香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九八年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1998

數學 試卷一
MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the Teachers' Centres.

Hong Kong Certificate of Education Examination
Mathematics Paper I

NOTES FOR MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, provided that the method used is sound.

2. In a question consisting of several parts each depending on the previous parts, marks may be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answers should NOT be awarded. In the marking scheme, marks are classified as:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

3. Use of notation different from those in the marking scheme should not be penalized.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

	<u>IA</u>	<u>IB</u>
5.	Each mark deducted for *poor presentation* (p.p.) should be denoted by [pp−1] : a. At most deduct 1 mark for (p.p.) in each question, up to a maximum of 2 marks for Sections A(1) and A(2). b. For similar (p.p.), deduct 1 mark for the first time that it occurs. i.e. do not penalize candidates twice for the same p.p	Each mark deducted for *poor presentation* (p.p.) should be denoted by [pp−1]. At most deduct 1 mark for Section B.
6.	Each Mark deducted for *wrong/no unit* (u.) should be denoted by [u−1]. At most deduct 1 mark for questions in Sections A(1) and A(2).	Each Mark deducted for *wrong/no unit* (u.) should be denoted by [u−1]. At most deduct 1 mark for Section B.
7.	Marks entered in the Question Total Box should be the NET total scored on that question.	Marks entered in the Page Total Box should be the NET total scored on that page.

Solution	Marks	Remarks
1. Area of cross-section $=\frac{(6+4)\times 3}{2}$ (cm^2)	1M	or $4\times 3+\frac{2\times 3}{2}$, $6\times 3-\frac{2\times 3}{2}$; can be omitted
$= 15$ (cm^2)		
Volume of the prism $= 15\times 8$ (cm^3)	1M	
$= 120$ cm^3	1A	
	(3)	
2. $x = 180 - 120$		
$= 60$	1A	*pp*–1 for $x° = 60°$, $\angle x = 60$
$y = 360 - 140 - 80 - 60$	1M	
$= 80$	1A	
	(3)	
3. $\tan x° = \frac{7}{5}$	1M	or $\tan y° = \frac{5}{7}$ etc.
$x \approx 54.5$	1A	r.t. 54.5 , *u*–1 for 54°28′
$y = 90 - x$		
≈ 35.5	1A	r.t. 35.5 , *u*–1 for 35°32′
	(3)	
4. $\frac{a^3a^4}{b^{-2}} = a^{3+4}b^{-(-2)}$ (or $\frac{a^{3+4}}{\frac{1}{b^2}}$)	1M+1M	1M for applying $x^m x^n = x^{m+n}$ 1M for applying $x^{-n} = \frac{1}{x^n}$ can be omitted
$= a^7b^2$	1A	
	(3)	
5. $b = 2x + (1-x)a$		
$b = 2x + a - ax$	1A	
$ax - 2x = a - b$	1M	for putting terms involving x on one side (can be omitted)
$x = \frac{a-b}{a-2}$ (or $x = \frac{b-a}{2-a}$)	1A	
	(3)	
6. (a) ΔEBA	1A	accept *EBA* or *ABE* etc.
(b) $\frac{y}{6} = \frac{3}{4}$	1M+1A	1M for setting up equation
$y = \frac{9}{2}$ (or 4.5)	1A	do not accept 4.50
	(4)	

Solution	Marks	Remarks
7. (a) Selling price = (\$) $29\times(1-20\%)$	1A	
$=$ \$ 23.2	1A	accept \$ 23.20
(b) Percentage profit = $\frac{23.2-18}{18}\times100$ (%)	1M	for $\frac{23.2-18}{18}$
≈ 28.9 (%)	1A	r.t. 28.9 or $28\frac{8}{9}$
	(4)	
8. (a) Slope of $AB=\frac{4-1}{0-(-2)}$	1M	can be omitted
$=\frac{3}{2}$	1A	
(b) Equation of the line:		
$\frac{y-3}{x-1}=-\frac{2}{3}$ (or $\left(\frac{y-3}{x-1}\right)\left(\frac{3}{2}\right)=-1$)	1M+1A	1M for the slope
$2x+3y-11=0$ (or $y=-\frac{2}{3}x+\frac{11}{3}$)	1A	or equivalent
	(5)	
9. (a) $f(2)=2^3+2(2^2)-5(2)-6$	1M	or by long division *pp*−1 for missing f(2)
$=0$ $\therefore$ $x-2$ is a factor of $f(x)$	1A	1A only for just writing f(2)=0
(b) $f(x)=(x-2)(x^2+4x+3)$	1A	1A for x^2+4x+3
$=(x-2)(x+1)(x+3)$	1A+1A	1A for $(x+1)(x+3)$
	(5)	

10. (a)

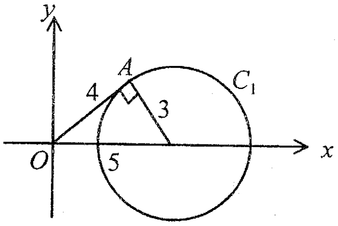
Test score (x)	Cumulative frequency
$x \le 50$	8
$x \le 60$	50
$x \le 70$	102
$x \le 80$	158
$x \le 90$	188
$x \le 100$	200

Test score (x)	Frequency
$40 < x \le 50$	8
$50 < x \le 60$	42
$60 < x \le 70$	52
$70 < x \le 80$	56
$80 < x \le 90$	30
$90 < x \le 100$	12

Solution	Marks	Remarks
10. (a)	1A+1A+1A	1A for any correct entry in c.f. column 1A for any correct entry in f. column 1A for all being correct
	(3)	
(b) Number of students whose score is below 55 $= 29$ (or 28 , 30)	1A	can be omitted (refer to the graph)
Passing percentage $= \dfrac{200-29}{200} \times 100$ (%)	1M+1M	1M for the numerator 1M for the denominator
$= 85.5$ (%) (or 86 , 85)	1A	
	(4)	
11. (a) The probability that the socks taken out are both white $= \dfrac{8}{14} \times \dfrac{7}{13}$	1A+1M	1A for $\dfrac{8}{14}$, 1M for $p_1 \times p_2$
$= \dfrac{4}{13}$ (or 0.308)	1A	r.t. 0.308 (p_3)
	(3)	
(b) The probability that the socks taken out are of the same colour $= \dfrac{4}{13} + \dfrac{4}{14} \times \dfrac{3}{13} + \dfrac{2}{14} \times \dfrac{1}{13}$	1M+1A+1A	1M for $p_3 + p_4 + p_5$ 1A for $\dfrac{4}{14} \times \dfrac{3}{13}$ or $\dfrac{2}{14} \times \dfrac{1}{13}$
$= \dfrac{5}{13}$ (or 0.385)	1A	r.t. 0.385
<u>Alternatively,</u> $1 - 2\left(\dfrac{8}{14} \times \dfrac{4}{13} + \dfrac{4}{14} \times \dfrac{2}{13} + \dfrac{2}{14} \times \dfrac{8}{13} \right)$	1M+1A+1A	1M for $1 - 2(p_6 + p_7 + p_8)$ 1A for $\dfrac{8}{14} \times \dfrac{4}{13}$ etc. (must have $1 - \cdots$)
$= \dfrac{5}{13}$ (or 0.385)	1A	
	(4)	

Solution	Marks	Remarks
12. (a) $S = a + bt$ for some constants a and b.	1M	
$\begin{cases} 230 = a + 100b & \text{.......... (1)} \\ 284 = a + 130b & \text{.......... (2)} \end{cases}$	1A	
(2) − (1) : $54 = 30b$ $b = 1.8$		
Sub. into (1) : $230 = a + 180$ $a = 50$	1M	
$\therefore \quad S = 50 + 1.8t$	1A	or $a = 50, b = 1.8$
	(4)	
(b) When $t = 110$,		
(i) the monthly service charge of network A		
= (\$) (50 + 1.8 × 110)	1M	or $230 + \frac{1}{3}(284 - 230)$
= (\$) 248		
(ii) the monthly service charge of network B = (\$) 2.2 × 110 = (\$) 242	1A	
Alternatively, (ii) The cost of using network A when $t = 110$ is \$2.25 per minute.	1A	r.t. 2.25
∴ The man should join network B as the monthly service charge (alternatively, the cost per minute) is less.	1	the values in (i) and (ii) must be correct
	(3)	
13. (a) $A_2B_2 = \sqrt{6^2 + 8^2}$ (cm)	1M	can be omitted
= 10 cm	1A	
	(2)	
(b) $A_2A_3 : A_1A_2 = 10 \times \frac{3}{7} : 6$	1M	for $10 \times \frac{3}{7}$
$= 5:7$	1A	accept $\frac{5}{7}$, $1:\frac{7}{5}$ or $\frac{5}{7}:1$
	(2)	
(c) $A_1A_2 + A_2A_3 + A_3A_4 + \ldots$		
$= 6[1 + \frac{5}{7} + (\frac{5}{7})^2 + \ldots]$ (cm)	1M	for the first 3 terms can be omitted *pp*−1 for missing '…'
$= \dfrac{6}{1 - \frac{5}{7}}$ (cm)	1A	no marks for using 0.714 instead of $\frac{5}{7}$
$= 21$ (cm) ∴ The total distance crawled by the ant cannot exceed 21 cm.	1	
	(3)	

Solution	Marks	Remarks
14. $\because$ $OB = OD$		
$\therefore$ $\angle OBD = \angle ODB$ (base $\angle$s, isos. Δ)		[等腰Δ底角]
$\because$ $BC = AB$		
$\therefore$ $\angle CDB = \angle ODB$ (equal chords, equal $\angle$s)		[等弦對等角] 或 [等角對等弦]
Hence $\angle OBD = \angle CDB$		
$BO \parallel CD$ (alt. $\angle$s equal)		[(內)錯角等]
Alternatively,		
$\because$ $BC = AB$		
$\therefore$ $\angle BDC = \angle ADB$ (equal chords, equal $\angle$s)		[等弦對等角] 或 [等角對等弦]
$\angle ODC = \angle ADB + \angle BDC$		
$= 2\angle ADB$		
$= \angle AOB$ ($\angle$ at centre twice $\angle$ at circumference)		[圓心角=2×圓周角]
Hence $BO \parallel CD$ (corr. $\angle$s equal)		[同位角等]
Marking Scheme :		
Case 1 Any correct proof with correct reasons.	5	
Case 2 Any correct proof without reasons.	3	
In addition, any relevant correct argument with correct reason.	1	Maximum 1 mark
Case 3 Any relevant correct argument with correct reason.	1	Maximum 2 marks
They refer to the above 5 arguments with reasons and		
$\angle AOB = \angle ODB + \angle OBD$ (ext. $\angle$ of Δ)		[Δ的外角]
	(5)	

Solution	Marks	Remarks
15. (a) Distance between the centres of C_1 and C_2		
$= \sqrt{(11-5)^2+(-8-0)^2}$	1A	
$= 10$		
Radius of $C_1 = 10-7=3$	1M	
Equation of C_1: $(x-5)^2+y^2=9$ (or $(x-5)^2+y^2=3^2$)	1A	or $x^2+y^2-10x+16=0$
	(3)	
(b) Let $y=mx$ be a tangent to C_1 from the origin.	1M	for constant term = 0
Sub. $y=mx$ into the equation of C_1, then		
$(x-5)^2+(mx)^2=9$		
$(1+m^2)x^2-10x+16=0$		
Since the discriminant is zero, therefore		
$100-64(1+m^2)=0$	1M+1A	
$1+m^2=\frac{25}{16}$		
$m^2=\frac{9}{16}$		
$m=\pm\frac{3}{4}$	1A	
The tangents to C_1 from the origin are $y=\pm\frac{3}{4}x$.		
Alternatively,		
Consider the right-angled triangle in the figure.		
OA is one of the tangents.		
Slope of $OA = \frac{3}{4}$	1A	
Slope of the other tangent $= -\frac{3}{4}$	1M	can be omitted
The tangents to C_1 from the origin are $y=\pm\frac{3}{4}x$.	1M+1A	1M for constant term = 0
	(4)	

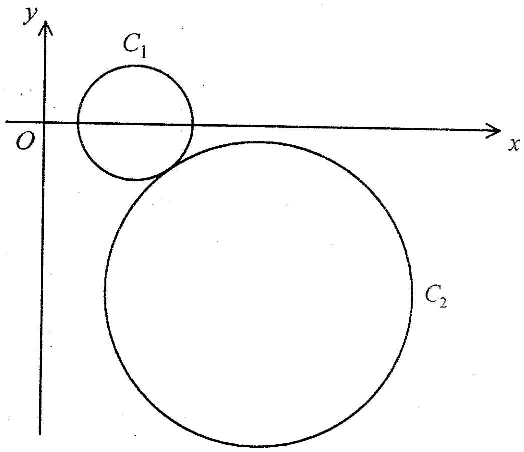


Figure 8

Solution	Marks	Remarks
(c) The tangent $y=-\frac{3}{4}x$ cuts C_2 at two points.	1M	
Sub. $y=-\frac{3}{4}x$ into the equation of C_2 , then		
$(x-11)^2+(-\frac{3}{4}x+8)^2=49$ $\frac{25}{16}x^2-34x+136=0$ $25x^2-544x+2176=0$ $(-\frac{4}{3}y-11)^2+(y+8)^2=49$ $\frac{25}{9}y^2+\frac{136}{3}y+136=0$ $25y^2+408y+1224=0$	1A	
Let (x_0,y_0) be the mid-point of AB , then		
$x_0=\frac{1}{2}(\frac{544}{25})=\frac{272}{25}$ $(10\frac{22}{25}$ or $10.9)$	1M	r.t. (10.9, –8.16)
$y_0=-\frac{3}{4}(\frac{272}{25})=-\frac{204}{25}$ $(-8\frac{4}{25}$ or $-8.16)$	1A	
Alternatively, Solving the equation for x : $x\approx 5.282$ or 16.48 $\therefore\ x_0\approx\frac{5.282+16.48}{2}\approx 10.9\ (10.88)$ Solving the equation for y : $y\approx -12.36$ or -3.962 $\therefore\ y_0\approx\frac{-12.36-3.962}{2}\approx -8.16$	1M	r.t. (10.9, –8.16)
$y_0\approx -\frac{3}{4}\times 10.88\approx 8.16$ $x_0\approx -\frac{4}{3}\times(-8.161)\approx 10.9$	1A	
$\therefore$ The mid-point of $AB=(\frac{272}{25},-\frac{204}{25})$		
Alternatively, (c) The tangent $y=-\frac{3}{4}x$ cuts C_2 at two points.	1M	
The line passing through (11, –8) and perpendicular to $y=-\frac{3}{4}x$ has eqtn.		
$\frac{y+8}{x-11}=\frac{4}{3}$ $4x-3y-68=0$	1A	
Sub. $y=-\frac{3}{4}x$ into the equation, we have $4x+\frac{9}{4}x-68=0$	1M	
$x=\frac{272}{25}$, $y=-\frac{204}{25}$	1A	r.t. (10.9, –8.16)
	(4)	

Solution	Marks	Remarks
16. (a) Let r cm be the radius of the surface of the melted ice-cream.		do not *pp* for not defining r
By considering the volume of the ice-cream, $\frac{4}{3}\pi(2^3)+\frac{4}{3}\pi x^3=\frac{1}{3}\pi r^2(2x+3)$	1A+1M	1A for the volume of the ice-cream balls 1M for equating the volumes (provided that 1A is awarded)
Using similar triangles, $\frac{r}{2x+3}=\frac{4}{8}$ $r=\frac{1}{2}(2x+3)$	2A	
Volume of the liquid $=\frac{1}{3}\pi\left[\frac{1}{2}(2x+3)\right]^2(2x+3)$ $=\frac{1}{12}\pi(2x+3)^3$	1M+1A	
<u>Alternatively</u>, $\frac{\text{Volume of the liquid}}{\frac{1}{3}\pi(4^2)(8)}=\left(\frac{2x+3}{8}\right)^3$	1A+1M+1A	1A for the ratio in length 1M for the ratio in volume 1A for the volume of the cone
Volume of the liquid $=\frac{1}{3}\pi(4^2)(8)\left(\frac{2x+3}{8}\right)^3$ $=\frac{1}{12}\pi(2x+3)^3$	1A	
Hence $\frac{4}{3}\pi(2^3)+\frac{4}{3}\pi x^3=\frac{1}{12}\pi(2x+3)^3$ $16(8+x^3)=(2x+3)^3$ $128+16x^3=8x^3+36x^2+54x+27$ $8x^3-36x^2-54x+101=0$	1 (7)	
(b) From (a), $8x^3-36x^2-54x+101=0$ $4(2x^3-9x^2)-54x+101=0$ $4y-54x+101=0$ (or $y=\frac{27}{2}x-\frac{101}{4}$)	2A	
Adding the line in Figure 9.2, we have	1A	for the graph (±1 grid at margin)
$x\approx 1.2$	1A (4)	

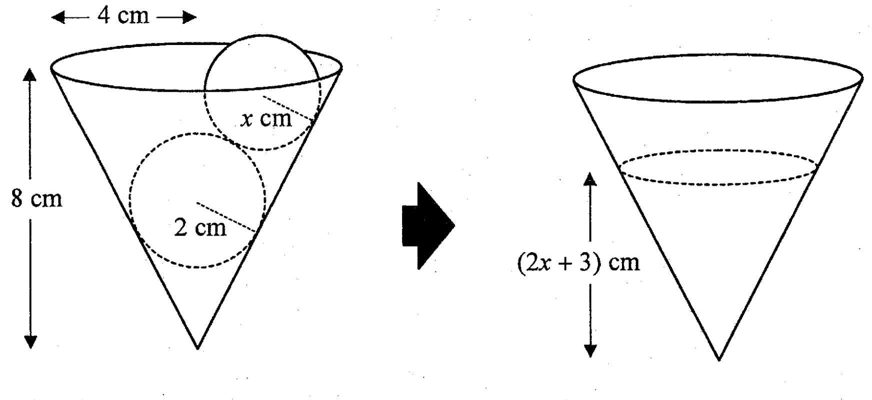
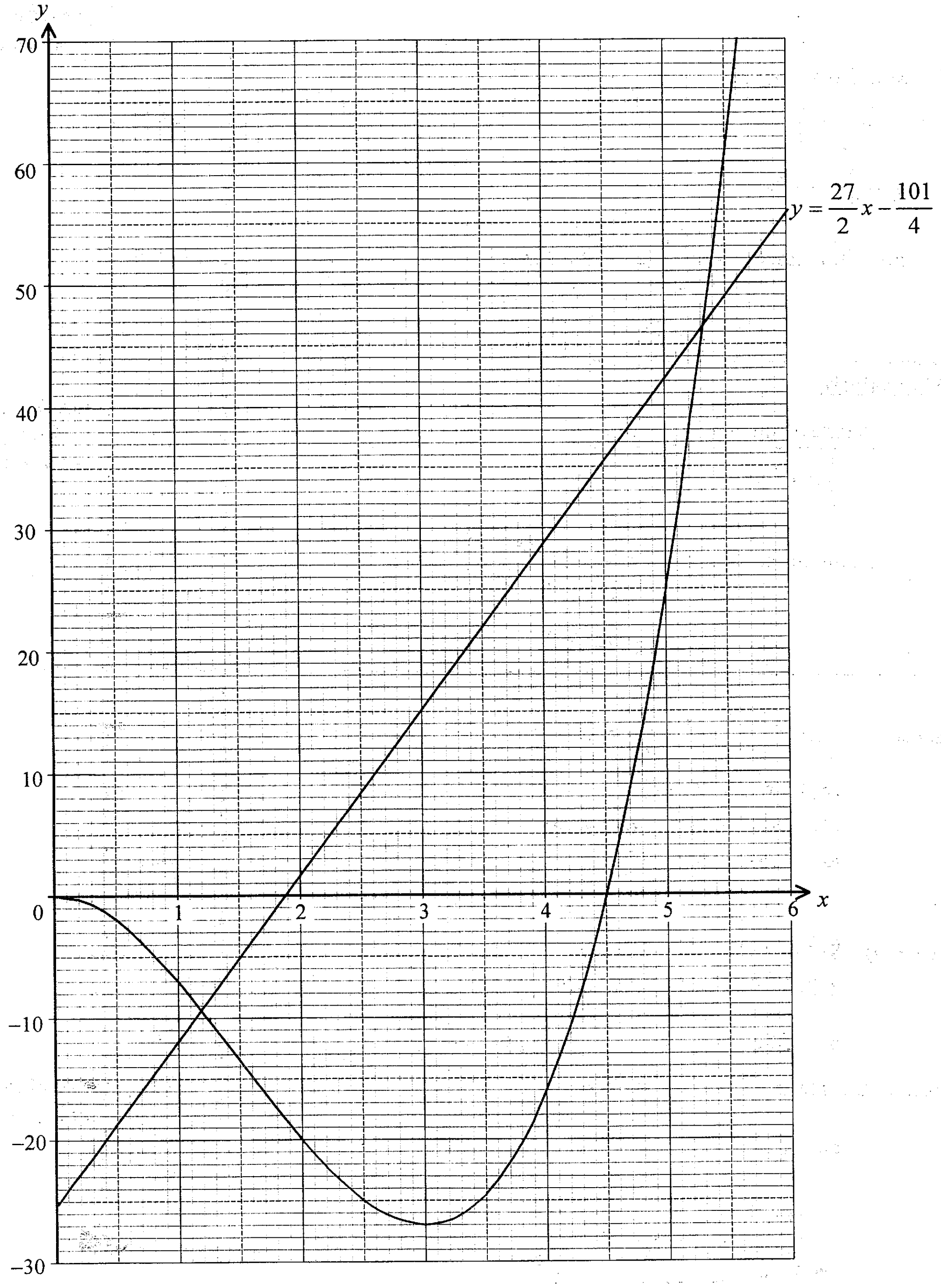


Figure 9.1


y
70
60
50
40
30
20
10
0
–10
–20
–30
1
2
3
4
5
6
x
$y = \frac{27}{2}x - \frac{101}{4}$

Solution	Marks	Remarks
17. (a) $AF = 4\sin 72^\circ$ (m)	1A	accept $\sin 72^\circ = \frac{AF}{4}$
≈ 3.80423 (m)		
≈ 3.80 m	1A	r.t. 3.80 (withhold 1A for 3.8)
$FD \approx \frac{3.80423}{\tan 35^\circ}$ (m)	1M	accept $\tan 35^\circ = \frac{3.80}{FD}$
≈ 5.43300 (m)		
≈ 5.43 m	1A (4)	r.t. 5.43

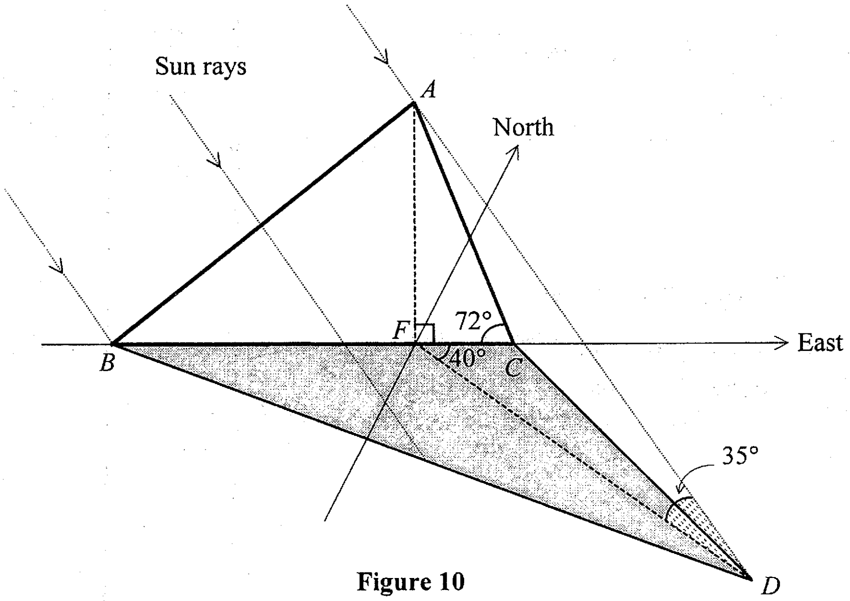


Figure 10

Solution	Marks	Remarks
(b) $CF = 4\cos 72^\circ$ (m) (or $\sqrt{4^2 - 3.80423^2}$)	1A	accept $\cos 72^\circ = \frac{CF}{4}$
≈ 1.23607 (m)		
Area of ΔDBC = Area of ΔBFD + Area of ΔFCD		
$\approx \left[\frac{1}{2}(6-1.23607)(5.43300)\sin 140^\circ\right.$		
$\left.+\frac{1}{2}(1.23607)(5.43300)\sin 40^\circ\right]$ (m^2)	1M+1M	
≈ 10.5 m^2	2A	r.t. 10.5
Alternatively,		
Let h m be the height of ΔDBC with BC as the base.		
$h \approx 5.43300\sin 40^\circ$	1M+1A	accept $\sin 40^\circ = \frac{h}{5.43}$
≈ 3.49227		
Area of $\Delta DBC \approx \frac{6\times 3.49227}{2}$ (m^2)	1M	
≈ 10.5 m^2	2A	r.t. 10.5

Solution	Marks	Remarks
<u>Alternatively</u>,		
$CF = 4\cos 72°$ (m) ≈ 1.23607 (m)	1A	accept $\cos 72° = \dfrac{CF}{4}$
$CD^2 \approx [(1.23607)^2 + (5.43300)^2 - 2(1.23607)(5.43300)\cos 40°]$ (m^2) $CD \approx 4.55593$ (m)	1M	for applying cosine rule
In ΔDCF, $\dfrac{\sin\angle DCF}{5.43300} \approx \dfrac{\sin 40°}{4.55593}$ $\sin\angle DCF \approx 0.76653$		
Area of $\Delta DBC \approx \dfrac{1}{2} \times 6 \times 4.55593 \times 0.76653$ (m^2)	1M	
≈ 10.5 m^2	2A	r.t. 10.5
	(5)	
(c) Area of the shadow $= \dfrac{1}{2} \times BC \times FD \sin\angle CFD$		
If $50 < x < 90$, $\angle CFD$ will be smaller (or less than 40°).	1A	or $\sin\angle CFD$ will be smaller
$\because$ BC, FD remain unchanged and $\sin\angle CFD$ is smaller		
$\therefore$ The area of the shadow will be smaller than the area obtained in (b).	1	
	(2)	

Solution	Marks	Remarks
18. (a) The inequalities representing the constraints for x and y:		
$0.32x+0.28y \le 4.48$ $(8x+7y \le 112)$	1A	deduct 1 mark for any strict inequality sign
$0.24x+0.36y \le 4.32$ $(2x+3y \le 36)$	1A	
$2x+10y \le 100$ $(x+5y \le 50)$	1A	
$x \ge 0$, $y \ge 0$		optional
Drawing the 3 straight lines.	1A+1A+1A	$\pm\frac{1}{2}$ grid
Shading the region R.	1A (7)	accept marking all lattice points
(b) Let $\$P$ be the profit.		
$P = 90x+120y$ $= 30(3x+4y)$	1A	
On the graph paper, draw the line $3x+4y=c$ for some constant c. From the graph, the maximum possible profit is obtained at (6, 8).	1M+1A	1M for +/– slope
The maximum possible profit $= \$(90\times 6+120\times 8)$ $= \$\ 1500$	1A	
Alternatively, $\because$ $P(0,10)=1200$, $P(4,9)=1440$, $P(6,8)=1500$ and $P(14,0)=1260$ $\therefore$ The maximum profit is \$ 1500.	1M+1A 1A	1M for testing these 4 points
	(4)