2003-CE
MATH
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2003

MATHEMATICS PAPER 1

Question-Answer Book

8.30 am – 10.30 am (2 hours)
This paper must be answered in English

1. Write your Candidate Number, Centre Number and Seat Number in the spaces provided on this cover.
2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 33 marks.
3. Attempt ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Supplementary answer sheets will be supplied on request. Write your Candidate Number on each sheet and fasten them with string inside this book.
4. Write the question numbers of the questions you have attempted in Section B in the spaces provided on this cover.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Candidate Number							
Centre Number							
Seat Number							

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Section A Question No.	Marks	Marks
1–3		
4–5		
6–7		
8		
9		
10		
11		
12		
13		
Section A Total		

Checker's Use Only	Section A Total		

Section B Question No.*	Marks	Marks
Section B Total		

To be filled in by the candidate.

Checker's Use Only	Section B Total		

Checker No.	

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Make m the subject of the formula $mx = 2(m+c)$. (3 marks)

2. Find the range of values of x which satisfy both $\frac{3-5x}{4} \geq 2-x$ and $x+8>0$. (3 marks)

3. Factorize

   (a) $x^2-(y-z)^2$,

   (b) $ab-ad-bc+cd$.

   (3 marks)

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4. Solve the equation $4^{x+1} = 8$. (3 marks)

5. A handbag costs \$ 400 . The marked price of the handbag is 20% above the cost. It is sold at a 25% discount on the marked price.

(a) Find the selling price of the handbag.

(b) Find the percentage profit or percentage loss.

(4 marks)

Page total

6. There are only two kinds of tickets for a cruise: first-class tickets and economy-class tickets. A total of 600 tickets are sold. The number of economy-class tickets sold is three times that of first-class tickets sold. If the price of a first-class ticket is \$850 and that of an economy-class ticket is \$500, find the sum of money for the tickets sold. (4 marks)

7. Consider the arithmetic sequence 2, 5, 8, ... Find

(a) the 10th term of this sequence,

(b) the sum of the first 10 terms of this sequence.

(4 marks)

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8. Figure 1 shows a parallelogram $ABCD$. The diagonals AC and BD cut at E.

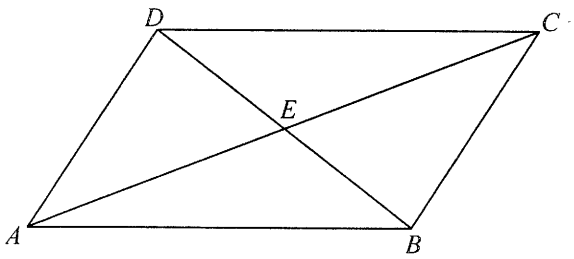

Figure 1

(a) Prove that the triangles ABC and CDA are congruent.

(b) Write down all other pairs of congruent triangles.

(4 marks)

9. At 1:00 a.m., a ship S is 100 km due east of a lighthouse L. S is moving at a speed of 20 km/h in the direction N 30° W as shown in Figure 2.

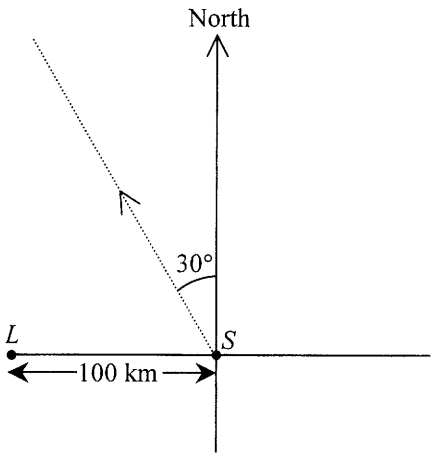

Figure 2

(a) Find the shortest distance between the ship and the lighthouse, correct to the nearest km.

(b) At what time will the ship be nearest to the lighthouse?

(5 marks)

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Section A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. The speed of a solar-powered toy car is V cm/s and the length of its solar panel is L cm, where $5 \leq L \leq 25$. V is a function of L. It is known that V is the sum of two parts, one part varies as L and the other part varies as the square of L. When $L = 10$, $V = 30$ and when $L = 15$, $V = 75$.

(a) Express V in terms of L. (3 marks)

(b) Find the range of values of L when $V \geq 30$. (4 marks)

11. (a) For the set of data 10 , 10 , 11 , 12 , 13 , 16 , find

(i) the mode,

(ii) the median,

(iii) the mean,

(iv) the range.

(4 marks)

(b) Four unknown data are combined with the six data in (a) to form a set of ten data.

(i) Find the least and the greatest possible values of the median of the combined set of ten data.

(ii) If the mean of the four unknown data is 11 , find the mean of the combined set of ten data.

(4 marks)

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12. In Figure 3 , AP is an altitude of the triangle ABC. It cuts the y-axis at H.

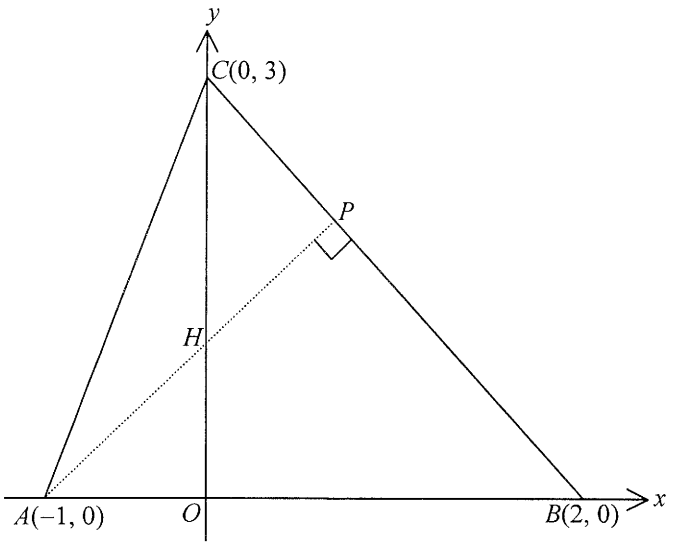

Figure 3

(a) Find the slope of BC. (1 mark)

(b) Find the equation of AP. (3 marks)

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(c) (i) Find the coordinates of H.

(ii) Prove that the three altitudes of the triangle ABC pass through the same point.

(5 marks)

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13. Sector OCD is a thin metal sheet. The sheet $ABCD$ is formed by cutting away sector OBA from sector OCD as shown in Figure 4(a).

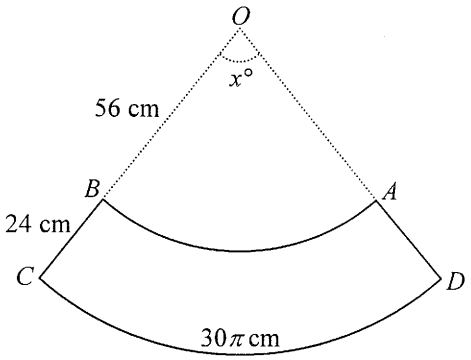

Figure 4(a)

It is known that $\angle COD = x°$, $AD = BC = 24$ cm, $OA = OB = 56$ cm and $\overset{\frown}{CD} = 30\pi$ cm.

(a) (i) Find x.

(ii) Find, in terms of π, the area of $ABCD$.

(4 marks)

(b)

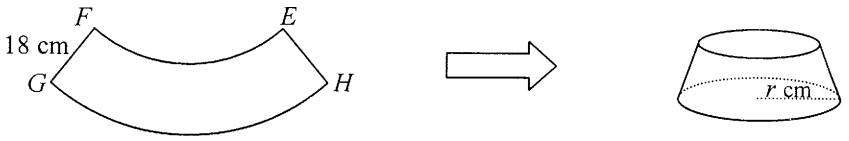

Figure 4(b)

Figure 4(c)

Figure 4(b) shows another thin metal sheet $EFGH$ which is similar to $ABCD$. It is known that $FG = 18$ cm.

(i) Find, in terms of π, the area of $EFGH$.

(ii) By joining EH and FG together, $EFGH$ is then folded to form a hollow frustum of base radius r cm as shown in Figure 4(c). Find r.

(5 marks)

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SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

14.

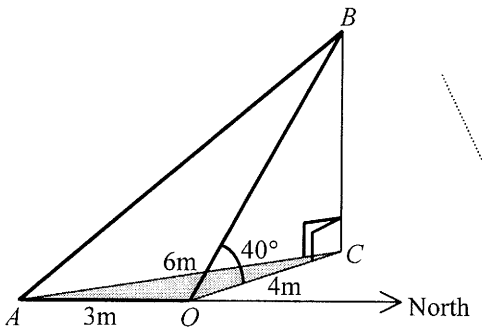

Figure 5(a)

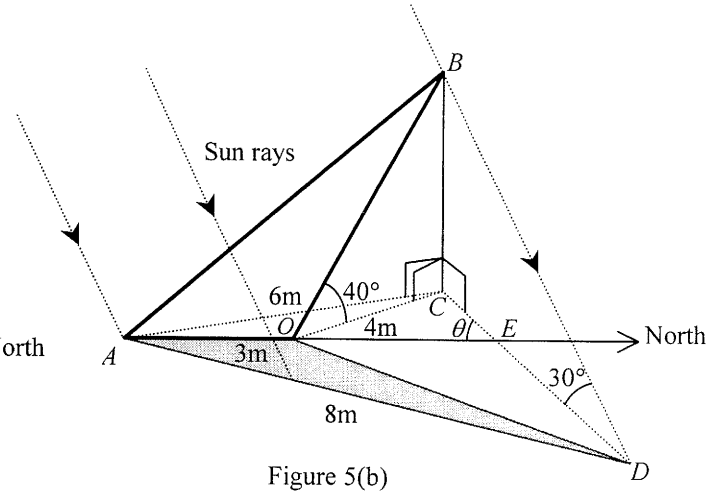

Figure 5(b)

Figure 5(a) shows a triangular metal plate OAB standing on the horizontal ground. The side OA lies along the north-south direction on the ground. OB is inclined at an angle of $40°$ to the horizontal. The overhead sun casts a shadow of the plate, OAC, on the ground. $OA = 3$ m, $OC = 4$ m and $AC = 6$ m.

(a) Find $\angle OAC$. (2 marks)

(b) In Figure 5(b), OAD is the shadow of the plate cast on the horizontal ground when the sun shines from S θ W with an angle of elevation $30°$. AO is produced to cut CD at E. $AD = 8$ m.

(i) Find CD.

(ii) Find $\angle CAD$.

(iii) Using $CE + ED = CD$, or otherwise, find θ.

(9 marks)

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15.

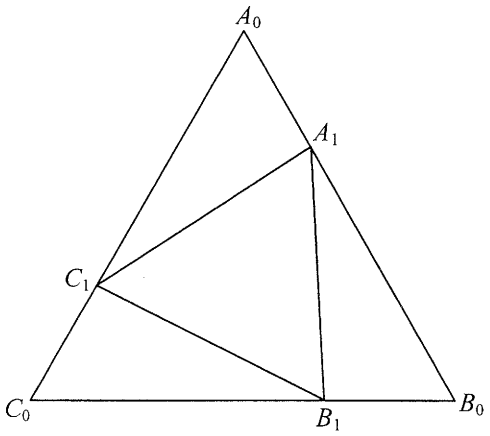

Figure 6(a)

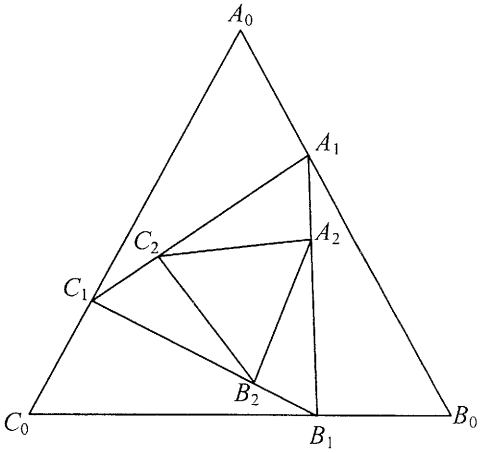

Figure 6(b)

Figure 6(a) shows an equilateral triangle $A_0B_0C_0$ of side 1 m. Another triangle $A_1B_1C_1$ is inscribed in triangle $A_0B_0C_0$ such that $\frac{A_0A_1}{A_0B_0} = \frac{B_0B_1}{B_0C_0} = \frac{C_0C_1}{C_0A_0} = k$, where $0 < k < 1$. Let $A_1B_1 = x$ m.

(a) (i) Express the area of triangle $A_1B_0B_1$ in terms of k.

(ii) Express x in terms of k.

(iii) Explain why $A_1B_1C_1$ is an equilateral triangle.

(5 marks)

(b) Another equilateral triangle $A_2B_2C_2$ is inscribed in triangle $A_1B_1C_1$ such that $\frac{A_1A_2}{A_1B_1} = \frac{B_1B_2}{B_1C_1} = \frac{C_1C_2}{C_1A_1} = k$ as shown in Figure 6(b).

(i) Prove that the triangles $A_1B_0B_1$ and $A_2B_1B_2$ are similar.

(ii) The above process of inscribing triangles is repeated indefinitely to generate equilateral triangles $A_3B_3C_3$, $A_4B_4C_4$, $A_5B_5C_5$, ... Find the total area of the triangles $A_1B_0B_1$, $A_2B_1B_2$, $A_3B_2B_3$, ...

(6 marks)

Page total

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16. John will participate in a contest to be held at a university. If John wins the contest, he will go to Canteen X for lunch. Otherwise, he will go to Canteen Y. The following table shows the types of set lunches and the prices served in the two canteens. He will choose one type of set lunch randomly.

Canteen	Set lunch	Price ($)
X	A	40
	B	50
Y	C	15
	D	20

(a) If the probability of John winning the contest is $\frac{1}{10}$, find the probability that he will spend \$15 for lunch. (2 marks)

(b) If John takes a bus leaving at 8:00 a.m. to the university, his probability of winning the contest will be $\frac{1}{10}$. If he misses the bus, he will take a train leaving at 8:20 a.m. Owing to his nervousness, his probability of winning will be reduced to $\frac{2}{25}$.

(i) Suppose John misses the bus, find the probability that he will spend \$15 for lunch.

(ii) The following table shows the cost of a single trip by bus or train:

Transportation	Cost of a single trip ($)
Bus	4.5
Train	7.5

It is known that the probability of John taking the bus is twice that of taking the train.

(1) Find the probability that John will spend \$15 for lunch after the contest.

(2) If John goes home by train after lunch, find the probability that he will spend more than a total of \$30 for the lunch and the transportation of the two trips.

(9 marks)

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17.

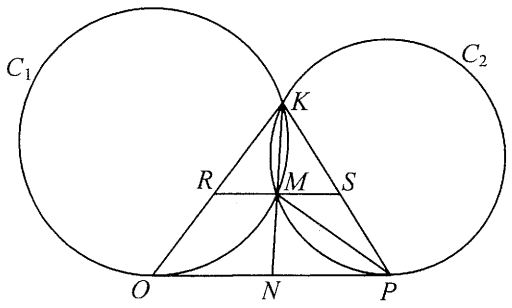

Figure 7(a)

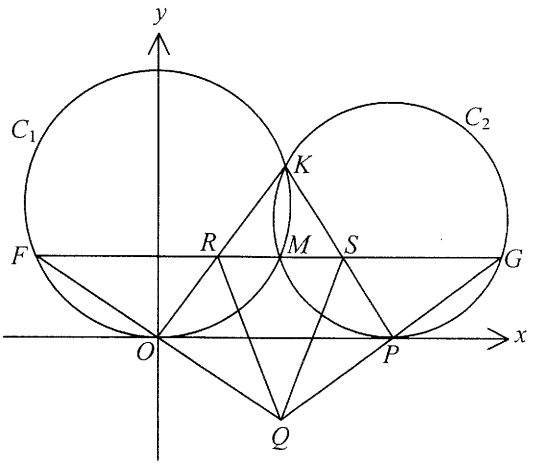

Figure 7(b)

(a) In Figure 7(a), OP is a common tangent to the circles C_1 and C_2 at the points O and P respectively. The common chord KM when produced intersects OP at N. R and S are points on KO and KP respectively such that the straight line RMS is parallel to OP.

(i) By considering triangles NPM and NKP, prove that $NP^2 = NK \cdot NM$.

(ii) Prove that $RM = MS$.

(5 marks)

(b) A rectangular coordinate system, with O as the origin, is introduced to Figure 7(a) so that the coordinates of P and M are $(p, 0)$ and (a, b) respectively (see Figure 7(b)). The straight line RS meets C_1 and C_2 again at F and G respectively while the straight lines FO and GP meet at Q.

(i) Express FG in terms of p.

(ii) Express the coordinates of F and Q in terms of a and b.

(iii) Prove that triangle QRS is isosceles.

(6 marks)