HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2000

MATHEMATICS PAPER 1

Question-Answer Book

8.30 am – 10.30 am (2 hours)

This paper must be answered in English

1. Write your candidate number, centre number and seat number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 33 marks.

3. Attempt ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Supplementary answer sheets will be supplied on request. Write your Candidate Number on each sheet and fasten them with string inside this book.

4. Write the question numbers of the questions you have attempted in Section B in the spaces provided on this cover.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

Candidate Number		
Centre Number		
Seat Number		

Section A Question No.	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
	Marks	Marks
1–2		
3–4		
5–6		
7–8		
9		
10		
11		
12		
13		
14		
Section A Total		

Checker's Use Only — Section A Total

Section B Question No.*	Marks	Marks
Section B Total		

Checker's Use Only — Section B Total

*To be filled in by the candidate.

Checker No.

FORMULAS FOR REFERENCE

SPHERE	Surface area	$= 4\pi r^2$
	Volume	$= \dfrac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	$= 2\pi rh$
	Volume	$= \pi r^2 h$
CONE	Area of curved surface	$= \pi rl$
	Volume	$= \dfrac{1}{3}\pi r^2 h$
PRISM	Volume	$=$ base area $\times$ height
PYRAMID	Volume	$= \dfrac{1}{3} \times$ base area $\times$ height

SECTION A(1) (33 marks)

Answer ALL questions in this section and write your answers in the spaces provided.

1.　Let $C = \dfrac{5}{9}(F - 32)$. If $C = 30$, find F .

(3 marks)

2.　Simplify $\dfrac{x^{-3}\,y}{x^2}$　and express your answer with positive indices.

(3 marks)

3. Find the area of the sector in Figure 1.

75°

6 cm

Figure 1

(3 marks)

4. In Figure 2, find a and x.

$x°$

10 cm

7 cm

a cm

Figure 2

(4 marks)

Page total

5. Solve $\dfrac{11-2x}{5} < 1$ and represent the solution in Figure 3.

(4 marks)

Figure 3

6. Let $f(x) = 2x^3 + 6x^2 - 2x - 7$. Find the remainder when $f(x)$ is divided by $x+3$.

(3 marks)

7. In Figure 4, AD and BC are two parallel chords of the circle. AC and BD intersect at E. Find x and y.

(4 marks)

Figure 4

8. On a map of scale $1:5\,000$, the area of the passenger terminal of the Hong Kong International Airport is $220\ \text{cm}^2$. What is the actual area, in m^2, occupied by the terminal on the ground?

(4 marks)

9. Let L be the straight line passing through $(-4, 4)$ and $(6, 0)$.

   (a) Find the slope of L.

   (b) Find the equation of L.

   (c) If L intersects the y-axis at C, find the coordinates of C.

   (5 marks)

Section A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. (a) Solve $10x^2 + 9x - 22 = 0$.

(2 marks)

(b) Mr. Tung deposited $\$10\,000$ in a bank on his 25th birthday and $\$9\,000$ on his 26th birthday. The interest was compounded yearly at $r\%$ p.a., and the total amount he received on his 27th birthday was $\$22\,000$. Find r.

(4 marks)

Page total

11. Figure 5 shows the cumulative frequency polygon of the distribution of the lengths of 75 songs.

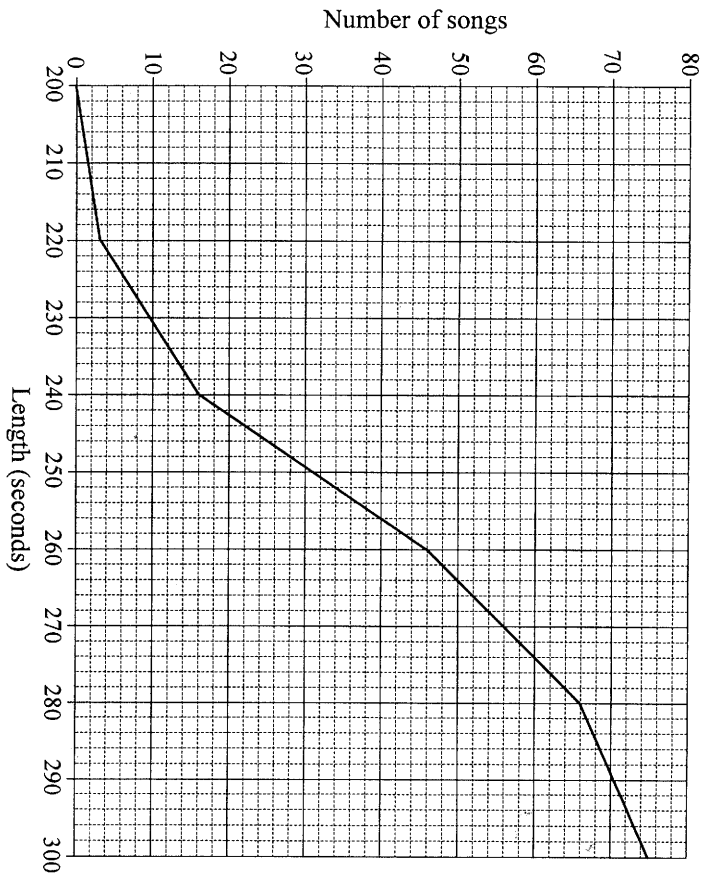**The cumulative frequency polygon of the distribution of the lengths of 75 songs**

Length (seconds)

Number of songs

Figure 5

(a) Complete the tables below.

Length (*t* seconds)	Cumulative frequency
$t \le 220$	3
$t \le 240$	16
$t \le 260$	46
$t \le 280$	
$t \le 300$	75

Length (*t* seconds)	Frequency
$200 < t \le 220$	3
$220 < t \le 240$	13
$240 < t \le 260$	30
$260 < t \le 280$	
$280 < t \le 300$	9

(2 marks)

(b) Find an estimate of the mean of the distribution.

(2 marks)

(c) Estimate from the cumulative frequency polygon the median of the distribution.

(1 mark)

(d) What percentage of these songs have lengths greater than 220 seconds but not greater than 260 seconds?

(2 marks)

12. A box contains nine hundred cards, each marked with a different 3-digit number from 100 to 999. A card is drawn randomly from the box.

(a) Find the probability that two of the digits of the number drawn are zero.

(2 marks)

(b) Find the probability that none of the digits of the number drawn is zero.

(2 marks)

(c) Find the probability that exactly one of the digits of the number drawn is zero.

(2 marks)

Go on to the next page ▶

13. In Figure 6, *ABCDE* is a regular pentagon and *CDFG* is a square. *BG* produced meets *AE* at *P* .

(a) Find $\angle BCG$, $\angle ABP$ and $\angle APB$.

(5 marks)

(b) Using the fact that $\dfrac{AP}{\sin \angle ABP} = \dfrac{AB}{\sin \angle APB}$, or otherwise, determine which line segment, *AP* or *PE* , is longer.

(3 marks)

Figure 6

14. An auditorium has 50 rows of seats. All seats are numbered in numerical order from the first row to the last row, and from left to right, as shown in Figure 7. The first row has 20 seats. The second row has 22 seats. Each succeeding row has 2 more seats than the previous one.

1st row

2nd row

3rd row

43 44   65 66

21 22   41 42

1 2   19 20

Figure 7

(a)   How many seats are there in the last row?

(2 marks)

(b)   Find the total number of seats in the first n rows.

Hence determine in which row the seat numbered 2000 is located.

(4 marks)

Page total

SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

15. A company produces two brands, A and B, of mixed nuts by putting peanuts and almonds together. A packet of brand A mixed nuts contains 40 g of peanuts and 10 g of almonds. A packet of brand B mixed nuts contains 30 g of peanuts and 25 g of almonds. The company has 2400 kg of peanuts, 1200 kg of almonds and 70 carton boxes. Each carton box can pack 1000 brand A packets or 800 brand B packets.

    The profits generated by a box of brand A mixed nuts and a box of brand B mixed nuts are $800 and $1000 respectively. Suppose x boxes of brand A mixed nuts and y boxes of brand B mixed nuts are produced.

    (a) Using the graph paper in Figure 8, find x and y so that the profit is the greatest.
    (8 marks)

    (b) If the number of boxes of brand B mixed nuts is to be smaller than the number of boxes of brand A mixed nuts, find the greatest profit.
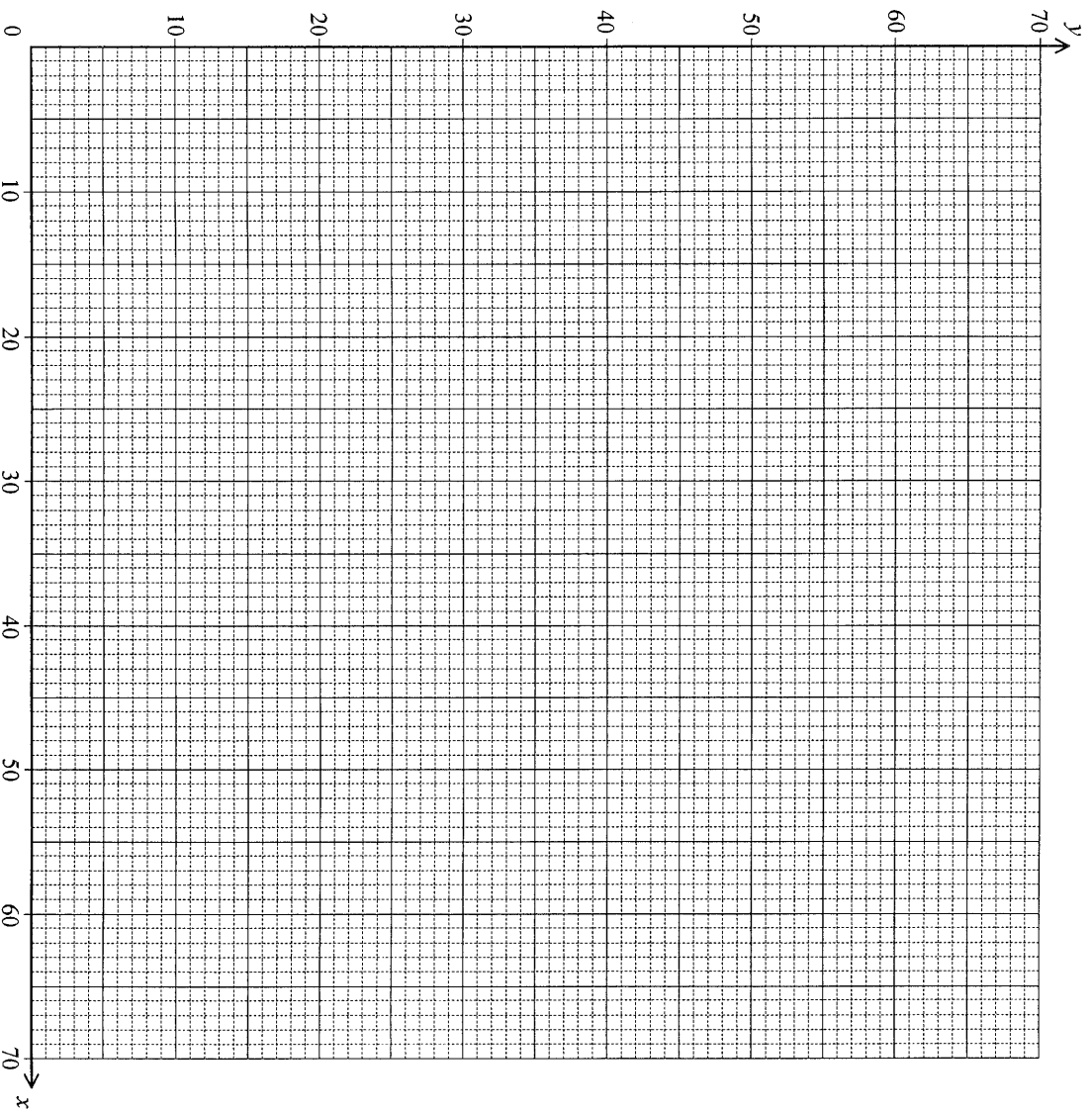   (3 marks)

Figure 8

16. In Figure 9, C is the centre of the circle PQS. OR and OP are tangent to the circle at S and P respectively. OCQ is a straight line and $\angle QOP = 30°$.

(a) Show that $\angle PQO = 30°$.   (3 marks)

(b) Suppose $OPQR$ is a cyclic quadrilateral.

   (i) Show that RQ is tangent to circle PQS at Q.

   (ii) A rectangular coordinate system is introduced in Figure 9 so that the coordinates of O and C are $(0, 0)$ and $(6, 8)$ respectively. Find the equation of QR.

                      (8 marks)

Figure 9

Page total

17. Figure 10 shows a circle with centre O and radius 10 m on a vertical wall which stands on the horizontal ground. A, B and C are three points on the circumference of the circle such that A is vertically below O, $\angle AOB = 90°$ and $\angle AOC = 20°$. A laser emitter D on the ground shoots a laser beam at B. The laser beam then sweeps through an angle of $30°$ to shoot at A. The angles of elevation of B and A from D are $60°$ and $30°$ respectively.

Figure 10

(a) Let A be h m above the ground.

    (i)   Express AD and BD in terms of h.

    (ii) Find h.

                                  (7 marks)

(b) Another laser emitter E on the ground shoots a laser beam at A with angle of elevation $25°$. The laser beam then sweeps through an angle of $5°$ to shoot at C. Find $\angle ACE$.

                                  (4 marks)

18. Figure 11.1 shows a solid hemisphere of radius 10 cm . It is cut into two portions, P and Q , along a plane parallel to its base. The height and volume of P are h cm and V cm^3 respectively.
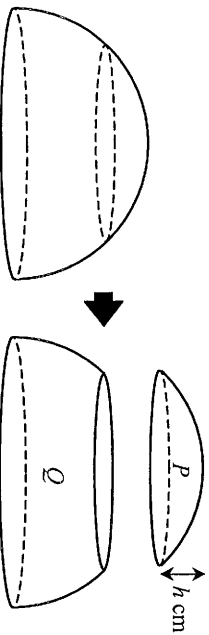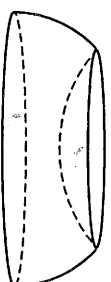

Figure 11.1

It is known that V is the sum of two parts. One part varies directly as h^3 . $V = \dfrac{29}{3}\pi$ when $h = 1$ and $V = 81\pi$ when $h = 3$.

(a) Find V in terms of h and π .

(3 marks)

(b) A solid congruent to P is carved away from the top of Q to form a container as shown in Figure 11.2 .

Figure 11.2

(i) Find the surface area of the container (excluding the base).

(ii) It is known that the volume of the container is $\dfrac{1400}{3}\pi$ cm^3 . Show that $h^3 - 30h^2 + 300 = 0$.

Using the graph in Figure 11.3 and a suitable method, find the value of h correct to 2 decimal places.

(8 marks)
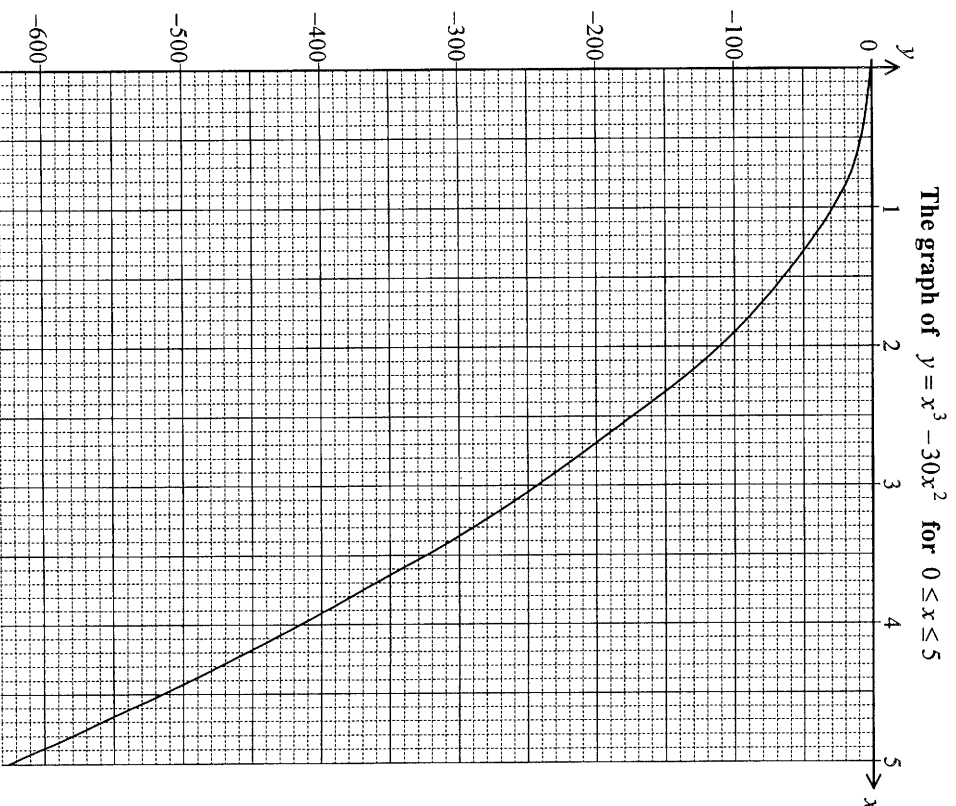

The graph of $y = x^3 - 30x^2$ **for** $0 \le x \le 5$

Figure 11.3

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