

只限教師參閱

FOR TEACHERS' USE ONLY

香港考試及評核局

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2002年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2002

數學 試卷一

MATHEMATICS PAPER 1

本評卷參考乃香港考試及評核局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the teachers' centre.

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2002-CB-MATH 1-1

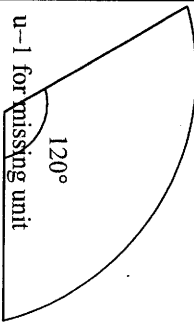
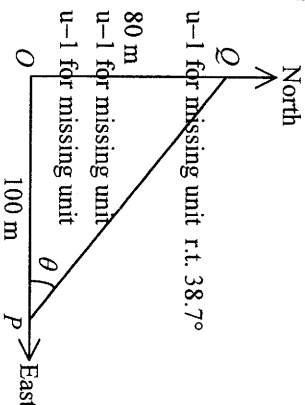
只限教師參閱

FOR TEACHERS' USE ONLY

Hong Kong Certificate of Education Examination
Mathematics Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:
 - 'M' marks awarded for correct methods being used;
 - 'A' marks awarded for the accuracy of the answers;
 - Marks without 'M' or 'A' awarded for correctly completing a proof or arriving at an answer given in a question.In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
 - a. The symbol $(u-1)$ should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* for the whole paper.
 - b. The symbol $(pp-1)$ should be used to denote 1 mark deducted for *pp*. At most deduct **2 marks** for *pp* for the whole paper. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
 - c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
 - d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.
7. Marks entered in the Page Total Box should be the NET total scored on that page.
8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to', 'f.t.' stands for 'follow through' and 'or equivalent' means 'accepting equivalent forms of the equation which has been simplified and without uncollected like terms'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
<p>1. $\frac{(ab^2)^2}{a^5} = \frac{(a^2)(b^2)^2}{a^5}$ $= \frac{a^4 b^4}{a^5}$ $= \frac{b^4}{a^{5-2}}$ $= \frac{b^4}{a^3}$</p>	<p>IM IM 1A</p>	<p>$(xy)^n = x^n y^n$ $\frac{x^m}{x^n} = x^{m-n}$</p>
<p>2. Area = $\frac{120}{360} \cdot \pi(6)^2$ $= 12\pi \text{ cm}^2$</p>	<p>IM + 1A 1A</p>	<p>IM for $\frac{120}{360}$, 1A for area of circle u-1 for missing unit</p>
<p>The angle at the centre is $120 \times \frac{\pi}{180} = \left(\frac{2\pi}{3}\right)$ Area = $\frac{1}{2} \cdot \frac{2\pi}{3} \cdot 6^2$ $= 12\pi \text{ cm}^2$</p>	<p>1A IM 1A</p>	 <p>u-1 for missing unit</p>
<p>3. (a) $\tan \theta = \frac{80}{100}$ $\theta \approx 38.66^\circ \approx 38.7^\circ$ (b) The bearing of P from Q is $90^\circ + 38.7^\circ = 128.7^\circ \approx 129^\circ$ S 51.3° E.</p>	<p>1A 1A 1A IM IM</p>	 <p>u-1 for missing unit r.t. 38.7° 80 m u-1 for missing unit 100 m u-1 for missing unit</p>
<p>4. (a) $f(2) = 2^3 - 2(2)^2 - 9(2) + 18$ $= 0$ (b) $x - 2$ is a factor of $f(x)$. $\therefore f(x) = (x - 2)(x^2 - 9)$ $= (x - 2)(x - 3)(x + 3)$</p>	<p>1A IM 1A</p>	<p>for $f(x) = (x - 2)(ax^2 + bx + c)$</p>
<p>5. (a) Mean = $\frac{4+4+5+6+8+12+13+13+13+18}{10} = 9.6$ (b) Mode = 13 (c) Median = $\frac{8+12}{2} = 10$ (d) Standard deviation = 4.59</p>	<p>1A 1A 1A 1A 1A</p>	<p>r.t. 4.59</p>

Solution	Marks	Remarks
11. (a) Let $A = aP + bP^2$, where a and b are constants. Sub. $P = 24$, $A = 36$, $24a + 576b = 36$ $2a + 48b = 3$ (1)	1A	
Sub. $P = 18$, $A = 9$, $18a + 324b = 9$ $2a + 36b = 1$ (2)	1M	for substitution (either)
Solving (1) and (2) $a = -\frac{5}{2}$ $b = \frac{1}{6}$ $\therefore A = -\frac{5}{2}P + \frac{1}{6}P^2$	1A	for both
(b) (i) When $A = 54$, $-\frac{5}{2}P + \frac{1}{6}P^2 = 54$ $P^2 - 15P - 324 = 0$ $P = 27$ or $P = -12$ (rejected) \therefore the required perimeter is 27 cm.	1A	
(ii) Let P' cm be the perimeter of the gold bookmark. $\left(\frac{P'}{27}\right)^2 = \frac{8}{54}$ $P' = 6\sqrt{3}$ (≈ 10.4) The perimeter of the gold bookmark is $6\sqrt{3}$ (≈ 10.4) cm.	1M+1A 1A -----(5)	1M for $\left(\frac{P'}{P}\right)^2 = \frac{8}{54}$ r.t. 10.4

Solution

Marks

Remarks

Number of books read (x)	Number of participants	Award
$0 < x \leq 5$	66	Certificate
$5 < x \leq 15$	34	Book coupon
$15 < x \leq 25$	64	Bronze medal
$25 < x \leq 35$	26	Silver medal
$35 < x \leq 50$	10	Gold medal

1A

for both

-----(1)

(b) Lower quartile = 3.8

Upper quartile = 22.8

Inter-quartile range = 22.8 - 3.8

= 19

1M
1A

(22→23) - (3→4)
r.t. 19

-----(2)

(c) (i) The number of participants who won medals,

$$64 + 26 + 10 = 100$$

The number of participants who won gold medals is 10.

The probability that they both won gold medals

$$= \frac{10}{100} \times \frac{9}{99}$$

$$= \frac{1}{110}$$

1M for $\frac{p}{q} \times \frac{p-1}{q-1}$, where $p < q$

1A 0.00909

(ii) Both won bronze medals

$$P_1 = \frac{64}{100} \times \frac{63}{99} = \frac{112}{275}$$

Both won silver medals

$$P_2 = \frac{26}{100} \times \frac{25}{99} = \frac{13}{198}$$

The probability that they won different medals

$$= 1 - \frac{1}{110} - \frac{112}{275} - \frac{13}{198}$$

$$= \frac{1282}{2475}$$

2M

for $1 - (c)(i) - P_1 - P_2$

1A

0.518

$$P(B \text{ and } S) = \frac{64}{100} \times \frac{26}{99} \times 2$$

$$P(B \text{ and } G) = \frac{64}{100} \times \frac{10}{99} \times 2$$

$$P(S \text{ and } G) = \frac{26}{100} \times \frac{10}{99} \times 2$$

P(different medals) = P(B and S) + P(B and G) + P(S and G)

$$= \frac{1282}{2475}$$

2M+1A

2M for sum of three different cases

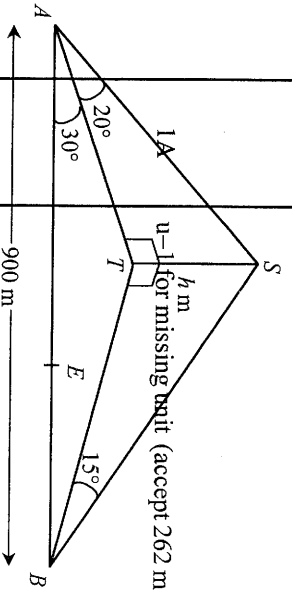
$$(P_1' \times 2 + P_2' \times 2 + P_3' \times 2)$$

1A

0.518

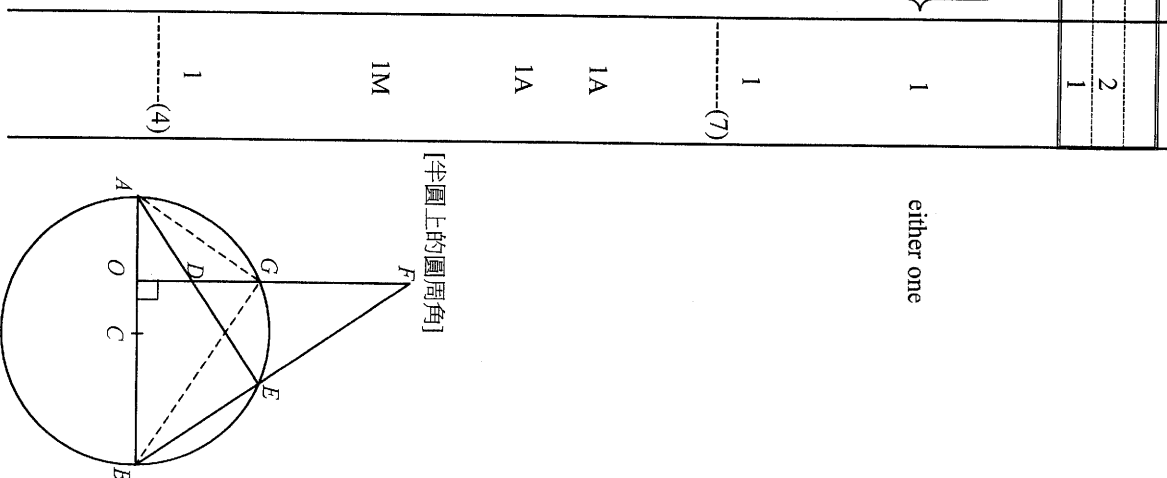
-----(6)

Solution		Marks	Remarks
13. (a)	Area of $\Delta C_1C_2C_3 = \frac{1}{2}(1)(1)\sin 60^\circ$ $= \frac{\sqrt{3}}{4} \text{ m}^2$	1A -----(2)	u-1 for missing unit
(b)	Each side of a smaller triangle = $\frac{1}{3} \text{ m}$ Area of each smaller triangle = $\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\sin 60^\circ = \frac{\sqrt{3}}{36} \text{ m}^2$ Total area = $4 \times \frac{\sqrt{3}}{36} + \frac{\sqrt{3}}{4}$ $= \frac{13\sqrt{3}}{36} \text{ m}^2$	1M+1M 1A -----(3)	1M for 4 times, 1M for + (a) u-1 for missing unit
(c)	The area $= \frac{\sqrt{3}}{4} + \frac{4}{9} \times \frac{\sqrt{3}}{4} + \left(\frac{4}{9}\right)^2 \times \frac{\sqrt{3}}{4} + \left(\frac{4}{9}\right)^3 \times \frac{\sqrt{3}}{4} + \dots$ $= \frac{\sqrt{3}}{4} \left[1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \left(\frac{4}{9}\right)^3 + \dots \right]$ $= \frac{9\sqrt{3}}{20} \text{ m}^2$	1M 1A	for $\frac{a}{1-r}$ u-1 for missing unit
The area $\frac{\sqrt{3}}{4}$ $= \frac{\sqrt{3}}{4} \left[\frac{1}{1-\frac{4}{9}} \right]$ $= \frac{9\sqrt{3}}{20} \text{ m}^2$		2M+1A 1A ----- (4)	2M for $\frac{(a)}{1-r}$ u-1 for missing unit

Solution	Marks	Remarks
<p>14. (a) $AT = \frac{h}{\tan 20^\circ}$ m and $BT = \frac{h}{\tan 15^\circ}$ m.</p> <p>$\therefore BT^2 = AB^2 + AT^2 - 2AB \cdot AT \cos 30^\circ$</p> <p>$\therefore \left(\frac{h}{\tan 15^\circ}\right)^2 = 900^2 + \left(\frac{h}{\tan 20^\circ}\right)^2 - 2(900)\left(\frac{h}{\tan 20^\circ}\right) \cos 30^\circ$</p> <p>$\left(\frac{1}{\tan^2 15^\circ} - \frac{1}{\tan^2 20^\circ}\right)h^2 + \frac{900\sqrt{3}}{\tan 20^\circ}h - 810000 = 0$</p> <p>$h \approx 153.86 \approx 154$</p>	<p>1A</p> <p>IM+1A</p> <p>IM</p> <p>1A</p> <p>------(5)</p>	<p>u-1 for missing unit for both $AT = 2.75h$ m and $BT = 3.73h$ m</p> <p>in the form of $ah^2 + bh + c = 0$</p> <p>r.t. 154</p>
<p>(b) (i) ES is minimum when $SELAB$ (or $TELAB$).</p> <p>When $TELAB$, $ET = AT \sin 30^\circ = \frac{h \sin 30^\circ}{\tan 20^\circ}$ (≈ 211.36)</p> <p>Shortest distance = $\sqrt{h^2 + (AT \sin 30^\circ)^2}$</p> <p>$= h\sqrt{1 + \left(\frac{\sin 30^\circ}{\tan 20^\circ}\right)^2}$</p> <p>$\approx 261.43$</p> <p>$\approx 261$ m.</p>	<p>IM</p> <p>IM</p> <p>IM</p>	<p>$\sqrt{153.86^2 + 211.36^2}$</p>
 <p>$AS = \frac{h}{\sin 20^\circ} \approx 449.86$ and $SB = \frac{h}{\sin 15^\circ} \approx 594.48$.</p> <p>$\cos \angle SAB = \frac{\left(\frac{h}{\sin 20^\circ}\right)^2 + (900)^2 - \left(\frac{h}{\sin 15^\circ}\right)^2}{2\left(\frac{h}{\sin 20^\circ}\right)(900)} \approx 0.8138$.</p> <p>$\angle SAB = 35.53^\circ$</p> <p>Shortest distance = $AS \sin \angle SAB$</p> <p>$\approx \left(\frac{h}{\sin 20^\circ}\right) \sin 35.53^\circ$</p> <p>$\approx 261$ m</p>	<p>1A</p> <p>IM</p> <p>IM</p> <p>IM</p> <p>1A</p> <p>------(3)</p>	<p>u-1 for missing unit (accept 262 m)</p> <p>accept $\angle SBA = 26.09^\circ$</p> <p>(Accept 262 m)</p>
<p>(ii) $\therefore \tan \theta = \frac{h}{ET}$</p> <p>$\therefore \theta$ is maximum when $TELAB$.</p> <p>$\tan \theta_{\max} = \frac{h}{AT \sin 30^\circ}$</p> <p>$= \frac{\tan 20^\circ}{\sin 30^\circ}$</p> <p>Maximum value of $\theta \approx 36.1^\circ$</p> <p>Hence $15^\circ \leq \theta \leq 36.1^\circ$.</p> <p>Accept using $\cos \theta = \frac{ET}{ES} = \frac{211.4}{261.4}$, $\theta \approx 36.0^\circ$</p>	<p>IM</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>------(3)</p>	<p>can be omitted</p> <p>$\tan \theta = \frac{h}{ET} = \frac{153.86}{211.36}$</p> <p>$\sin \theta = \frac{h}{ES} = \frac{153.86}{261.43}$</p> <p>$\cos \theta = \frac{ET}{ES} = \frac{211.36}{261.43}$</p> <p>u-1 for missing unit</p> <p>(Accept $\theta \approx 36.2^\circ$)</p>

Solution	Marks	Remarks																								
<p>15. (a) (i) Total amount of water = $\frac{1}{3}\pi \cdot 9^2 \cdot 24 = 648\pi$ cm³</p> <p>Volume of water in the cylinder = $\pi \cdot 6^2 h = 36\pi h$ cm³</p> <p>Volume of water in the cone = $\frac{1}{3}\pi \cdot 9^2 \cdot 24 \cdot \left(\frac{h+5}{24}\right)^3$ cm³</p> <p>Let r cm be the radius of the water surface in the cone when water is being poured into the cylinder.</p> <p>Then $\frac{r}{h+5} = \frac{9}{24}$.</p> <p>Volume of water remains in the cone</p> $= \frac{\pi}{3} \left[\frac{3}{8}(h+5) \right]^2 (h+5) = \frac{3\pi}{64}(h+5)^3 \text{ cm}^3.$	<p>IM+1A</p> <p>1A</p> <p>IM</p>	<p>IM for $V = V' \cdot \left(\frac{h+5}{24}\right)^3$</p>																								
<p>$\therefore \frac{3\pi}{64}(h+5)^3 + 36\pi h = 648\pi$</p> $1 - \left(\frac{h+5}{24}\right)^3 = \frac{h}{18}$ $h^3 + 15h^2 + 75h + 125 = 768(18 - h)$ $h^3 + 15h^2 + 75h + 125 + 768h = 13824$ $h^3 + 15h^2 + 843h - 13699 = 0$ <p>(ii) Let $f(h) = h^3 + 15h^2 + 843h - 13699$</p> <p>$\therefore f(11) = -1280 < 0$ and $f(12) = 305 > 0$</p> <p>\therefore The value of h lies between 11 and 12.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>a [$f(a) < 0$]</th> <th>b [$f(b) > 0$]</th> <th>$m = \frac{a+b}{2}$</th> <th>$f(m)$</th> </tr> </thead> <tbody> <tr> <td>11</td> <td>12</td> <td>11.5</td> <td>-500</td> </tr> <tr> <td>11.5</td> <td>12</td> <td>11.75</td> <td>-101</td> </tr> <tr> <td>11.75</td> <td>12</td> <td>11.875</td> <td>+101</td> </tr> <tr> <td>11.75</td> <td>11.875</td> <td>11.8125</td> <td>+0.224</td> </tr> <tr> <td>11.75</td> <td>11.8125</td> <td></td> <td></td> </tr> </tbody> </table> <p>$\therefore 11.75 < h < 11.8125$</p> <p>$h \approx 11.8$ (correct to 1 decimal place)</p>	a [$f(a) < 0$]	b [$f(b) > 0$]	$m = \frac{a+b}{2}$	$f(m)$	11	12	11.5	-500	11.5	12	11.75	-101	11.75	12	11.875	+101	11.75	11.875	11.8125	+0.224	11.75	11.8125			<p>IM</p> <p>1A</p> <p>1</p> <p>IM</p> <p>IM</p> <p>IM</p>	<p>can be absorbed</p> <p>for expanding $(h+5)^3$</p> <p>Testing sign of mid-value Choosing the correct interval</p>
a [$f(a) < 0$]	b [$f(b) > 0$]	$m = \frac{a+b}{2}$	$f(m)$																							
11	12	11.5	-500																							
11.5	12	11.75	-101																							
11.75	12	11.875	+101																							
11.75	11.875	11.8125	+0.224																							
11.75	11.8125																									
<p>(b) The situation in Figure 9(b) is the same as the situation in Figure 9(a) if the lower part (5 cm height) of the water of the cone is ignored.</p> <p>Thus the depth of water in the frustum is h cm</p> <p>≈ 11.8 cm</p>	<p>2M</p> <p>----- (2)</p>	<p>2M for the answer in (a)(ii) u-1 for missing unit</p>																								

Solution	Marks	Remarks
<p>16. (a) (i) In $\triangle AOD$ and $\triangle FOB$, $\angle AOD = \angle FOB = 90^\circ$ $\therefore \angle AEB = 90^\circ$ $\therefore \angle DAO = 90^\circ - \angle ABE$ On the other hand, $\angle BFO = 90^\circ - \angle ABE$ $\therefore \angle DAO = \angle BFO$ Hence, $\triangle AOD \sim \triangle FOB$</p>		[已知] [半圓上的圓周角] [內角和] [內角和] [等角] (AA) (equiangular)
<p>(ii) In $\triangle AOG$ and $\triangle GOB$, $\angle AOG = \angle GOB = 90^\circ$ $\therefore \angle AGB = 90^\circ$ $\therefore \angle AGO = 90^\circ - \angle BGO$ Thus, $\triangle AOG \sim \triangle GOB$</p>		[已知] [半圓上的圓周角] [內角和] [等角] (AA) (equiangular)
<p>Marking Scheme : Case 1 Any correct proof with correct reasons. 3 Case 2 Any correct proof without reasons. 2 Case 3 Incomplete proof with any one correct angle and correct reason. 1</p>		
<p>(iii) Hence $\frac{OD}{OA} = \frac{OB}{OF}$ Since $OD \cdot OF = OA \cdot OB$ $\frac{OA}{OG} = \frac{OG}{OB}$ i.e. $OA \cdot OB = OG^2$. Thus $OD \cdot OF = OA \cdot OB = OG^2$</p>	1	either one
<p>(b) (i) $A = (c-r, 0)$ and $B = (c+r, 0)$ $m_{AD} = \frac{p}{r-c}$ $m_{BF} = -\frac{q}{r+c}$</p>	1A 1A	
<p>(ii) $\therefore \angle AEB = 90^\circ$ (\angle in semi circle) $\therefore m_{AD} \cdot m_{BF} = \frac{p}{r-c} \cdot \left(-\frac{q}{r+c}\right) = -1$ $pq = r^2 - c^2$ Since $pq = OD \cdot OF$ and $r^2 - c^2 = CG^2 - OC^2 = OG^2$, therefore $OD \cdot OF = OG^2$</p>	1M 1 1	[半圓上的圓周角]



Solution	Marks	Remarks
<p>17. (a) Equation of L_1: $\frac{y-9k}{x} = -\frac{9}{5}$ $9x+5y = 45k$</p> <p>Equation of L_2: $\frac{y-5k}{x} = -\frac{5}{12}$ $5x+12y = 60k$</p>	<p>1M 1A ------(2)</p>	<p>$\frac{x}{5k} + \frac{y}{9k} = 1$ $\frac{x}{12k} + \frac{y}{5k} = 1$ for both equations</p>
<p>(b) (i) Let x and y be respectively the number of articles produced by lines A and B. The constraints are</p> $\begin{cases} 45x+25y \leq 225 & (\text{or } 9x+5y \leq 45), \\ 50x+120y \leq 600 & (\text{or } 5x+12y \leq 60), \end{cases}$ <p>x and y are non-negative integers. The profit is \$ 1 000 ($3x+2y$).</p> <p>Using the graph in Figure 11 with $k = 1$, the feasible solutions are represented by the lattice points in the shaded region below.</p>	<p>1A 1A 1A</p>	<p>withhold 1 mark for strict inequality</p>
<p>From the graph, the most profitable combinations are (3, 3) and (5, 0)</p> <p>At (3, 3), the profit is \$ 1 000 ($9 + 6$) = \$ 15 000 At (5, 0), the profit is \$ 1 000 ($15 + 0$) = \$ 15 000 At (0, 5), the profit is \$ 1 000 (10) = \$ 10 000 At (2, 4), the profit is \$ 1 000 ($6 + 8$) = \$ 14 000</p> <p>The greatest possible profit is \$ 15 000 .</p>	<p>1M 1A</p>	<p>Testing u-1 for missing unit</p>

Solution	Marks	Remarks
<p>(ii) Let x and y be respectively the number of articles produced by production lines A and B. The constraints are</p> $\begin{cases} 45x + 25y \leq 450 & (\text{or } 9x + 5y \leq 90), \\ 50x + 120y \leq 1200 & (\text{or } 5x + 12y \leq 120), \end{cases}$ <p>x and y are non-negative integers.</p>	1A	
	1M 1A 1A	<p>can be absorbed</p> <p>can be absorbed</p>
<p>Using the same graph as in (i) and taking $k = 2$, the feasible solutions are represented by the lattice points in the shaded region.</p> <p>From the graph, the most profitable combinations is $(6, 7)$.</p> <p>The greatest possible profit is \$ 1 000 $(18 + 14) = \\$32\,000$</p>	1A -----(9)	<p>u-1 for missing unit (accept drawing 2 lines on Figure 11 with correct labels)</p>