt{3}}{3}$

(b) By **(a)**, $\tan\theta+\cot\theta=\dfrac{4\sqrt{3}}{3}$
$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{4\sqrt{3}}{3}$
$\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\dfrac{4\sqrt{3}}{3}$
$3=4\sqrt{3}\sin\theta\cos\theta$
$3=2\sqrt{3}\sin 2\theta$
$\sin 2\theta=\dfrac{3}{2\sqrt{3}}$
$=\dfrac{\sqrt{3}}{2}$

(c) $\sin 2\theta=\dfrac{\sqrt{3}}{2}$
$2\theta=n\pi+(-1)^n\dfrac{\pi}{3}$
$\theta=\dfrac{n\pi}{2}+(-1)^n\dfrac{\pi}{6}$
where n is any integer.

12 – 13. No solutions are provided for the H.K.C.E.E. questions because of the copyright reasons.

Enrichment 8 (p.190)

1. (a) $\cos 4\theta=2\cos^2 2\theta-1$
$=2(2\cos^2\theta-1)^2-1$
$=2(4\cos^4\theta-4\cos^2\theta+1)-1$
$=8\cos^4\theta-8\cos^2\theta+1$

(b) $16x^4-16x^2+1=0$
$8x^4-8x^2+\dfrac{1}{2}=0$
Let $x=\cos\theta$
$\cos 4\theta-1+\dfrac{1}{2}=0$
$\cos 4\theta=\dfrac{1}{2}$
$4\theta=2n\pi\pm\dfrac{\pi}{3}$
$\theta=\dfrac{n\pi}{2}\pm\dfrac{\pi}{12}$
where n is any integer.
$x=\cos(\dfrac{n\pi}{2}\pm\dfrac{\pi}{12})$
$x=\pm 0.966,\ \pm 0.259$ (corr. to 3 sig.fig.)

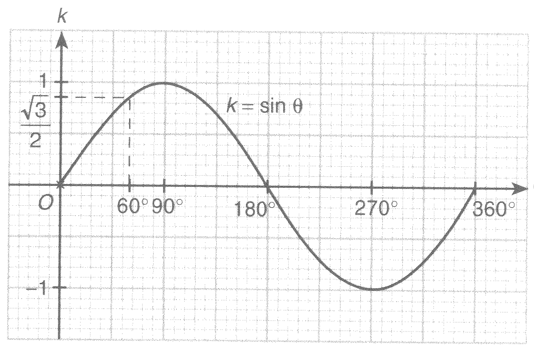
2. (a) $1+m=1+\dfrac{\cos\beta}{\cos(2\alpha+\beta)}=\dfrac{\cos(2\alpha+\beta)+\cos\beta}{\cos(2\alpha+\beta)}$
$=\dfrac{2\cos(\alpha+\beta)\cos\alpha}{\cos(2\alpha+\beta)}$

(b) $(1+m)\tan\alpha\tan(\alpha+\beta)$
$=\dfrac{2\cos(\alpha+\beta)\cos\alpha}{\cos(2\alpha+\beta)}\cdot\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$
$=\dfrac{2\sin\alpha\sin(\alpha+\beta)}{\cos(2\alpha+\beta)}$
$=\dfrac{2(-\frac{1}{2})[\cos(2\alpha+\beta)-\cos\beta]}{\cos(2\alpha+\beta)}$
$=\dfrac{\cos\beta-\cos(2\alpha+\beta)}{\cos(2\alpha+\beta)}$
$=\dfrac{\cos\beta}{\cos(2\alpha+\beta)}-1$
$=m-1$

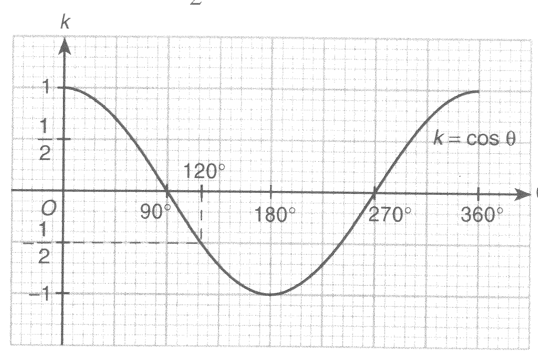
(c) Let $\alpha=x$, $\beta=\dfrac{\pi}{4}$
$m=\dfrac{\cos\frac{\pi}{4}}{\cos(2x+\frac{\pi}{4})}=5$
By **(b)**,
$(1+5)\tan x\tan(x+\dfrac{\pi}{4})=5-1$
$\tan x\tan(x+\dfrac{\pi}{4})=\dfrac{2}{3}$
$\tan x\cdot\dfrac{1+\tan x}{1-\tan x}=\dfrac{2}{3}$
$3\tan x+3\tan^2 x=2-2\tan x$
$3\tan^2 x+5\tan x-2=0$
$(3\tan x-1)(\tan x+2)=0$
$\tan x=\dfrac{1}{3}$ or $\tan x=-2$
$x=n\pi+0.322$ or $x=n\pi-1.11$
(corr. to 3 sig.fig.)
where n is any integer.

Classwork 1 (p.178)

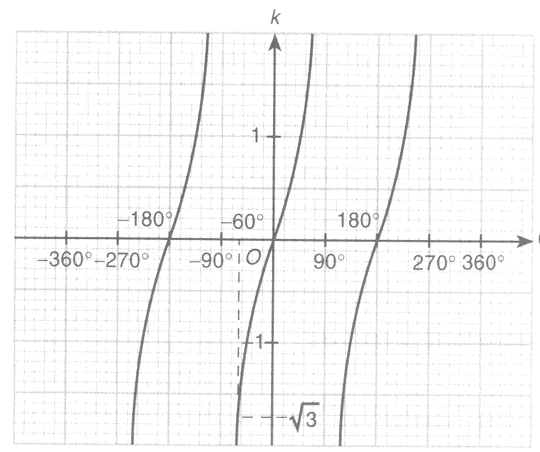
1. $\sin\theta=\dfrac{\sqrt{3}}{2}$
$\theta=\sin^{-1}(\dfrac{\sqrt{3}}{2})=60°$



2. $\cos\theta=-\dfrac{1}{2}$
$\theta=\cos^{-1}(-\dfrac{1}{2})=120°$



3. $\tan\theta=-\sqrt{3}$
$\theta=\tan^{-1}(-\sqrt{3})=-60°$



Classwork 2 (p.181)

	Principal value	General solution
$\sin\theta=\frac{1}{2}$	30°	$180°n+(-1)^n 30°$
$\sin\theta+\frac{\sqrt{3}}{2}=0$	−60°	$180°n-(-1)^n 60°$
$\cos\theta=-\frac{1}{2}$	120°	$360°n\pm 120°$
$\frac{1}{3}\tan\theta+\frac{1}{2}=0$	−56.3°	$180°n-56.3°$

Classwork 3 (p.183)

1. $\sin\theta+\cos\theta=0$
$\sin\theta=-\cos\theta$
$\tan\theta=-1$
$\theta=n\pi-\dfrac{\pi}{4}$
where n is any integer.

2. $1+2\sqrt{3}\sin x\cos x+2\cos^2 x=0$
$\sin^2 x+\cos^2 x+2\sqrt{3}\sin x\cos x+2\cos^2 x=0$
$\sin^2 x+2\sqrt{3}\sin x\cos x+3\cos^2 x=0$
$(\sin x+\sqrt{3}\cos x)^2=0$
$\sin x+\sqrt{3}\cos x=0$
$\sin x=-\sqrt{3}\cos x$
$\tan x=-\sqrt{3}$
$x=n\pi-\dfrac{\pi}{3}$
where n is any integer.

Classwork 4 (p.184)

$$\begin{aligned}\tan 4x - \cot 3x &= 0\\ \tan 4x &= \cot 3x\\ \tan 4x &= \tan\left(\frac{\pi}{2} - 3x\right)\\ 4x &= n\pi + \frac{\pi}{2} - 3x\\ x &= \underline{\underline{\frac{n\pi}{7} + \frac{\pi}{14}}}\end{aligned}$$

where n is any integer.

Classwork 5 (p.184)

$$\begin{aligned}\cos\frac{\pi}{5}\cos\theta - \sin\frac{\pi}{5}\sin\theta &= \frac{1}{2}\\ \cos\left(\frac{\pi}{5} + \theta\right) &= \frac{1}{2}\\ \frac{\pi}{5} + \theta &= 2n\pi \pm \frac{\pi}{3}\end{aligned}$$

$\therefore\quad \theta = 2n\pi + \frac{\pi}{3} - \frac{\pi}{5}$ or $\theta = 2n\pi - \frac{\pi}{3} - \frac{\pi}{5}$

$\theta = \underline{\underline{2n\pi + \frac{2\pi}{15}}}$ or $\theta = \underline{\underline{2n\pi - \frac{8\pi}{15}}}$

Classwork 6 (p.185)

Let α be an acute angle such that $\tan\alpha = \frac{1}{2}$. Then,

$1 = \sqrt{5}\sin\alpha$

$2 = \sqrt{5}\cos\alpha$

$\alpha = 0.463\,6$

The given equation becomes

$$\begin{aligned}\sqrt{5}\sin\alpha\sin\theta - \sqrt{5}\cos\alpha\cos\theta &= \sqrt{5}\\ \cos\alpha\cos\theta - \sin\alpha\sin\theta &= -1\\ \cos(\theta + \alpha) &= -1\\ \theta + 0.463\,6 &= 2n\pi \pm \pi\\ &= (2n+1)\pi\end{aligned}$$

where n is any integer.

$\therefore\quad \theta + 0.463\,6 = 2n\pi + \pi$

$\theta = \underline{\underline{2n\pi + 2.68}}$ (corr. to 2 d.p.)

Classwork 7 (p.186)

$$\begin{aligned}\cos 2\theta\cos 4\theta &= \frac{1}{2}\cos 6\theta - \frac{1}{2}\cos 2\theta\\ \frac{1}{2}(\cos 6\theta + \cos 2\theta) &= \frac{1}{2}\cos 6\theta - \frac{1}{2}\cos 2\theta\\ \frac{1}{2}\cos 2\theta &= -\frac{1}{2}\cos 2\theta\\ \cos 2\theta &= 0\end{aligned}$$

$2\theta = 2n\pi \pm \frac{\pi}{2}$, where n is any integer.

$\theta = \underline{\underline{n\pi \pm \frac{\pi}{4}}}$