香港考試局
HONG KONG EXAMINATIONS AUTHORITY

2001年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2001

數學 試卷一
MATHEMATICS PAPER 1

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員／教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the teachers' centre.

Hong Kong Certificate of Education Examination
Mathematics Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
   a. The symbol (*u-1*) should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* for the whole paper.
   b. The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deduct ***2 marks*** for *pp* for the whole paper. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
   c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
   d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.

7. Marks entered in the Page Total Box should be the NET total scored on that page.

8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to', 'f.t.' stands for 'follow through' and 'or equivalent' means 'accepting equivalent forms of the equation which has been simplified and without uncollected like terms'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

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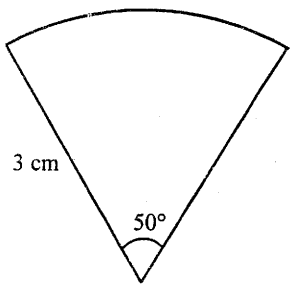
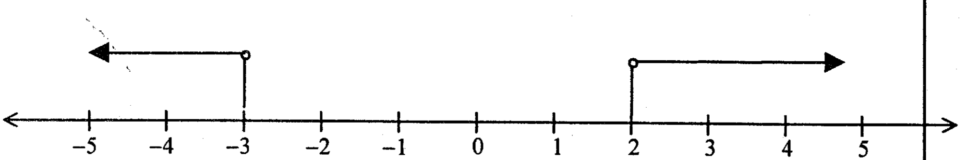
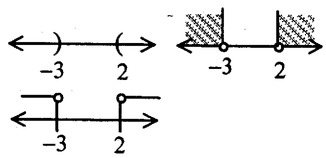
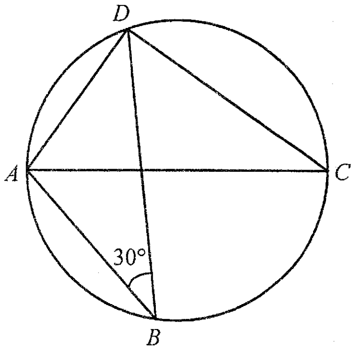
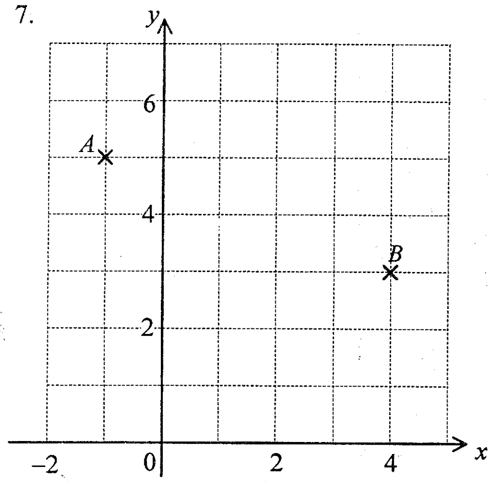
Solution	Marks	Remarks
1. $\dfrac{m^3}{(mn)^2} = \dfrac{m^3}{m^2n^2}$	1M	applying $(ab)^p = a^p b^p$
$= \dfrac{m^{3-2}}{n^2}$	1M	applying $\dfrac{a^p}{a^q} = a^{p-q}$
$= \dfrac{m}{n^2}$	1A	
	----------(3)	
2. $\because$ f(2) $\boxed{2^3 - 2^2 + 2 - 1}$	2A	
$= 5$	1A	
$\therefore$ the remainder is 5 .		
$\begin{array}{r} 1+1+3 \\ 1-2\,\overline{)1-1+1-1} \\ 1-2 \\ \hline 1+1-1 \\ 1-2 \\ \hline 3-1 \\ 3-6 \\ \hline 5 \end{array}$	$\boxed{1A}$ $\boxed{1A}$	
$\therefore$ remainder = 5	$\boxed{1A}$	
	----------(3)	
3.  Perimeter $= [3+3+\dfrac{50}{360}(2\times3\times\pi)]$ (cm)	1M+1A	1M for summation $(3+3+?)$ 1A for arc length
Perimeter $= [3+3+3\times\dfrac{50}{180}\pi]$ (cm)	$\boxed{1M}$+$\boxed{1A}$	1M for summation 1A for arc length
≈ 8.62 cm $\boxed{[6+\frac{5}{6}\pi] \text{ cm}}$	1A	r.t. 8.62 , u–1 for missing unit
	----------(3)	
4. $x^2 + x - 6 > 0$		
$(x+3)(x-2) > 0$	1A	For factorization only
$x < -3$ or $x > 2$	1M	Distinct roots and ">0" No mark for $x < -3$, $x > 2$ etc.
	1M	or 
	----------(3)	

Solution	Marks	Remarks
5. *Ignore writings on the diagram when marking this question.*		
 		
$\because$ $\angle ADC = 90°$	1A	Can be absorbed below
$\angle DCA = 30°$	1A	Can be absorbed below
$\therefore$ $\angle DAC = 180° - 90° - 30°$	1M	
$= 60°$	1A	u−1 for missing unit
Joint BC,		
$\angle ABC = 90°$	1A	Can be absorbed below
$\angle DBC = 90° - 30°$	1A	
$\angle DAC = \angle DBC = 60°$	1M+1A	1M for $\angle DAC = \angle DBC$
	----------(4)	
6. $\because$ $y = \frac{1}{2}(x+3)$		
$\therefore$ $2y = x+3$　　$y = \frac{1}{2}x + \frac{3}{2}$	1A	removing brackets
$x = 2y - 3$	1A	
In the above linear relation, the coefficient of y is 2.	1M	
Let $x_0 = 2y_0 - 3$. If $y = y_0 + 1$, then $x = 2(y_0+1) - 3 = x_0 + 2$.	1M	Putting $y = y_0 + 1$ or substituting two particular values of y which differ by 1
$\therefore$ x will be increased by 2 if y is increased by 1.	1M	
	----------(4)	
7.  		
(a) The coordinates of A and B are $(-1, 5)$ and $(4, 3)$ resp.	1A	Irrespective of order Accept writing on the diagram
(b) Slope of $AB = \frac{5-3}{-1-4}$	1M	
Equation of AB:		
$\frac{y-3}{x-4} = -\frac{2}{5}$	1M	
$2x + 5y - 23 = 0$	1A	or equivalent
$y = -\frac{2}{5}x + \frac{23}{5}$		
	----------(4)	
8. (a) New price $= (\$)80 \times (1 + 20\%)$	1A	
$= \$96$	1A	u−1 for missing unit
(b) Peter pays : $(\$)96 \times (1 - 20\%)$	1M	
$= \$76.8$	1A	u−1 for missing unit
	----------(4)	

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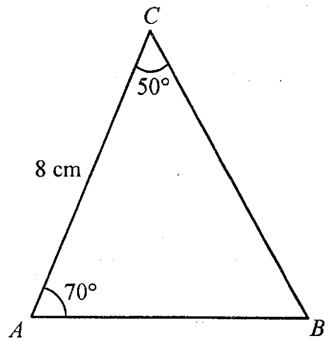
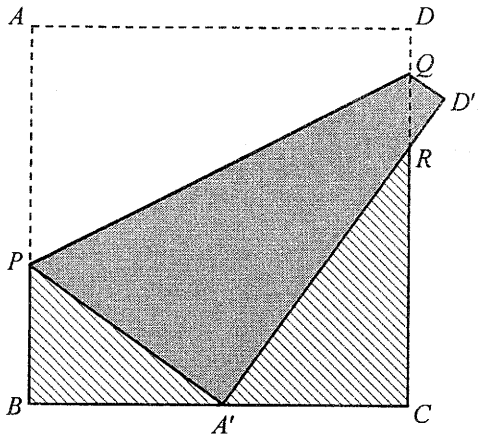
Solution	Marks	Remarks
9.  $\angle ABC = 60°$	1A	Can be absorbed below
Using sine rule, $\dfrac{AB}{\sin 50°} = \dfrac{8\text{ (cm)}}{\sin 60°}$	1M	
$AB \approx 7.0764$ (cm) ≈ 7.08 cm	1A	r.t. 7.08, $u-1$ for missing unit
Area of $\Delta ABC \approx \dfrac{1}{2}(8)(7.07642)\sin 70°$ (cm^2)	1M	
$\approx 26.6\text{ cm}^2$	1A -----------(5)	r.t. 26.6, $u-1$ for missing unit

10. (a)

Score (x)	Class mid-value (Class mark)	Frequency
$44 \le x < 52$	48	3
$52 \le x < 60$	56	9
$60 \le x < 68$	64	15
$68 \le x < 76$	72	11
$76 \le x < 84$	80	2

Solution	Marks	Remarks
(a)	1A+1A+1A -----------(3)	1A for each column
(b) Mean = 64	1A	
Standard deviation = 8	1A -----------(2)	
(c) Standard score $= \dfrac{76-64}{8}$	1M	
$= 1.5$	1A -----------(2)	
(d) Let her score in the second test be y, then $\dfrac{y-58}{10} = 1.5$	1M	
$y = 73$	1A -----------(2)	

Solution	Marks	Remarks
11. 		
(a) Since $A'P = x$ (cm),	1A	Can be absorbed below or written on the diagram
$\therefore \quad (12-x)^2 + 6^2 = x^2$	1A	
$144 - 24x + x^2 + 36 = x^2$		
$x = 7.5$	1A	u−1 for writing $x = 7.5$ cm
	----------(3)	
(b) (L1)... In Δs PBA' and $A'CR$,		
(L2)... (i) $\angle PBA' = \angle A'CR = 90°$		
(L3)... Since $\angle A'PB + 90° + \angle BA'P = 180°$ (∠ sum of Δ)		[Δ內角和]
(L4)... and $\angle RA'C + 90° + \angle BA'P = 180°$ (adj. ∠s on st. line)		[直線上的鄰角]
(L5)... ∴ (ii) $\angle A'PB = \angle RA'C$		
(L6)... Hence $\Delta PBA' \sim \Delta A'CR$ (AAA)		(equiangular), (AA) [等角]

Marking Scheme :		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	1	
In addition, any relevant correct argument with correct reason – L3, L4 or L6 (at most 1 mark)	1	At most 2 marks
Case 3 Any relevant correct argument with correct reason – L2, L3 or L4 (at most 1 mark)	1	At most 1 mark

Solution	Marks	Remarks
	----------(3)	
(c) Let $A'R = y$ cm and use the result of (b),		
$\dfrac{A'R}{A'C} = \dfrac{PA'}{PB}$		
$\dfrac{y}{6} = \dfrac{7.5}{12-7.5}$	1M	
$y = 10$		
i.e. $A'R = 10$ cm	1A	u−1 for missing unit
	----------(2)	

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Solution	Marks	Remarks
12. (a) (i) Perimeter of F_{40} $= [10+(40-1)\times 1]$ (cm)	1A	
$= 49$ cm	1A	$u-1$ for missing unit
(ii) The sum of the perimeters of the 40 figures		
$= [40\times\frac{10+49}{2}]$ (cm) $[40\times\frac{2\times 10+(40-1)\times 1}{2}]$ (cm)	1M	
$= 1180$ cm	1A	$u-1$ for missing unit
	----------(4)	
(b) (i) Area of F_2 $= [4\times\left(\frac{11}{10}\right)^2]$ (cm^2)	1M	Using the square of linear ratio
$= 4.84$ cm^2 $\frac{121}{25}$ cm^2	1A	r.t. 4.84, $u-1$ for missing unit
(ii) Area of F_3 $= 4\times\left(\frac{12}{10}\right)^2$ (cm^2) $= 5.76$ (cm^2)		
$\because$ Area of F_2 $-$ Area of F_1 $\neq$ Area of F_3 $-$ Area of F_2 (0.84 cm^2 $\neq$ 0.92 cm^2)	1M	
$\therefore$ the areas of figures $F_1, F_2, \cdots, F_{40}$ do not form an arithmetic sequence.	1	
	----------(4)	

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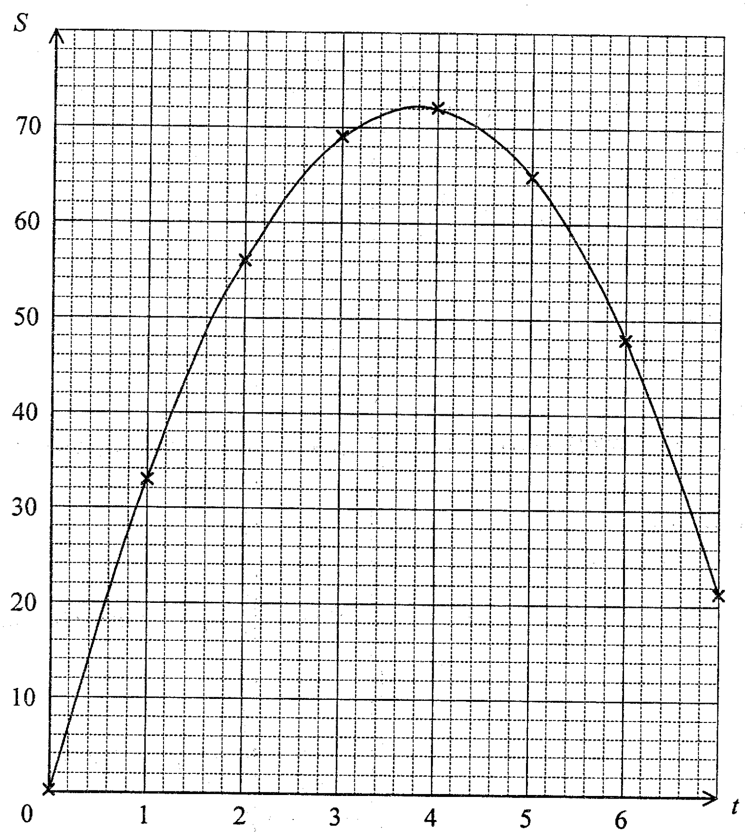
Solution	Marks	Remarks
13. (a) Let $S = at + bt^2$ for some non-zero constants a and b.	1A	
Solving $\begin{cases} 33 = a + b \\ 56 = 2a + 4b \end{cases}$, we have	1M	For substituting one pair of no - zero values of S and t
$a = 38$ and $b = -5$	1A	
$\therefore$ $S = 38t - 5t^2$		
	----------(3)	
(b) When $S = 40$, $5t^2 - 38t + 40 = 0$	1M	
$t = 1.26$ or 6.34 $\boxed{\dfrac{19 \pm \sqrt{161}}{5}}$	1A	r.t. 1.26, 6.34
	----------(2)	
(c) 	1A+1M	1A for the 8 pts., ±½ grid lines 1M for the curve, joining at least 6 pts. smoothly
From the graph, S is greatest when $t \approx 3.8$.	1A ----------(3)	Accept 3.6 – 3.9, read from the graph

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Solution	Marks	Remarks
14. (a) (i)		

x	$f(x)$
1	0
1.05	−0.0237
1.1	0.0105
1.15	0.111

1A — r.t. −0.02, 0.01

(ii) From (i), the root lies in the interval [1.05, 1.1]. — 1M — Can be absorbed in the table below

Using the method of bisection,

a [$f(a) < 0$]	b [$f(b) > 0$]	$m = \frac{a+b}{2}$	$f(m)$	Marks	Remarks
1.0500	1.1000	1.0750	−0.0144	1M	Testing sign of mid-value
1.0750	1.1000	1.0875	−0.0039	1M	Choosing the correct interval
1.0875	1.1000	1.0938	0.0028		
1.0875	1.0938	1.0907	−0.0006		
1.0907	1.0938	1.0923	0.0011		
1.0907	1.0923	1.0915	0.0002		
1.0907	1.0915				

$\therefore\ 1.0907 < h < 1.0915$

$x \approx 1.091$ (correct to 3 decimal places) — 1A — f.t.

----------(5)

(b) The given conditions lead to the equation

$1000(1+r\%)^4 + 1000(1+r\%)^3 + 1000(1+r\%)^2 + 1000(1+r\%) = 5000$ — 1M+1A — 1M for compound interest

Let $x = 1 + r\%$, then

$1000x^4 + 1000x^3 + 1000x^2 + 1000x = 5000$

$x^4 + x^3 + x^2 + x = 5$

$\frac{x(x^4-1)}{x-1} = 5$ — 1M+1A — 1M for sum of the sequence

$x^5 - x = 5x - 5$

$x^5 - 6x + 5 = 0$

From (a), $x \approx 1.091$ — 1M — Using the result of (a)(ii)

i.e. $r \approx 9.1$ — 1A — f.t.

----------(6)

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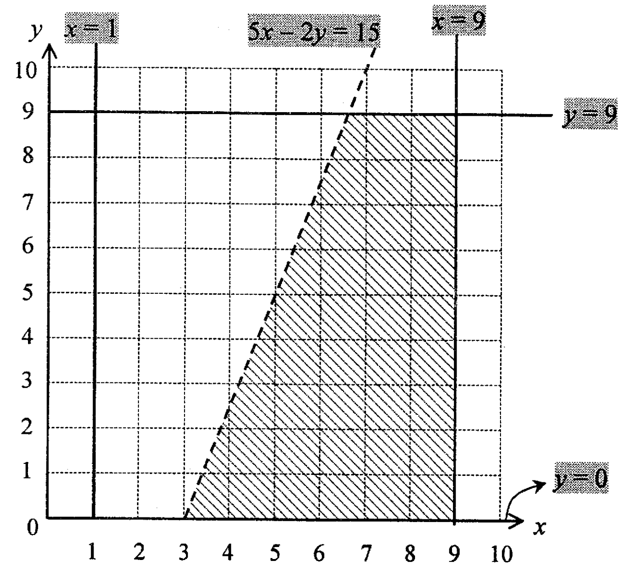
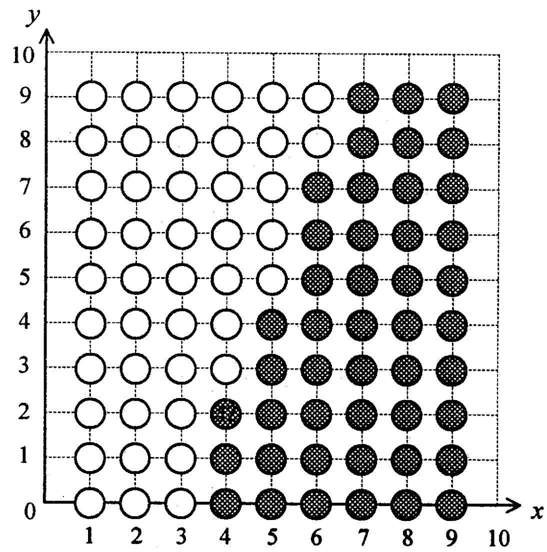
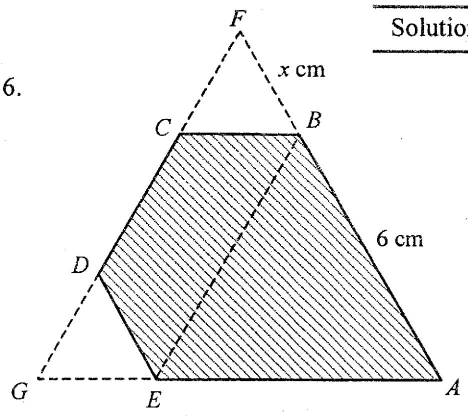
Solution	Marks	Remarks
15. (a) 	1A+1A	For drawing the lines $x=1$, $x=9$, $y=9$ and $5x-2y=15$. 1A for any one being correct. 1A for all.
	1A	For the shaded region, excluding the boundaries
	1A	For the dotted line
	----------(4)	
(b) (i) 	1A	Independent of (a)

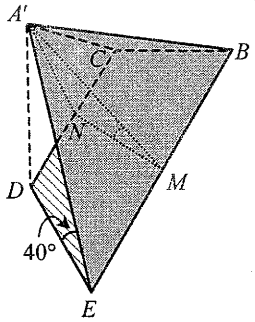
(ii)

No. of tables	Table no. is a multiple of 3	Table no. is not a multiple of 3	Row sum
Non-smoking area	14	30	44
Smoking area	16	30	46
Column sum	30	60	

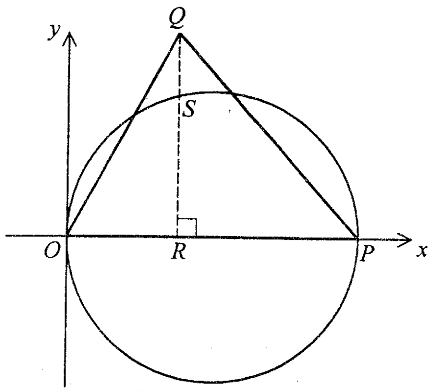
Solution	Marks	Remarks
(I) Required probability $=\frac{46}{90}$	1M	
$=\frac{23}{45}$ $\boxed{0.511}$ (*p*)	1A	r.t. 0.511
(II) Required probability $=\frac{46}{90}\times\frac{14}{89}+\frac{14}{90}\times\frac{46}{89}$ $\boxed{\text{or } \frac{46}{90}\times\frac{14}{89}\times 2}$	1M+1M+1A	1M for $p\times\frac{n}{89}$ 1M for multiplying by 2
$=\frac{644}{4005}$ $\boxed{0.161}$	1A	r.t. 0.161
	----------(7)	

16.





Solution	Marks	Remarks
(a) In the trapezium $BCDE$,		
height $= x\sin 60°$ (cm) [$\frac{\sqrt{3}}{2}x$ (cm)]	1A	For either, can be absorbed below
$CD = (6-x)$ (cm)		
$\therefore\ \frac{6+(6-x)}{2}\times\frac{\sqrt{3}}{2}x = 5\sqrt{3}$	1A	
$\frac{\sqrt{3}(12-x)x}{4} = 5\sqrt{3}$		
[$2\left(\frac{1}{2}x^2\sin 60°\right)+5\sqrt{3}+\frac{1}{2}(6^2)\sin 60° = \frac{1}{2}(x+6)^2\sin 60°$	[1A]+[1A]	1A for the area of one triangle 1A for all
$2x^2+20+36=(x+6)^2$]		
$x^2-12x+20=0$	1	
$(x-2)(x-10)=0$		
$x=2$ or $x=10$ (rejected)	1A ----------(4)	Accept "$x=2$ only" $u-1$ for writing $x=2$ cm
(b) (i) $A'D^2 = [6^2+2^2-2(6)(2)\cos 40°]$ (cm²)	1M	
≈ 21.6149 (cm²)		
$A'D \approx 4.6492$ (cm)		
≈ 4.65 cm	1A	r.t. 4.65, $u-1$ for missing unit
(ii) Let M, N be the mid-points of EB and DC respectively, then		
$A'M = 6\sin 60°$ (cm) $= 3\sqrt{3}$ (cm),		
$MN = 2\sin 60°$ (cm) $= \sqrt{3}$ (cm), and		
$A'N = \sqrt{A'D^2 - DN^2}$		
$\approx \sqrt{21.6149-2^2}$ (cm)	1M	
$\approx \sqrt{17.6149}$ (cm) [4.197 (cm)]		
The angle between the planes $BCDE$ and $A'BE$ is $\angle A'MN$.	1A	
$\cos\angle A'MN \approx \frac{(3\sqrt{3})^2+(\sqrt{3})^2-17.6149}{2(3\sqrt{3})(\sqrt{3})}$		
≈ 0.6881		
$\angle A'MN \approx 46.5°$	1A	r.t. 46.5, $u-1$ for missing unit
(iii) Required volume $= \frac{1}{3}$(area of trapezium $CDEB$)($A'M\sin\angle A'MN$)		
$\approx \frac{1}{3}(5\sqrt{3})(3\sqrt{3}\sin 46.5°)$ (cm³)	1M	
≈ 10.9 cm³	1A ----------(7)	r.t. 10.9, $u-1$ for missing unit

Solution	Marks	Remarks
17. (a) (i) Centre = $\left(\frac{p}{2},\ 0\right)$, radius = $\frac{p}{2}$	1A+1A	Can be absorbed below
		
Equation of the circle OPS :		
$\left(\frac{y}{x}\right)\left(\frac{y}{x-p}\right)=-1$ $x(x-p)+y^2=0$	2A (boxed)	
$\left(x-\frac{p}{2}\right)^2+y^2=\left(\frac{p}{2}\right)^2$ $x^2+y^2-px=0$ $x^2+y^2-px+pa-a^2-b^2=0$	1A	
Let the equation of the circle OPS be $x^2+y^2+Dx+Ey+F=0$, then $\begin{cases}F=0\\p^2+pD+F=0\\a^2+b^2+aD+bE+F=0\end{cases}$	1A (boxed)	for anyone
which gives $F=0$, $D=-p$ and $E=-\dfrac{a^2+b^2-pa}{b}$.	1A (boxed)	for any two being correct
$\therefore$ the equation is $x^2+y^2-px-\dfrac{a^2+b^2-pa}{b}y=0$.	1A (boxed)	
(a) (ii) $\because$ S lies on the circle OPS , $\therefore$ $a^2+b^2-pa=0$	1M	Sub. (a, b) into eqtn. in (a)
Since $OS\perp SP$, $\therefore$ $\left(\frac{b}{a-p}\right)\left(\frac{b}{a}\right)=-1$ or $a^2+b^2-pa=0$.	1M (boxed)	
$OR=OQ\cos\angle POQ$	1A	
Using Pythagoras' Theorem, $OS^2=a^2+b^2$ $=pa$ $=OP\cdot OR$	1A	
$=OP\cdot OQ\cos\angle POQ$	1	
Since $\cos\angle SOR=\frac{OR}{OS}=\frac{OS}{OP}$	1M+1A (boxed)	1M for considering $\cos\angle SOR$ or the similar triangles
$OR=OQ\cos\angle POQ$	1A (boxed)	
$\therefore$ $OS^2=OP\cdot OR$ $=OP\cdot OQ\cos\angle POQ$	1 (boxed)	
Using cosine rule, $OP\cdot OQ\cos\angle POQ$ $=\frac{1}{2}\left(OP^2+OQ^2-PQ^2\right)$	1A (boxed)	Cosine rule
$=\frac{1}{2}\left[\left(OS^2+PS^2\right)+\left(OR^2+QR^2\right)-\left(QR^2+PR^2\right)\right]$	1M (boxed)	Pythagoras' Theorem
$=\frac{1}{2}\left(OS^2+PS^2+OR^2-PR^2\right)$	1A (boxed)	
$=\frac{1}{2}\left[OS^2+\left(PR^2+RS^2\right)+\left(OS^2-RS^2\right)-PR^2\right]$ $=OS^2$	1 (boxed)	
	----------(7)	

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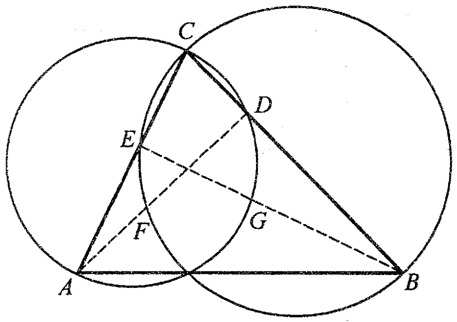
Solution	Marks	Remarks
(b) 		
(i) ∵ BC is a diameter of the circle $BCEF$, ∴ $\angle BEC = 90°$ ($\angle$ in semicircle) i.e. BE is an altitude of ΔABC .	1	[半圓上的圓周角] Stating that BC is a diameter or using $\angle$ in semicircle
(ii) Since the points C , A , B , G and E are defined analogously as the points O , P , Q , S and R in (a), ∴ $CG^2 = CA \cdot CB \cos \angle ACB$.	1A	
Similarly, AD is also an altitude of ΔABC and $CF^2 = CB \cdot CA \cos \angle ACB$.	1A	
Hence $CG = CF$.	1 ----------(4)	

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