30) Let $k > 0$.

(a) (i) Find the common ratio of the geometric progression

$$k,\ 10k,\ 100k.$$

(ii) Find the sum of the first n terms of the geometric progression

$$k,\ 10k,\ 100k,\ \ldots .$$

(b) (i) Show that

$$\log_{10}k,\ \log_{10}10k,\ \log_{10}100k$$

is an arithmetic progression.

(ii) Find the sum of the first n terms of the arithmetic progression

$$\log_{10}k,\ \log_{10}10k,\ \log_{10}100k,$$

Also, if $n = 10$, what is the sum?

2) (a) (i) Find the sum of all the multiples of 3 from 1 to 1000

(ii) Find the sum of all the multiples of 4 from 1 to 1000

(including 1000).

(6 marks)

(b) Hence, or otherwise, find the sum of all the integers from 1 to 1000 which are neither multiples of 3 nor multiples of 4.

(including 1 and 1000)

(6 marks)

3(81).

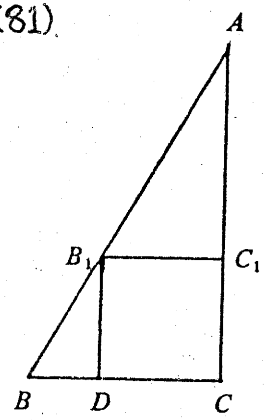


Figure (a)

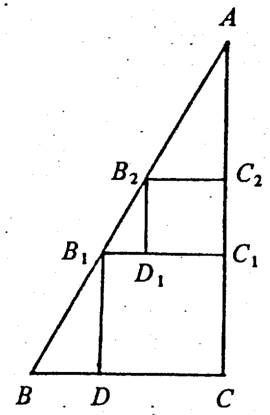


Figure (b)

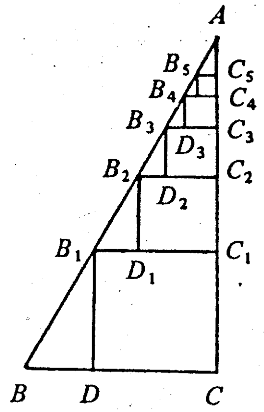


Figure (c)

In Figure (a), B_1C_1CD is a square inscribed in the right-angled triangle ABC. $\angle C = 90°$, $BC = a$, $AC = 2a$, $B_1C_1 = b$.

(a) Express b in terms of a.

(3 marks)

(b) $B_2C_2C_1D_1$ is a square inscribed in ΔAB_1C_1 (see Figure (b)).

(i) Express B_2C_2 in terms of b.

(ii) Hence express B_2C_2 in terms of a.

(2 marks)

(c) If squares $B_3C_3C_2D_2$, $B_4C_4C_3D_3$, $B_5C_5C_4D_4$, ... are drawn successively as indicated in Figure (c),

(i) write down the length of B_5C_5 in terms of a,

(ii) find, in terms of a, the sum of the areas of the infinitely many squares drawn in this way.

(7 marks)

4(82)

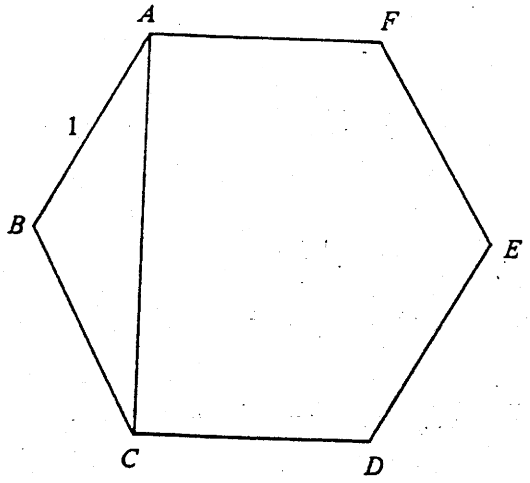


Figure (a)

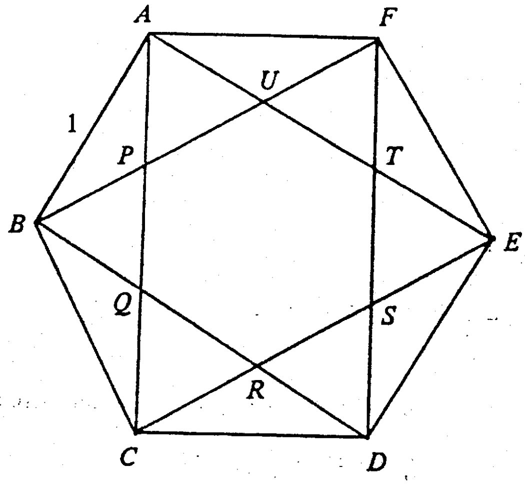


Figure (b)

(In this question, answers should be given in surd form.)

In Figures (a) and (b), $ABCDEF$ is a regular hexagon with $AB = 1$.

(a) Calculate the area of the hexagon in Figure (a) and the length of its diagonal AC.

(6 marks)

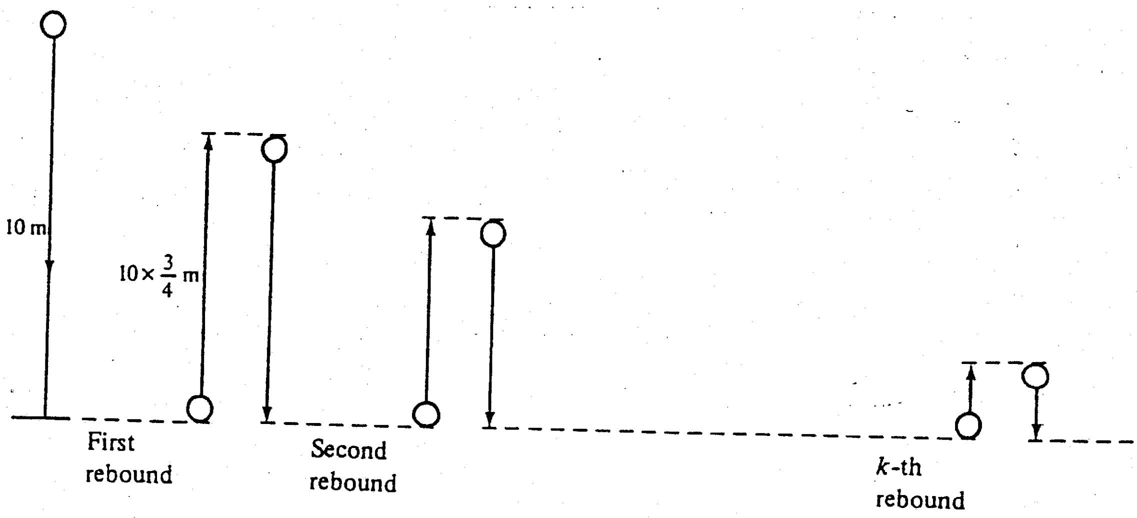
(b) In Figure (b), $PQRSTU$ is another regular hexagon formed by the diagonals of $ABCDEF$.

(i) Calculate the length of PQ.

(ii) Calculate the area of the hexagon $PQRSTU$.

(6 marks)

5(83) A ball is dropped vertically from a height of 10 m, and when it reaches the ground, it rebounds to a height of $10 \times \frac{3}{4}$ m. The ball continues to fall and rebound again and again, each time rebounding to $\frac{3}{4}$ of the height from which it previously fell (see Figure).



Figure

(a) Find the total distance travelled by the ball just before it makes its second rebound. (3 marks)

(b) Find, in terms of k, the total distance travelled by the ball just before it makes its $(k + 1)$th rebound.

(6 marks)

(c) Find the total distance travelled by the ball before it comes to rest. (3 marks)

6(84) a and b are positive numbers. a , -2 , b form a geometric progression and -2 , b , a form an arithmetic progression.

(a) Find the value of ab .

(2 marks)

(b) Find the values of a and b .

(5 marks)

(c) (i) Find the sum to infinity of the geometric progression a , -2 , b , ...

(ii) Find the sum to infinity of all the terms that are positive in the geometric progression a , -2 , b , ...

(5 marks)

7(85) \$ P is deposited in a bank at the interest rate of $r\%$ per annum compounded annually. At the end of each year, $\frac{1}{3}$ of the amount in the account (including principal and interest) is drawn out and the remainder is redeposited at the same rate.

Let \$ Q_1 , \$ Q_2 , \$ Q_3 , . . . denote respectively the sums of money drawn out at the end of the first year, second year, third year, ...

(a) (i) Express Q_1 and Q_2 in terms of P and r.

(ii) Show that $Q_3 = \frac{4}{27}P(1 + r\%)^3$.

(5 marks)

(b) Q_1 , Q_2 , Q_3 , . . . form a geometric progression. Find the common ratio in terms of r.

(2 marks)

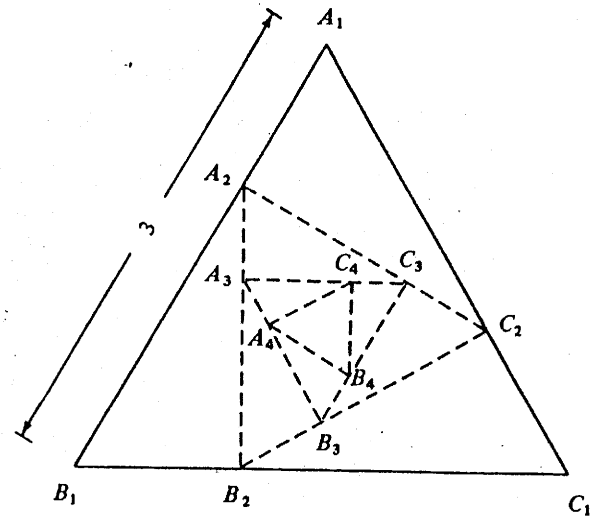
(c) Suppose $Q_3 = \frac{27}{128}P$.

(i) Find the value of r .

(ii) If $P = 10\,000$, find $Q_1 + Q_2 + Q_3 + \ldots + Q_{10}$. (Give your answer correct to the nearest integer.)

(5 marks)

8(87)



Figure

In this question you should leave your answers in surd form.

In Figure , $A_1B_1C_1$ is an equilateral triangle of side 3 and area T_1 .

(a) Find T_1 .

(2 marks)

(b) The points A_2 , B_2 and C_2 divide internally the line segments A_1B_1 , B_1C_1 and C_1A_1 respectively in the same ratio 1 : 2. The area of $\Delta A_2B_2C_2$ is T_2 .

(i) Find A_2B_2 .

(ii) Find T_2 .

(4 marks)

(c) Triangles $A_3B_3C_3$, $A_4B_4C_4$, . . . are constructed in a similar way. Their areas are T_3 , T_4 , . . . , respectively. It is known that T_1 , T_2 , T_3 , T_4 , ... form a G.P.

(i) Find the common ratio.

(ii) Find T_n .

(iii) Find the value of $T_1 + T_2 + \ldots + T_n$.

(iv) Find the sum to infinity of the G.P.

(6 marks)

9(86) 2, −1, −4, . . . are in A.P.

(a) Find (i) the n th term,

(ii) the sum of the first n terms,

(iii) the sum of the progression from the 21st term to the 30th term.

(7 marks)

(b) If the sum of the first n terms of the progression is less than −1000, find the least value of n.

(5 marks)

10(88)(a) Write down the smallest and the largest multiples of 7 between 100 and 999.

(2 marks)

(b) How many multiples of 7 are there between 100 and 999?

Find the sum of these multiples.

(6 marks)

(c) Find the sum of all positive three-digit integers which are **NOT** divisible by 7.

(4 marks)

11.(89) **The positive numbers $1, k, \frac{1}{2}, \ldots$ are in geometric progression.**

(a) Find the value of k, leaving your answer in surd form. **(2 marks)**

(b) Express the nth term $T(n)$ in terms of n. **(2 marks)**

(c) Find the sum to infinity, expressing your answer in the form $p + \sqrt{q}$, where p and q are integers. **(4 marks)**

(d) Express the product $T(1) \times T(3) \times T(5) \times \ldots \times T(2n-1)$ in terms of n. **(4 marks)**

12(90) The positive integers 1, 2, 3, . . . are divided into groups $G_1, G_2, G_3, \ldots$, so that the k^{th} group G_k consists of k consecutive integers as follows:

G_1 : 1

G_2 : 2 , 3

G_3 : 4 , 5 , 6

. . . .

. . . .

. . . .

G_{k-1} : $u_1, u_2, \ldots, u_{k-1}$

G_k : $v_1, v_2, \ldots, v_{k-1}, v_k$

. . . .

. . . .

. . . .

(a) (i) Write down all the integers in the 6^{th} group G_6.

(ii) What is the total number of integers in the first 6 groups $G_1, G_2, \ldots, G_6$?

(4 marks)

(b) Find, in terms of k,

(i) the last integer u_{k-1} in G_{k-1} and the first integer v_1 in G_k,

(ii) the sum of all the integers in G_k.

(8 marks)

13(91)

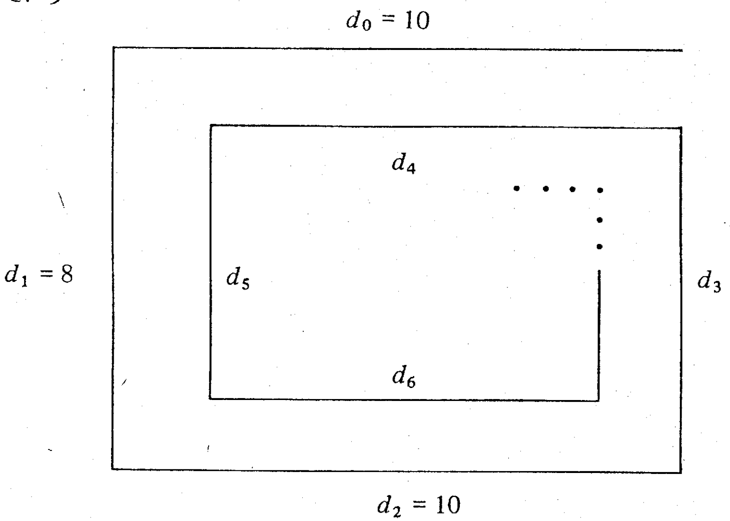


Figure 6

A maze is formed by line segments of lengths $d_0, d_1, d_2, \cdots, d_n, \cdots$, with adjacent line segments perpendicular to each other as shown in Figure 6.

Let $d_0 = 10$, $d_1 = 8$, $d_2 = 10$ and $\dfrac{d_{n+2}}{d_n} = 0.9$ when $n \geq 1$,

i.e. $\dfrac{d_3}{d_1} = \dfrac{d_5}{d_3} = \cdots = 0.9$ and $\dfrac{d_4}{d_2} = \dfrac{d_6}{d_4} = \cdots = 0.9$.

(a) Find d_3 and d_5, and express d_{2n-1} in terms of n. (4 marks)

(b) Find d_6 and express d_{2n} in terms of n. (2 marks)

(c) Find, in terms of n, the sums

(i) $d_1 + d_3 + d_5 + \cdots + d_{2n-1}$,

(ii) $d_2 + d_4 + d_6 + \cdots + d_{2n}$. (3 marks)

(d) Find the value of the sum $d_0 + d_1 + d_2 + d_3 + \cdots$ to infinity. (3 marks)

1a i) $k, 10k, 100k \ldots$ in G.P.

the common ratio

$$= \frac{10k}{k} = 10.$$

ii) the sum of the first n terms.

$$= \frac{a(1-R^n)}{1-R}$$ { a - first term, R - common ratio.

$$= \frac{k(1-10^n)}{1-10}$$

$$= \frac{k(10^n-1)}{9}.$$

b i) $\log_{10}k, \log_{10}10k, \log_{10}100k,$

$\log_{10}k + \log_{10}100k$

$= \log_{10}100k^2$

$= \log_{10}(10k)^2$

$= 2\log_{10}(10k).$

$\therefore \log_{10}k, \log_{10}10k, \log_{10}100k$ in A.P.

ii. the common difference.

$= \log_{10}10k - \log_{10}k$

$= \log_{10}\left(\frac{10k}{k}\right)$

$= \log_{10}10 = 1.$

$\therefore$ the sum of the first n terms.

$= \frac{n}{2}[2a+(n-1)(d)]$

$= \frac{n}{2}[2\log_{10}k + (n-1)(1)]$

$= n\log_{10}k + \frac{n(n-1)}{2}.$

if $n = 10$.

the sum $= 10\log_{10}k + \frac{10(10-1)}{2}$

$= 10\log_{10}k + 45.$

2a) i) In 1 to 1000,

the first term $= 3$.

the last term $= 999$.

no. of terms $= \frac{999}{3} = 333$.

the sum of all the multiples of 3.

$= \frac{n}{2}[a+l]$.

$= \frac{333}{2}[3+999]$

$= 166833.$

a ii) the first term $= 4$.

the last term $= 1000$

no. of terms $= \frac{1000}{4} = 250.$

the sum of all the multiples of 4

$= \frac{250}{2}[4+1000]$

$= 125500.$

b. the sum of all the integers which are neither multiples of 3 nor multiples of 4.

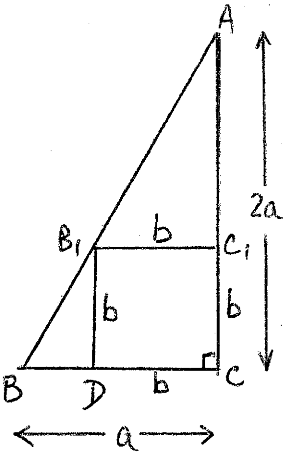
= sum of integers (1 to 1000) − sum of multiples of 3 − sum of multiples of 4 + sum of multiples of 12.

$= \frac{1000}{2}[1+1000] - 166833 - 125500 + \frac{83}{2}[12+996]$

$= 500500 - 166833 - 125500 + 41832$

$= 249999.$

3.a) $\Delta ABC \sim \Delta AB_1BD$.



$$\therefore \frac{AC}{BC} = \frac{B_1D}{BD}$$

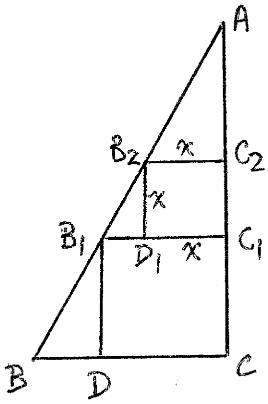
$$\frac{2a}{a} = \frac{b}{(a-b)}$$

$$2(a-b) = b.$$

$$\therefore 3b = 2a$$

$$b = \frac{2}{3}a.$$

b) i) Let B_2C_2 be x



$\therefore$ In $\Delta B_2B_1D_1 \sim \Delta ABC$

$$\frac{AC}{BC} = \frac{B_2D_1}{B_1D_1}$$

$$\frac{2a}{a} = \frac{x}{b-x}$$

$$2(b-x) = x.$$

$$\therefore 3x = 2b$$

$$x = \frac{2}{3}b.$$

$$\therefore B_2C_2 = \frac{2}{3}b.$$

ii) $B_2C_2 = \frac{2}{3}\left(\frac{2}{3}a\right)$ from (a)

$$= \left(\frac{2}{3}\right)^2 a. = \frac{4}{9}a.$$

c) i) deduction from (a) & (b)

$$B_5C_5 = \left(\frac{2}{3}\right)^5 a$$

$$B_5C_5 = \frac{32}{243}a.$$

ii) the area of $B_1C_1DC = b^2 = \left(\frac{2}{3}a\right)^2 = \frac{4}{9}a^2$

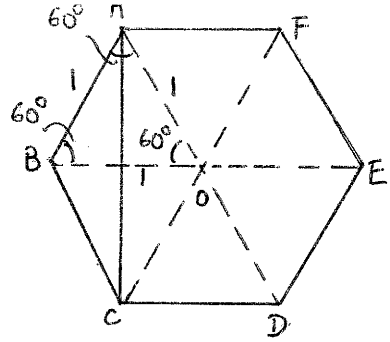
the area of $B_2C_2D_1C_1 = \left(\frac{4}{9}a\right)^2 = \left(\frac{4}{9}\right)^2 a^2.$

$\therefore$ the sum of the areas.

$$= \frac{a}{1-R}. \quad \text{if } |R|<1$$

$$= \frac{\frac{4}{9}a^2}{1-\left(\frac{4}{9}\right)} = \frac{4/9\, a^2}{5/9} = \frac{4}{5}a^2.$$

4.



a) area of ABCDEF

$$= 6 \times \text{area of } \Delta OAB$$

$$= 6 \times \frac{1}{2}\cdot(OA)(OB)\sin 60°.$$

$$= 6 \times \frac{1}{2}(1)(1)\cdot \sqrt{3}/2$$

$$= \frac{3\sqrt{3}}{2} \text{ sq. unit.}$$

the length of AC, (In ΔABC

By cosine rule,

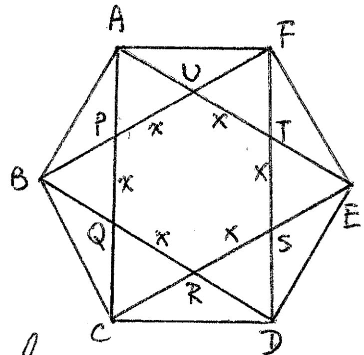
$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle B$$

$$AC^2 = 1^2 + 1^2 - 2(1)(1)\cos 120°$$

$$AC^2 = 1+1+1$$

$$AC = \sqrt{3}$$

b) i) Let PQ be x



$$\angle P = 120°.$$

$$\therefore \angle APU = 180° - 120° = 60°.$$

$$\angle AUP = 60°.$$

$\therefore \Delta APU$ is equilateral.

$$AP = PQ = QC = x.$$

$$\therefore AC = 3x$$

$$3x = \sqrt{3}$$

$$x = \sqrt{3}/3.$$

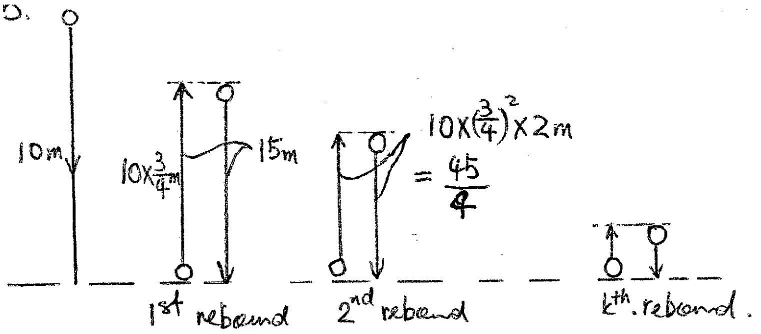
ii) area of PQRSTU.

$$= 6 \times \frac{1}{2}\left(\frac{\sqrt{3}}{3}\right)\left(\sqrt{3}/3\right)\cdot \sin 60°.$$

$$= 6 \times \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\cdot\frac{\sqrt{3}}{2}$$

$$= \sqrt{3}/2 \text{ sq. unit.}$$

5.



a) the total distance (just before 2nd rebound)

$= 10\text{m} + (10\times\frac{3}{4}\text{m})\times 2$

$= 10\text{m} + 15\text{m}.$

$= 25\text{m}.$

b) the total distance (just before (k+1) rebound)

$= 10\text{m} + \dfrac{a(1-R^k)}{1-R}.$

$= 10\text{m} + \dfrac{15[1-(\frac{3}{4})^k]}{1-\frac{3}{4}}\text{m}$

$= 10\text{m} + 15[1-(\frac{3}{4})^k]/\frac{1}{4}\text{m}$

$= \{10 + 60[1-(\frac{3}{4})^k]\}\text{m}.$

c) As it comes to rest,

$k \to \infty.$

$\therefore (\frac{3}{4})^k \to 0.$

the total distance.

$= [10 + 60(1-0)]\text{m}$

$= 70\text{m}.$

b) $\begin{cases} a, -2, b & \text{in G.P.} \\ -2, b, a & \text{in A.P.} \end{cases}$ where $a, b > 0$.

In G.P.

$\dfrac{-2}{a} = \dfrac{b}{-2}$

$\therefore (-2)^2 = ab$

$ab = 4.$ —— ①

In. A.P.

$b = \frac{1}{2}(-2+a)$

$2b = -2 + a.$ —— ②.

From ① $a = \dfrac{4}{b}.$ —— ③

sub into ②.

$2b = -2 + \dfrac{4}{b}.$

$2b^2 = -2b + 4$

$b^2 + b - 2 = 0$

$(b+2)(b-1) = 0$

$b = 1$ or -2 (rejected $b>0$)

when $b = 1$

$a = \dfrac{4}{1} = 4.$

c) i) the common ratio.

$R = \dfrac{-2}{4} = -\frac{1}{2}.$

the sum to infinity.

$= \dfrac{a}{1-R}$

$= \dfrac{4}{1-(-\frac{1}{2})} = \dfrac{4}{3/2} = \dfrac{8}{3}.$

ii) $4, -2, 1, \ldots\ldots$

the common ratio.

$R' = \dfrac{1}{4}$

$\therefore$ the sum to infinity (+ve terms.)

$= \dfrac{a}{1-R'}$

$= \dfrac{4}{1-\frac{1}{4}} = \dfrac{4}{3/4} = \dfrac{16}{3}.$

7. a) the amount at end of 1st year

$= P \times (1 + r\%)$.

$Q_1 = \frac{1}{3} \cdot P \times (1 + r\%)$

$= \frac{P}{3}(1 + r\%)$.

the amount at end of 2nd year.

$= [\frac{2}{3}P(1+r\%)](1+r\%)$

$= \frac{2}{3}P(1+r\%)^2$.

$Q_2 = \frac{1}{3}[\frac{2}{3}P(1+r\%)^2]$

$= \frac{2}{9}P(1+r\%)^2$.

money remainder

$= \frac{2}{3}[\frac{2}{3}P(1+r\%)^2]$

$= \frac{4}{9}P(1+r\%)^2$

amount at the end of 3rd year.

$= [\frac{4}{9}P(1+r\%)^2](1+r\%)$

$= \frac{4}{9}P(1+r\%)^3$.

$\therefore Q_3 = \frac{1}{3}[\frac{4}{9}P(1+r\%)^3]$

$= \frac{4}{27}P(1+r\%)^3$.

b) Q_1, Q_2, Q_3 form a G.P.

$\therefore$ the common ratio,

$R = \frac{Q_2}{Q_1} = \frac{\frac{2}{9}P(1+r\%)^2}{\frac{P}{3}(1+r\%)}$

$R = \frac{2}{3}(1+r\%)$.

c) if $Q_3 = \frac{27}{128}P$.

from a ii) $\frac{27}{128}P = \frac{4}{27}P(1+r\%)^3$.

$(1+r\%)^3 = \frac{729}{512}$

$1 + r\% = 9/8$.

$1 + r\% = 1.25$.

$r\% = 0.25$

$r = 25$.

ii) if $P = 10000$,

$Q_1 = \frac{2}{9}(10000)(1+25\%)$

$= \frac{25000}{9}$.

$R = \frac{2}{3}(1+25\%)$

$= 5/6$.

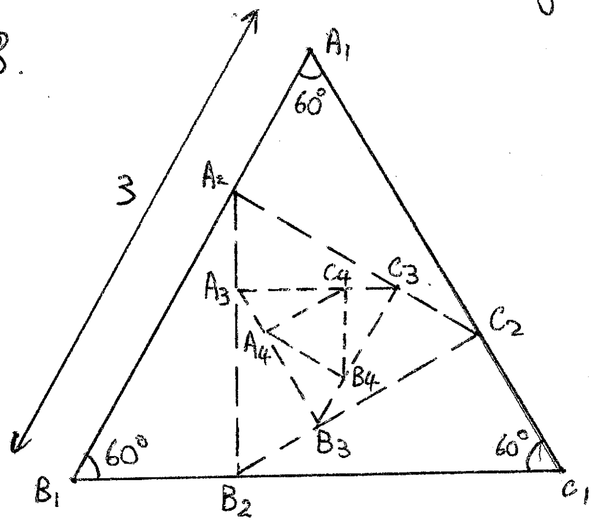
$\therefore Q_1 + Q_2 + Q_3 + \cdots + Q_{10}$.

$= \frac{Q_1(1-R)}{1-R}$

$= \frac{25000}{9}\left[\frac{1-(5/6)^{10}}{1-5/6}\right]$

$= 13975$. (nearest integer.)

8.



a) since $\Delta A_1B_1C_1$ are equilateral

$\therefore \angle A_1 = \angle B_1 = \angle C_1 = 60°$.

$T_1 = \frac{1}{2}(A_1B_1)(B_1C_1)\sin\angle B_1$

$= \frac{1}{2}(3)(3)\sin 60°$

$= \frac{9}{4}\sqrt{3}$ sq. unit.

6b) Since A_1B_1, B_1C_1 & C_1A_1 divided into

i) $2:1$.

$\therefore A_1A_2 = 1,\quad A_2B_1 = 2$.

In $\Delta A_2B_1B_2$.

By cosine rule,

$$A_2B_2^2 = A_2B_1^2 + B_1B_2^2 - 2(A_2B_1)(B_1B_2)\cos\angle B_1$$

$$A_2B_2^2 = 2^2 + 1^2 - 2(2)(1)\cos 60^\circ.$$

$$A_2B_2 = \sqrt{4+1-2}$$

$$= \sqrt{3}.$$

b ii) T_2 = area of $\Delta A_2B_2C_2$

$$= \frac{1}{2}(A_2B_2)(A_2C_2)\sin\angle A_2$$

$$= \frac{1}{2}(\sqrt{3})(\sqrt{3})\sin 60^\circ$$

$$= \frac{3}{4}\sqrt{3} \text{ sq. unit.}$$

c, i) $T_1, T_2, T_3, T_4, \ldots$ in G.P.

the common ratio.

$$R = \frac{T_2}{T_1} = \frac{3/4\sqrt{3}}{9/4\sqrt{3}} = \frac{1}{3}.$$

ii) $T_n = aR^{n-1}$ a - first term.

$$T_n = (9/4\sqrt{3})\left(\frac{1}{3}\right)^{n-1}$$

$$T_n = \frac{\sqrt{3}}{4(3)^{n-3}}$$

iii) $T_1 + T_2 + T_3 + \cdots + T_n$.

$$= \frac{a(1-R^n)}{1-R}$$

$$= \frac{\frac{9}{4}\sqrt{3}\left[1-\left(\frac{1}{3}\right)^n\right]}{\left(1-\frac{1}{3}\right)}$$

$$= \frac{27}{8}\sqrt{3}\left[1-\left(\frac{1}{3}\right)^n\right]$$

iv) sum to infinity

$$= \frac{a}{1-R} = \frac{\frac{9}{4}\sqrt{3}}{1-\frac{1}{3}} = \frac{27}{8}\sqrt{3}.$$

7. a) $2, -1, -4, \ldots$ are in A.P.

i) the common difference

$$d = -1-2$$

$$d = -3.$$

the n^{th} term.

$$= a+(n-1)d$$

$$= 2+(n-1)(-3)$$

$$= 2-3n+3$$

$$= 5-3n.$$

ii) the sum of the first n terms.

$$= \frac{n}{2}[a+l] \qquad l \text{ - last term.}$$

$$= \frac{n}{2}[2+5-3n]$$

$$= \frac{n}{2}[7-3n]$$

iii) the sum of 21^{st} term to 30^{th} term.

= the sum of first 30 terms − the sum of first 20 terms

$$= \frac{30}{2}[7-3(30)] - \frac{20}{2}[7-3(20)]$$

$$= -1245 + 530$$

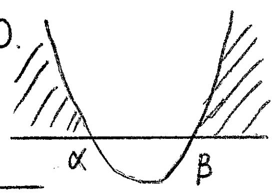
$$= -715.$$

b) sum of first n terms < -1000

$$\frac{n}{2}[7-3n] < -1000.$$

$$7n - 3n^2 < -2000.$$

$$3n^2 - 7n - 2000 > 0.$$



$$n < \frac{7-\sqrt{7^2-4(3)(-2000)}}{2(3)}$$

$$\text{or} \quad n > \frac{7+\sqrt{7^2-4(3)(-2000)}}{2(3)}.$$

$n < -24.6$ or $n > 27.01$

(rejected $n>0$)

$\therefore$ the least value of $n = 28$.

10. a) In 100 & 999.

the smallest multiples of 7 = 105.

the largest multiples of 7 = 994.

b) Let no. of term be n.

$\therefore 994 = 105 + (n-1)(7)$

$889 = 7n - 7$

$7n = 896$

$n = 128$.

the sum of multiples

$= \frac{n}{2}[a + l]$

$= \frac{128}{2}[105 + 994]$

$= 70336$.

c) the sum of all +ve. 3-digit integers which are not divisible by 7.

= sum of all 3-digit integers − sum of multiples of 7

$= \frac{1000}{2}[100 + 999] - 70336$

$= 549500 - 70336$

$= 479164$.

11. a) $1, k, \frac{1}{2}, \cdots$ are in G.P. where $k > 0$.

$\therefore \frac{k}{1} = \frac{1/2}{k}$

$\therefore k^2 = \frac{1}{2}$

$k = \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$. (rejected $k>0$.)

b) the common ratio.

$R = \frac{\frac{1}{\sqrt{2}}}{1} = \frac{1}{\sqrt{2}}$.

$T(n) = aR^{n-1}$

$= (1)(\frac{1}{\sqrt{2}})^{n-1} = (\frac{\sqrt{2}}{2})^{n-1}$.

c) The sum to infinity.

$= \frac{a}{1-R}$

$= \frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}-1}$.

$= \frac{\sqrt{2}}{\sqrt{2}-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$

$= \frac{2+\sqrt{2}}{2-1}$

$= 2+\sqrt{2}$.

d) $T(1) \times T(3) \times T(5) \times \cdots \times T(2n-1)$

$= 1 \times \frac{1}{2} \times (\frac{\sqrt{2}}{2})^{5-1} \times \cdots \times (\frac{\sqrt{2}}{2})^{2n-1-1}$

$= 1 \times \frac{1}{2} \times \frac{1}{4} \times \cdots \times (\frac{1}{2})^{n-1}$

$= 1 \times \frac{1}{2} \times (\frac{1}{2})^2 \times \cdots \times (\frac{1}{2})^{n-1}$

$= (\frac{1}{2})^{1+2+\cdots+n-1}$

$= (\frac{1}{2})^{\frac{n-1}{2}(n-1+1)}$

$= (\frac{1}{2})^{\frac{(n-1)n}{2}}$.

12. a i) $G_6 = 16, 17, 18, 19, 20, 21$.

ii) total no. of integers.

$= 1+2+3+4+5+6$

$= 21$

b In $G_1 : 1$.

$G_2 : 2, 3$.

$G_3 : 4, 5, 6$.

the last of $G_2 = 1+2 = 3$.

the last of $G_3 = 1+2+3 = 6$

the last u_{k-1} in G_{k-1}

$= 1+2+3+\cdots+k-1$

$= \frac{k-1}{2}[1+k-1]$

$= \frac{(k-1)}{2}k$.

the first term in G_k

$= (\frac{k-1}{2})k+1$

the last term in G_k

$= (\frac{k-1}{2})k+k$

sum of integers in G_k

$= (\frac{k-1}{2})k+1+\cdots+(\frac{k-1}{2})k+k$

$= \frac{k^2(k-1)}{2} + \frac{k(k+1)}{2}$

$= \frac{k}{2}[k^2-k+k+1]$

$= k(k^2+1)/2$

1. (83) The sixth term and the eleventh term of an arithmetic progression are 10 and 30 respectively. The first term is
A. −14 B. −10 C. 10 D. 50 E. 54

2. (83) In an arithmetic progression, the first term is 3 and the common difference is 2. If the sum of the first n terms of the arithmetic progression is 143, then n =
A. 10 B. 11 C. 12 D. 13 E. 14

3. (83) Three positive numbers a, b and c are in geometric progression. Which of the following are true ?
(1) $1/a$, $1/b$, $1/c$ are in geometric progression
(2) a^2, b^2, c^2 are in geometric progression
(3) $\log_{10}a$, $\log_{10}b$, $\log_{10}c$ are in arithmetic progression
A. (1) and (2) only B. (1) and (3) only
C. (2) and (3) only D. (1), (2) and (3) E. none of them

4. (84) The sum of the first ten terms of an arithmetic progression is 120. If the common difference is 4, then the first term is
A. −12 B. −6 C. −2 D. 2 E. 6

5. (84) If $a \neq \pm 1$, then $1 + a^2 + a^4 + \dots + a^{2n} =$
A. $\dfrac{1 - a^{2n}}{1 - a}$ B. $\dfrac{1 - a^{2n}}{1 - a^2}$ C. $\dfrac{1 - a^{2n+1}}{1 - a}$
D. $\dfrac{1 - a^{2n+1}}{1 - a^2}$ E. $\dfrac{1 - a^{2n+2}}{1 - a^2}$

6. (84) Which of the following must be geometric progression(s) ?
(1) $\log_{10}3$, $\log_{10}9$, $\log_{10}27$, $\log_{10}81$
(2) 0.9, 0.99, 0.999, 0.9999
(3) 1, −3, 9, −27
A. (1) only B. (3) only C. (1) and (3) only
D. (1) and (2) only E. (1), (2) and (3)

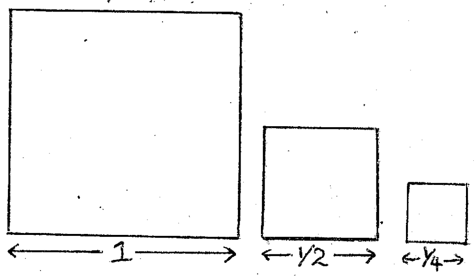
7. (85) The second term and the fifth term of a geometric progression are −12 and 40½ respectively. The first term is
A. 1½ B. 6 C. 8 D. 15 E. 18

8. (85) If $1/a$, $1/b$, $1/c$ are in geometric progression, then which of the following is true ?
A. $b^2 = ac$ B. $b^2 = 1/(ac)$ C. $b = (a+c)/2$ D. $b = (a+c)/2$
E. $b = 2ac/(a+c)$

9. (85) Three distinct numbers x, y and z are in arithmetic progression. Which of the following is/are also in arithmetic progression ?
I. $x+10$, $y+10$, $z+10$ II. $10x$, $10y$, $10z$ III. x^2, y^2, z^2
A. I and II only B. I and III only C. II and III only
D. I, II and III E. None of I, II and III-

10. (86) The figure shows an infinite number of squares. The length of a side of the first square is 1. The side of each subsequent square is equal to half of the side of the preceding one. Find the sum of the areas of the infinite number of squares.

A. 4 B. 2 C. 5/3
D. 3/2 E. 4/3



11. (86) If the five interior angles of a convex pentagon form an A.P. with a common difference of $10°$, then the smallest interior angle of the pentagon is

A. $52°$ B. $72°$ C. $88°$ D. $98°$ E. $108°$

12. (87) Given that $x \neq 0$ and $-x$, x, $3x^2$ are in G.P., find x.

A. -1 B. $-1/3$ C. 3 D. $1/3$ E. 1

13. (87) If $\log_{10}x$, $\log_{10}y$, $\log_{10}z$ are in A.P., then

A. $y = 10^{(x+z)/2}$ B. $y = (x+z)/2$ C. $y^2 = x + z$
D. $y^2 = xz$ E. $y = 10^{xz}$

14. (88) Which of the following is a G.P./are G.P.'s ?

(1) 5, 0.5, 0.05, 0.005, 0.0005
(2) log5, log50, log500, log5000, log50000
(3) 5, $5\sin 70°$, $5(\sin 70°)^2$, $5(\sin 70°)^3$, $5(\sin 70°)^4$

A. (1) only B. (2) only C. (3) only
D. (1) and (3) only E. (1), (2) and (3)

15. (88) p, q, r, s are in A.P. If $p + q = 8$ and $r + s = 20$, then the common difference is

A. 3 B. 4 C. 6 D. 7 E. 12

ANSWERS

1.B 2.B 3.D 4.B 5.E 6.B 7.C 8.A 9.A 10.E
11.C 12.B 13.D 14.D 15.A

Arithmetic and Geometric Progressions

1. Let a be first term, d be common different.

$\therefore \begin{cases} a+5d = 10 & \text{——①} \\ a+10d = 30 & \text{——②} \end{cases}$

① × 2 − ②

$2a - a = 20 - 30$

$a = -10$ (B.)

2. the sum.

$\frac{n}{2}[3+3+(n-1)2] = 143$.

$\frac{n}{2}[4+2n] = 143$.

$n(2+n) = 143$

$n^2+2n-143 = 0$.

$(n+13)(n-11) = 0$.

$n = 11$ or -13 (rejected.)

(B.)

3. a, b, c are in G.P.

$\therefore \frac{a}{b} = \frac{b}{c}$.

1). $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$.

$\frac{1/a}{1/b} = \frac{1/b}{1/c}$

$\frac{b}{a} = \frac{c}{b}$.

$\therefore \frac{a}{b} = \frac{b}{c}$.

$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P.

2). a^2, b^2, c^2.

$\frac{a^2}{b^2} = \frac{b^2}{c^2}$

$(\frac{a}{b})^2 = (\frac{b}{c})^2$.

$\frac{a}{b} = \frac{b}{c}$.

$\therefore a^2, b^2, c^2$ are in G.P.

(3.) $\log_{10} a, \log_{10} b, \log_{10} c$.

$\frac{a}{b} = \frac{b}{c}$

$b^2 = ac$

$2\log b = \log(a \cdot c)$

$\log b = \frac{1}{2}(\log a + \log c)$

$\therefore \log a, \log b, \log c$ in A.P.

(1), (2) & (3) are true. (D.)

4. $\frac{10}{2}[2a+(10-1)4] = 120$

$5[2a+36] = 120$

$2a+36 = 24$

$a = -6$. (B.)

5. If $a \neq \pm 1$.

$1 + a^2 + a^4 + \cdots + a^{2n}$,

a^2 is common ratio.

$= \frac{(1)(1-a^{2n+2})}{1-a^2}$

$= \frac{1-a^{2n+2}}{1-a^2}$. (E.)

6. (1). $\log_{10}3, \log_{10}9, \log_{10}27$,

$\frac{\log_{10}3}{\log_{10}9} = \frac{\log_{10}3}{2\log_{10}3} = \frac{1}{2}$.

$\frac{\log_{10}9}{\log_{10}27} = \frac{2\log 3}{3\log 3} = \frac{2}{3} \neq \frac{1}{2}$.

$\therefore \frac{\log_{10}3}{\log_{10}9} \neq \frac{\log_{10}9}{\log 27}$.

they are not in G.P.

(2). 0.9, 0.99, 0.999, 0.9999.

$\frac{0.9}{0.99} = \frac{10}{11}$.

$\frac{0.99}{0.999} = \frac{0.11}{0.111} = \frac{110}{111} \neq \frac{10}{11}$

$\therefore \frac{0.9}{0.99} \neq \frac{0.99}{0.999}$.

$\therefore$ they are not in G.P.

(3). 1, −3, 9, −27.

$\frac{1}{-3} = -\frac{1}{3}$

$\frac{-3}{9} = -\frac{1}{3}$

$\frac{9}{-27} = -\frac{1}{3}$.

$\therefore \frac{1}{-3} = \frac{-3}{9} = \frac{9}{-27}$.

$\therefore$ 1, −3, 9, −27 are in G.P.

(B.)

7. Let a be first term. R be common ratio.

$aR = -12$.

$aR^4 = 40\frac{1}{2}$.

$\therefore \frac{aR^4}{aR} = \frac{81/2}{-12}$

$R^3 = -27/8$.

$R = -3/2$.

$\therefore a(-3/2) = -12$

$a = 8$. (C.)

8. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P.

$\therefore \frac{1/a}{1/b} = \frac{1/b}{1/c}$.

$\frac{b}{a} = \frac{c}{b}$

$\therefore b^2 = ac$. (A.)

9. x, y, z are in A.P.

$2y = x + z$.

(I) $x+10, y+10, z+10$.

$x+10+z+10$.

$= x+z+20$.

$= 2y+20$.

$= 2(y+10)$.

$\therefore$ they are in A.P.

7.eem.

(II) $10x, 10y, 10z$.

$10x + 10z$

$= 10(x+z)$

$= 10(2y)$

$= 2(10y)$

$\therefore$ they are in A.P.

(III). $x^2, y^2 + z^2$.

$x^2 + z^2$

$= (x+z)^2 - 2xz$

$= (2y)^2 - 2xz$

$= 2(2y^2 - xz)$

$\neq 2 \cdot y^2$

$\therefore$ they are not in A.P.

(I) & (II) only. (A.)

10. area of 1st square $= 1^2 = 1$

" 2nd square $= (\frac{1}{2})^2 = \frac{1}{4}$

" 3rd square $= (\frac{1}{4})^2 = \frac{1}{16}$

$\therefore$ the sum of areas.

$= 1 + \frac{1}{4} + \frac{1}{16} + \cdots$

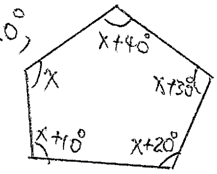
$= \dfrac{1}{1 - \frac{1}{4}}$

$= \dfrac{1}{3/4} = \frac{4}{3}$ (E.)

11. Let the smallest angles be x

$x, x+10°, x+20°,$

$x+30°, x+40°$.



$\therefore 5x + 100° = 180° \times 3$

$5x = 440°$

$x = 88°$ (C.)

..., x, x, 3x are in G.P.

$\dfrac{-x}{x} = \dfrac{x}{3x^2}$

$-1 = \dfrac{1}{3x}$

$3x = -1$

$x = -\frac{1}{3}$ (B.)

13. $\log_{10} x, \log_{10} y, \log_{10} z$.

$\log_{10} x + \log_{10} z = 2\log_{10} y$

$\log_{10}(x \cdot z) = \log_{10} y^2$

$\therefore y^2 = x \cdot z$ (D.)

14.(1). $5, 0.5, 0.05, 0.005, \ldots$

$\dfrac{5}{0.5} = 10$.

$\dfrac{0.5}{0.05} = 10 = \dfrac{5}{0.5}$.

$\therefore$ they are in G.P.

(2). $\log 5, \log 50, \log 500, \ldots$

$\dfrac{\log 5}{\log 50} = \dfrac{\log 5}{\log 5 + \log 10} = \dfrac{\log 5}{\log 5 + 1}$

$\dfrac{\log 50}{\log 500} = \dfrac{\log 5 + 1}{\log 5 + 2} \neq \dfrac{\log 5}{\log 50}$

$\therefore$ they are not in G.P.

(3). $5, 5\sin 70°, 5(\sin 70°)^2, \ldots$

$\dfrac{5}{5\sin 70°} = \dfrac{1}{\sin 70°}$,

$\dfrac{5\sin 70°}{5(\sin 70°)^2} = \dfrac{1}{\sin 70°}$

$= \dfrac{5}{5\sin 70°}$

$\therefore$ they are in G.P.

(1) & (3) only (D.)

15. p, q, r, s. are in A.P.

$p, p+d, p+2d, p+3d$.

$p + p + d = 8$

$2p + d = 8$ —— ①

$(p+2d) + (p+3d) = 20$

$2p + 5d = 20$ —— ②

② − ①.

$4d = 12$

$d = 3$. (A.)