

Figure 7 shows the graph of  $y = 25x - x^3$  for  $0 < x < 5$ . By adding a suitable straight line to the graph, solve the equation

$$30 = 25x - x^3,$$

where  $0 < x < 5$ . Give your answers correct to 2 significant figures.

Figure 8 shows a right pyramid with a square base  $ABCD$ .  $AB = b$  units and  $AE = 5$  units. The height of the pyramid is  $h$  units and its volume is  $V$  cubic units.

(i) Express  $b$  in terms of  $h$ .

Hence show that  $V = \frac{2}{3}(25h - h^3)$ . (3 marks)

(ii) Using (a), find the two values of  $h$  such that  $V = 20$ .

(Your answers should be correct to 2 significant figures.) (2 marks)

(iii) Use the "method of magnification" to find the smaller value of  $h$  in (b) (ii) correct to 3 significant figures. (3 marks)

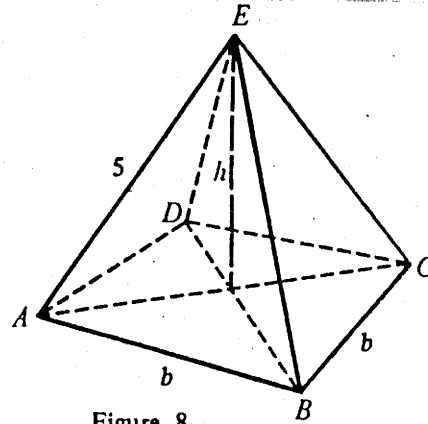


Figure 8

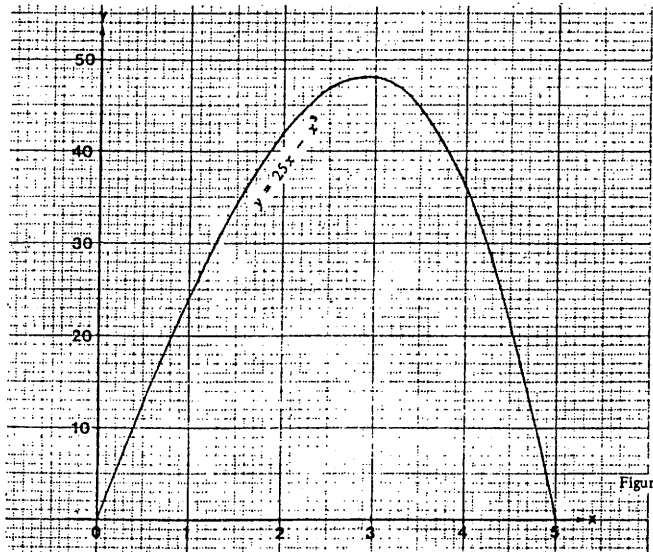


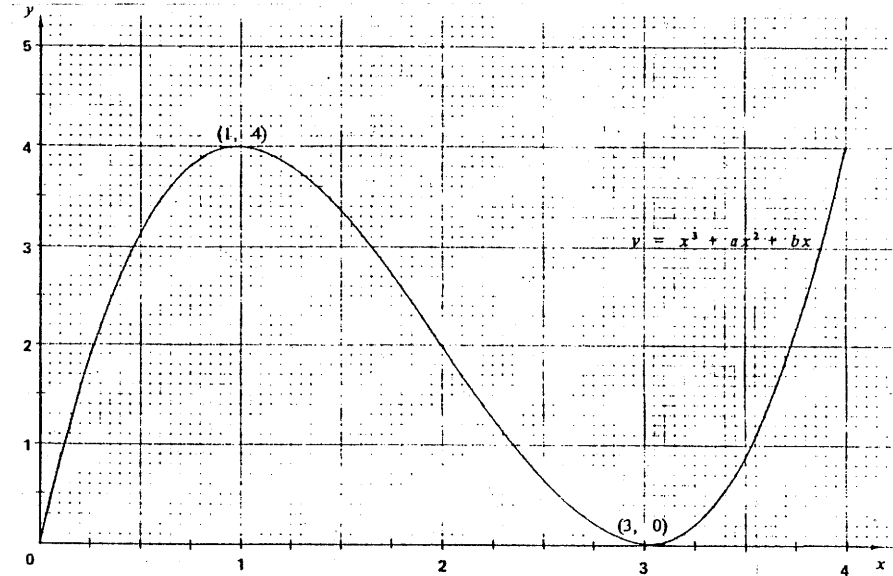
Figure 7

2(81) The relationship between the height  $y$  of a flying object and time  $x$  is given by

$$y = x^3 + ax^2 + bx,$$

where  $y$  is in kilometres above sea-level and  $x$  is the number of hours after 12:00 noon.

Figure 5 shows the graph of  $y = x^3 + ax^2 + bx$ .



Using Figure

- find the values of  $a$  and  $b$ ,
- write down the time interval in which the flying object is descending.

At 1:00 p.m., a balloon rises vertically from sea-level with a constant speed of 4 km/h.

- Add a straight line to Figure 5 to show the relationship between the height of the balloon and time  $x$ .
- Hence, write down the value of  $x$  to 2 significant figures, for which the balloon and the flying object are at the same height.

Use the method of magnification to find the value of  $x$  in (b) (ii) to 3 significant figures.

Three gold cubes have sides of length  $(x + 1)$  cm,  $x$  cm and  $(x - 1)$  cm respectively (see Figure 8(a)).

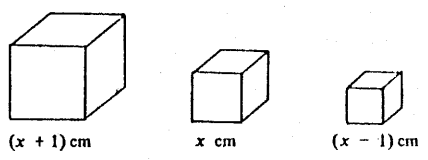
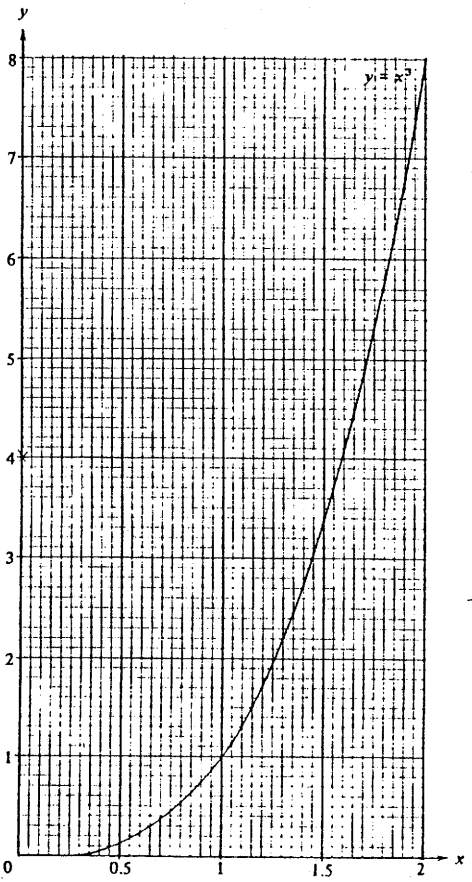


Figure 8(a)

- (a) (i) Find, in terms of  $x$ , the total volume of these three cubes.
  - (ii) If the total volume of these three cubes is  $12 \text{ cm}^3$ , show that  $x^3 + 2x - 4 = 0$ .
- (5 marks)



- (b) Figure 8(b) shows the graph of  $y = x^3$  for  $0 \leq x \leq 2$ .
  - (i) Draw a suitable straight line in Figure 8(b) to solve the equation  $x^3 + 2x - 4 = 0$  for  $0 \leq x \leq 2$ . Give the root of the equation correct to 2 significant figures.
  - (ii) Use the method of magnification to find the root in (b)(i) correct to 3 significant figures.
- (7 marks)

4(83) Equal squares each of side  $k$  cm are cut from the four corners of a square sheet of paper of side 7 cm (see Figure 7(a)). The remaining part is folded along the dotted lines to form a rectangular box as shown in Figure 7(b).

- (a) Show that the volume  $V$  of the rectangular box, in  $\text{cm}^3$ , is  $V = 4k^3 - 28k^2 + 49k$ . (3 marks)
- (b) Figure 7(c) shows the graph of  $y = 4x^3 - 28x^2 + 49x$  for  $0 \leq x \leq 5$ . Draw a suitable straight line in Figure 7(c) and use it to find all the possible values of  $x$  such that  $4x^3 - 28x^2 + 49x - 20 = 0$ . (Give the answers to 1 decimal place.) (4 marks)
- (c) Using the results of (a) and (b), deduce the values of  $k$  such that the volume of the box is  $20 \text{ cm}^3$ . (Give the answers to 1 decimal place.) (2 marks)
- (d) By the method of magnification, find the smaller value of  $k$  in (c) to two decimal places. (3 marks)

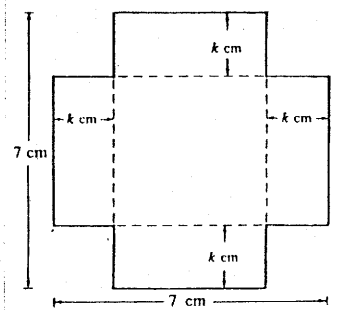


Figure 7(a)

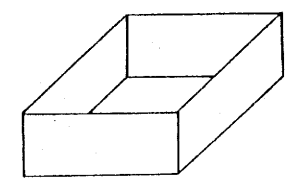


Figure 7(b)

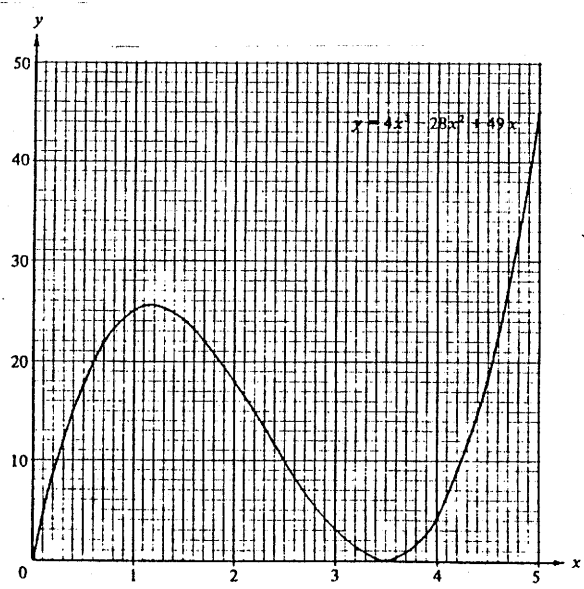


Figure 7(c)

54)(a) Figure 5 shows the graph of  $y = x^3 + x^2$  for  $-1 \leq x \leq 2$ .

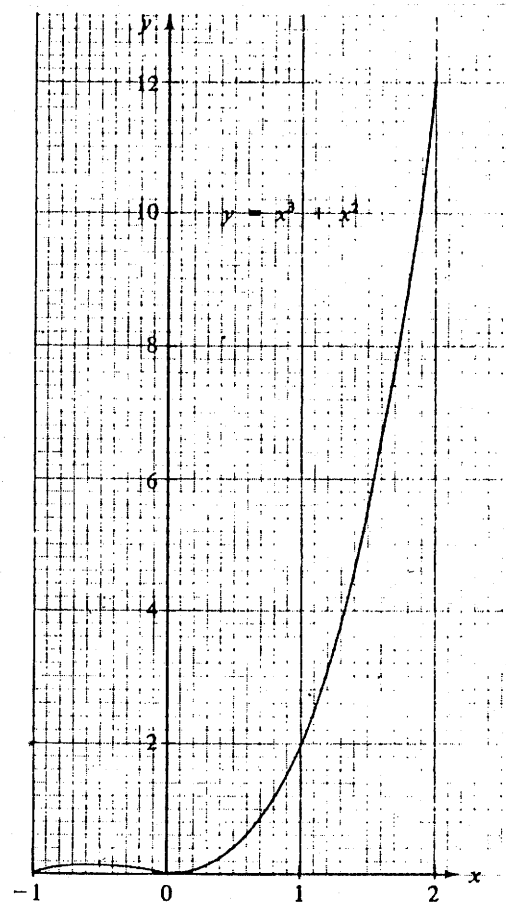
- (i) Draw a suitable straight line in Figure 5 and use it to find a root of the equation

$$x^3 + x^2 + x - 4 = 0.$$

(Give your answer correct to 1 decimal place.)

- (ii) By the method of magnification, find the root obtained in (i) correct to 2 decimal places.

(7 marks)



6(85) Figure 6 shows the graph of

$$y = x^3 + x \text{ for } -1 \leq x \leq 2.$$

- (a) (i) Draw a suitable straight line in Figure 6 and hence find, correct to 1 decimal place, the real root of the equation

$$x^3 + x - 1 = 0.$$

- (ii) By the method of magnification, find the real root of the equation in (i), correct to 2 decimal places.

(7 marks)

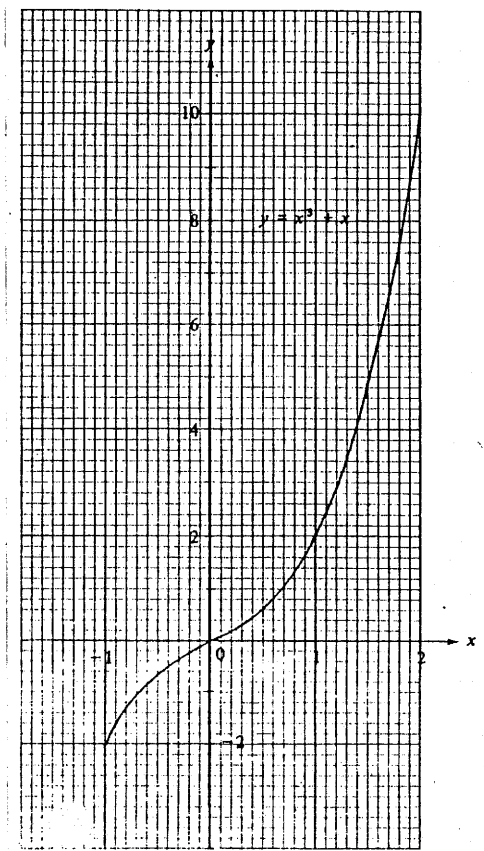
- (b) (i) Expand and simplify the expression

$$(x + 1)^4 - (x - 1)^4.$$

- (ii) Using the result in (a)(ii), find, correct to 2 decimal places, the real root of the equation

$$(x + 1)^4 - (x - 1)^4 = 8.$$

(5 marks)



1(06)

Figure 6 shows the graph of  $y = x^4 + x$  for  $0 \leq x \leq 2$ .

- (a) Draw a suitable straight line on the figure and use it to find the approximate value of the root of the equation

$$x^4 - x - 1 = 0$$

in the interval  $0 \leq x \leq 2$ , correct to 1 decimal place.

(4 marks)

- (b) By the method of magnification, find the approximate value of the root in (a), correct to two decimal places.

(4 marks)

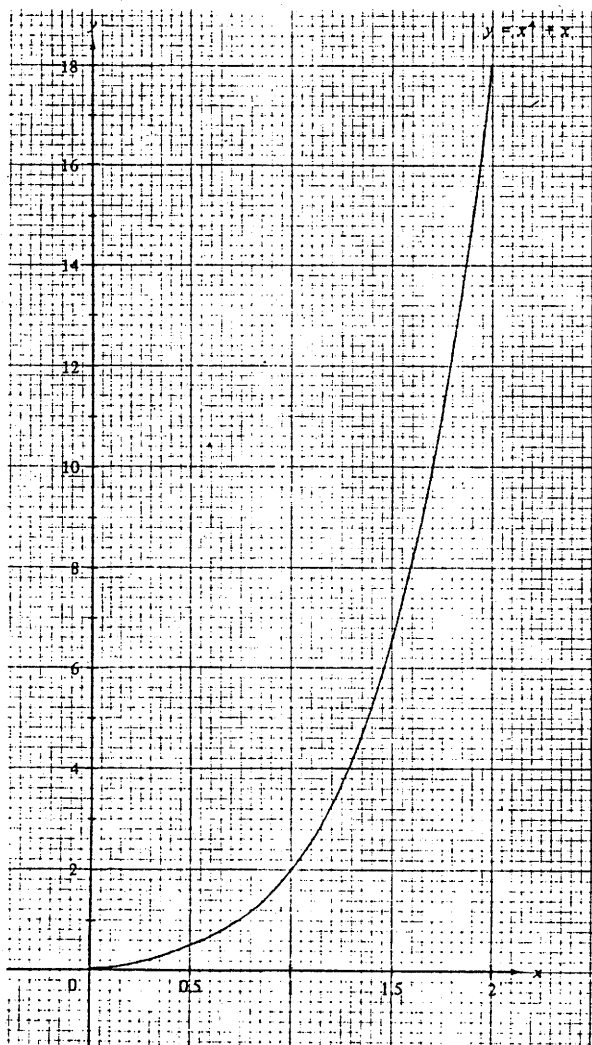
- (c) Use the result in (b) to find the approximate value of the root of the equation

$$(x - 1)^4 = x$$

in the interval  $1 \leq x \leq 3$ , correct to two decimal places.

[Hint: Put  $x = y + 1$ ]

(4 marks)



2(01)

K4

Figure 7 shows the graph of  $y = x^3 - 6x^2 + 9x$ .

- (a) By adding suitable straight lines to the figure, find, correct to 1 decimal place, the real roots of the following equations:

(i)  $x^3 - 6x^2 + 9x - 1 = 0$ ,

(ii)  $x^3 - 6x^2 + 10x - 6 = 0$ .

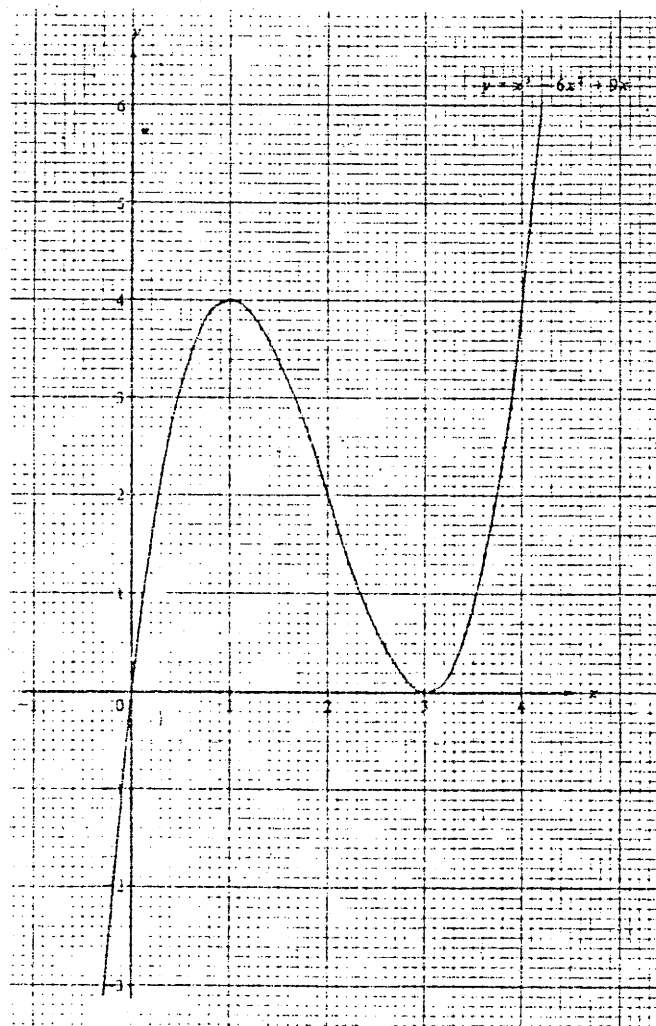
(6 marks)

- (b) By using the method of magnification, find, correct to 2 decimal places, the real root(s) of (a)(ii).

(3 marks)

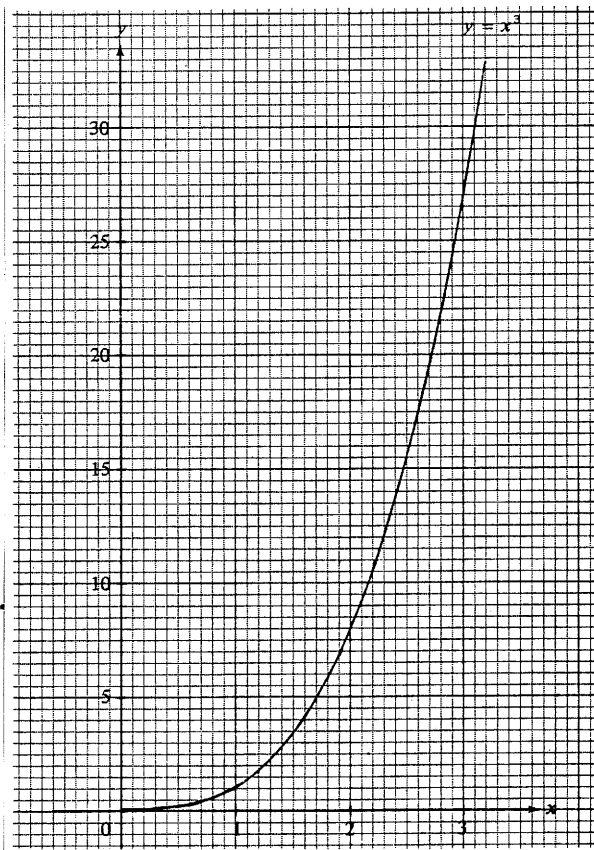
- (c) From the graph in Figure 7, find the range of values of  $k$  such that the equation  $x^3 - 6x^2 + 9x - k = 0$  has three distinct real roots.

(3 marks)



(88) Figure 7 shows the graph of  $y = x^3$  for  $x \geq 0$ .

- (a) Let  $r$  be the real root of the equation  $x^3 - \frac{4}{3}x - 6 = 0$ .
- (i) By adding a suitable straight line to the figure, find an interval of width 0.1 which contains  $r$ .
  - (ii) Use the method of bisection to find the value of  $r$  correct to two decimal places. Show your working in the form of a table. (9 marks)
- (b) Use (a) to find, correct to two decimal places, the real root of the equation  $3(t + 1)^3 - 4(t + 1) - 18 = 0$ . (3 marks)



10(89)

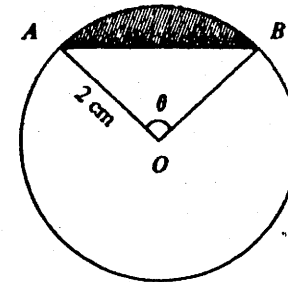


Figure 5

In Figure 5,  $O$  is the centre of a circle of radius 2 cm.  $A$  and  $B$  are two points on the circle such that  $\angle AOB = \theta$  radians, where  $0 < \theta < \pi$ .

- (a) (i) Find the area of  $\triangle OAB$  in terms of  $\theta$ .
  - (ii) Find the value of  $\theta$  for which the area of  $\triangle OAB$  is the greatest. (2 marks)
- (b) If the area of the shaded segment is  $2 \text{ cm}^2$ , show that
- $$\theta - \sin \theta - 1 = 0.$$
- (3 marks)
- (c) Let  $f(\theta) = \theta - \sin \theta - 1$  and  $\alpha$  be the root of  $f(\theta) = 0$ . Show that  $\alpha$  lies between 0 and 3. (2 marks)
- (d) Using the method of bisection, find the value of  $\alpha$  correct to one decimal place. (5 marks)

(90) A solid right circular cylinder has radius  $r$  and height  $h$ . The volume of the cylinder is  $V$  and the total surface area is  $S$ .

(a) (i) Express  $S$  in terms of  $r$  and  $h$ .

(ii) Show that  $S = 2\pi r^2 + \frac{2V}{r}$ .

(3 marks)

(b) Given that  $V = 2\pi$  and  $S = 6\pi$ , show that  $r^3 - 3r + 2 = 0$ .

Hence find the radius  $r$  by factorization.

(4 marks)

(c) Given that  $V = 3\pi$  and  $S = 10\pi$ , find the radius  $r$  ( $1 < r < 2$ ) by the method of bisection, correct to 1 decimal place.

(5 marks)

Method of Bisection.

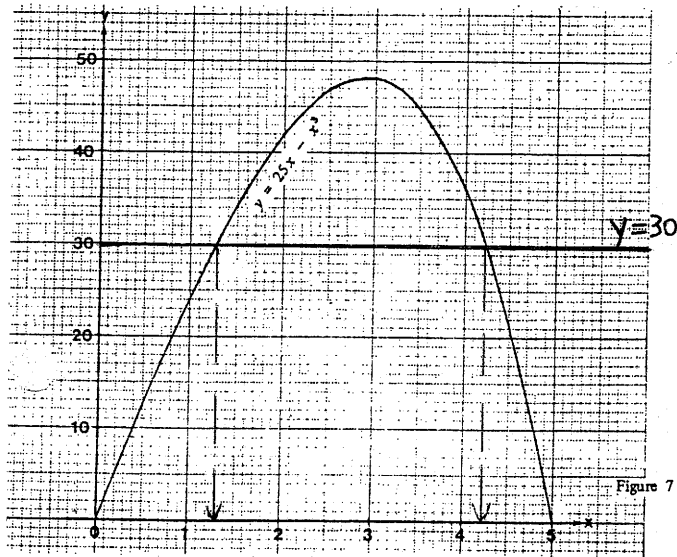
80) 1.  $y = 25x - x^3$  — (1)

$30 = 25x - x^3$  — (2)

(1) - (2)

$\therefore y - 30 = 0$

which is the line added on the graph,



From the graph, the soln. of eqn.

$30 = 25x - x^3$

are 1.3 and 4.2 (2 sig. fig.)

b) since ABCD is a square,

$\therefore \angle AOB = 90^\circ$

$OA = \sqrt{AE^2 - OE^2}$   
 $= \sqrt{25 - h^2}$

In  $\triangle OAB$

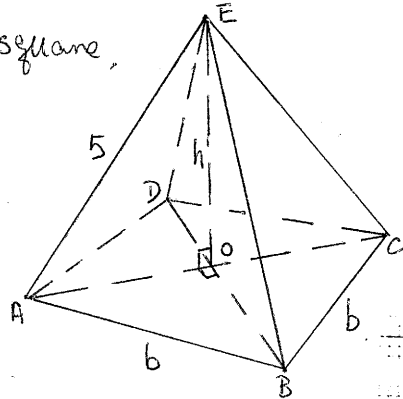
$AB^2 = OA^2 + OB^2$

$b = \sqrt{2(25 - h^2)}$

$V = \frac{1}{3} \times h \times \text{base area.}$

$V = \frac{1}{3} (h) (2(25 - h^2))$

$V = \frac{2}{3} (25h - h^3)$



b) For  $V = 20$

f.1

$\therefore 20 = \frac{2}{3} (25h - h^3)$

$\Rightarrow 30 = 25h - h^3$

By (a)  $h = 1.3$  or  $4.2$  (2 sig. fig.)

b) For the value of  $h = 1.3$

Let  $f(h) = h^3 - 25h + 30$

$f(1.2) = 1.728 > 0$

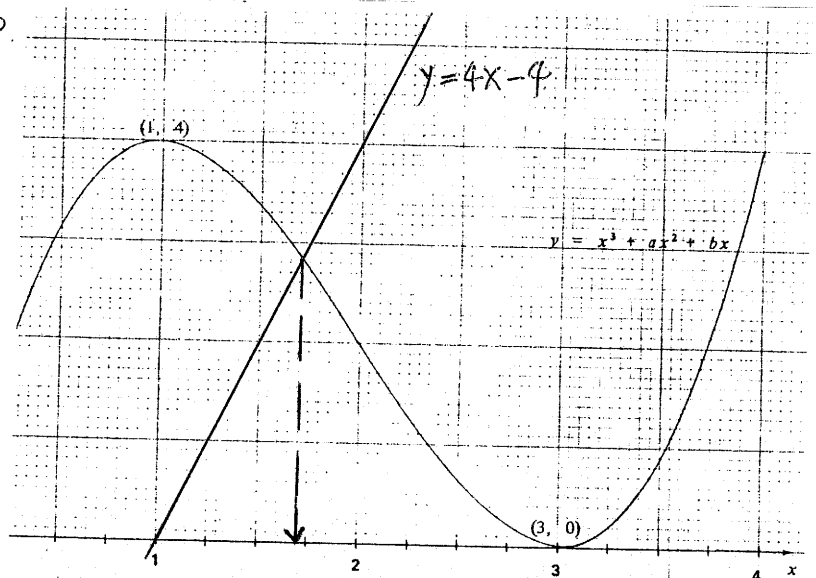
$f(1.4) = -2.256 < 0$

By method of Bisection.

interval of x.	mid-value	sign of f(x)
$1.2 \leq x \leq 1.4$	1.3	-
$1.2 \leq x \leq 1.3$	1.25	+
$1.25 \leq x \leq 1.3$	1.275	+
$1.275 \leq x \leq 1.3$	1.2875	-
$1.275 \leq x \leq 1.2875$	1.28125	+
$1.28125 \leq x \leq 1.2875$	1.284375	+
$1.284375 \leq x \leq 1.2875$	1.2859375	-
$1.284375 \leq x \leq 1.2859375$	1.28515625	-
$1.284375 \leq x \leq 1.28515625$	1.284765625	+
$1.284765625 \leq x \leq 1.28515625$	1.284960938	-
$1.284765625 \leq x \leq 1.284960938$		

$\therefore h \approx 1.28$  (3 sig. fig.)

2(81).



a) i)  $y = x^3 + ax^2 + bx$

From the graph,

For (1, 4)

$$4 = (1)^3 + a(1)^2 + b(1)$$

$$\therefore a + b - 3 = 0 \quad \text{--- ①}$$

For (3, 0)

$$0 = 3^3 + a(3)^2 + b(3)$$

$$3a + b + 9 = 0 \quad \text{--- ②}$$

$$\text{②} - \text{①}$$

$$2a = -12$$

$$a = -6$$

$$\therefore b = 9$$

out. of syllabus!

b) Let  $h$  be height of the balloon.

$$\therefore h = 4x \quad \leftarrow (\text{distance} = \text{speed} \times \text{time})$$

Let  $y = mx + c$  be the line graph.

At 1:00 p.m.,  $h = 0, x = 1$ .

$$0 = m(1) + c$$

$$m + c = 0 \quad \text{--- ①}$$

At 2:00 p.m.  $h = 4 = x = 2$ .

$$4 = m(2) + c$$

$$2m + c = 4 \quad \text{--- ②}$$

$$\text{②} - \text{①}$$

$$\therefore m = 4$$

$$c = -4$$

$\therefore y = 4x - 4$  is the line added on the graph.

From the graph,

the balloon and the flying object are at the same height at  $x = 1.7$  (2 sig. fig.)

$$\begin{cases} y = x^3 - 6x^2 + 5x + 4 & \text{--- ①} \\ y = 4x - 4 & \text{--- ②} \end{cases}$$

$$\text{①} - \text{②}$$

$$\therefore x^3 - 6x^2 + 5x + 4 = 0$$

$$\text{Let } f(x) = x^3 - 6x^2 + 5x + 4$$

$$f(1.65) = 0.407 > 0$$

$$f(1.75) = -0.266 < 0$$

interval	mid-value	sign of $f(x)$
$1.65 \leq x \leq 1.75$	1.7	+
$1.7 \leq x \leq 1.75$	1.725	-
$1.7 \leq x \leq 1.725$	1.7125	-
$1.7 \leq x \leq 1.7125$	1.70625	+
$1.70625 \leq x \leq 1.7125$		

By method of Bisection,

$$x = 1.71 \quad (3 \text{ sig. fig.})$$

3(82)

a) the total volume of the cubes.

$$V = (x+1)^3 + x^3 + (x-1)^3$$

$$V = x^3 + 3x^2 + 3x + 1 + x^3 + x^3 - 3x^2 + 3x - 1$$

$$V = 3x^3 + 6x$$

ii) For  $V = 12$ .

$$\therefore 3x^3 + 6x = 12$$

$$x^3 + 2x - 4 = 0$$

$$\text{b) } \begin{cases} y = x^3 \\ x^3 + 2x - 4 = 0 \end{cases}$$

$$\begin{cases} x^3 = y & \text{--- ①} \\ x^3 + 2x - 4 = 0 & \text{--- ②} \end{cases}$$

$$\text{②} - \text{①}$$

$\therefore 2x + y - 4 = 0$  is the line added on the graph.



b) From the graph,  
the soln. of eqn. is 1.2.

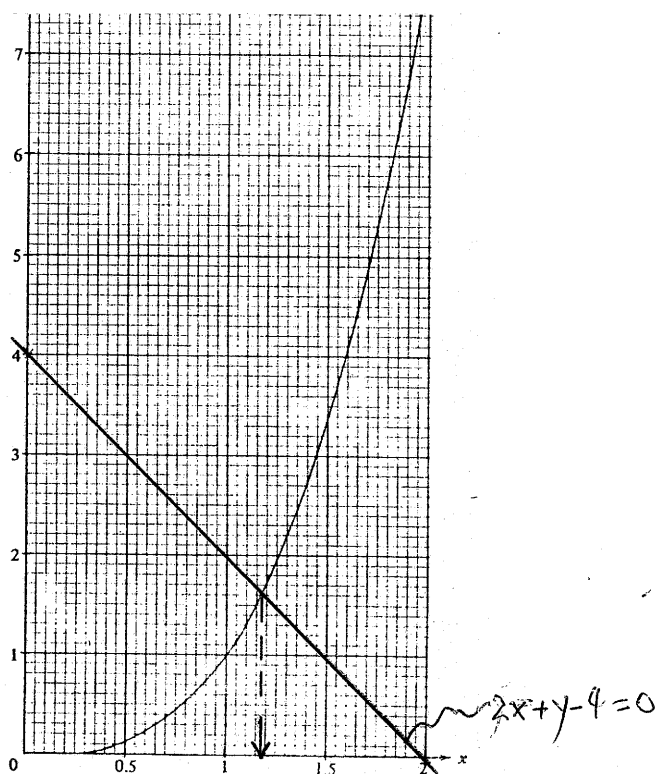
b.2) Let.  $f(x) = x^3 + 2x - 4$ .

$$f(1.15) = -0.179 < 0$$

$$f(1.25) = 0.453 > 0$$

interval	mid-value	sign of $f(x)$
$1.15 \leq x \leq 1.25$	1.2	+
$1.15 \leq x \leq 1.2$	1.175	-
$1.175 \leq x \leq 1.2$	1.1875	+
$1.175 \leq x \leq 1.1875$	1.18125	+
$1.175 \leq x \leq 1.18125$		

By method of Bisection,  
 $x = 1.18$  (3 sig. fig.)



4(83). a) the length & the width of the box  
 $= 7 - 2k$ .

$\therefore$  the volume of the box

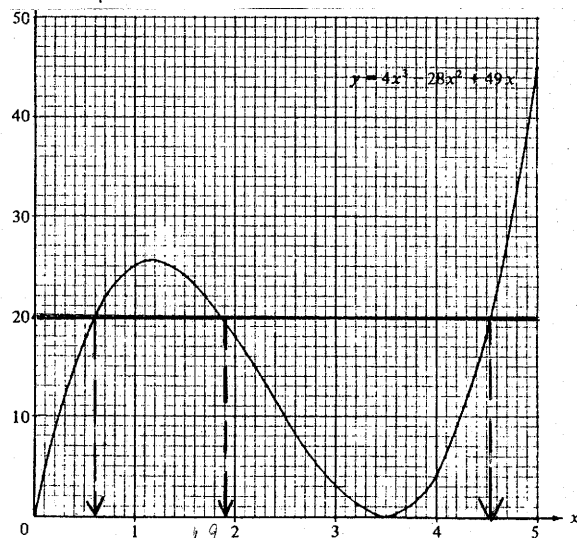
$$V = (7 - 2k)^2 k$$

$$V = 4k^3 - 28k^2 + 49k$$

$$b) \begin{cases} y = 4x^3 - 28x^2 + 49x & \text{--- (1) P.3.} \\ 0 = 4x^3 - 28x^2 + 49x - 20 & \text{--- (2)} \end{cases}$$

(1) - (2)

$y = 20$  is the line added on the graph.



From the graph, the roots of eqn. are 0.6, 1.9 or 4.5. (1 dec. pl.)

c) For  $V = 20$ .

$$\text{From (a)} \quad 20 = 4k^3 - 28k^2 + 49k$$

$$\therefore 4k^3 - 28k^2 + 49k - 20 = 0$$

From (b)

$$\therefore k = 0.6, 1.9 \text{ or } 4.5 \text{ (1 dec. pl.)}$$

d) Let.  $f(k) = 4k^3 - 28k^2 + 49k - 20$ .

For  $k \approx 0.6$ .

$$f(0.5) = -2 < 0$$

$$f(0.7) = 1.95 > 0$$

interval	mid-value	sign of $f(k)$
$0.5 \leq k \leq 0.7$	0.6	+
$0.5 \leq k \leq 0.6$	0.55	-
$0.55 \leq k \leq 0.6$	0.575	-
$0.575 \leq k \leq 0.6$	0.5875	-
$0.5875 \leq k \leq 0.6$	0.59375	+
$0.5875 \leq k \leq 0.59375$		

$$\therefore k \approx 0.59 \text{ (2 dec. pl.)}$$

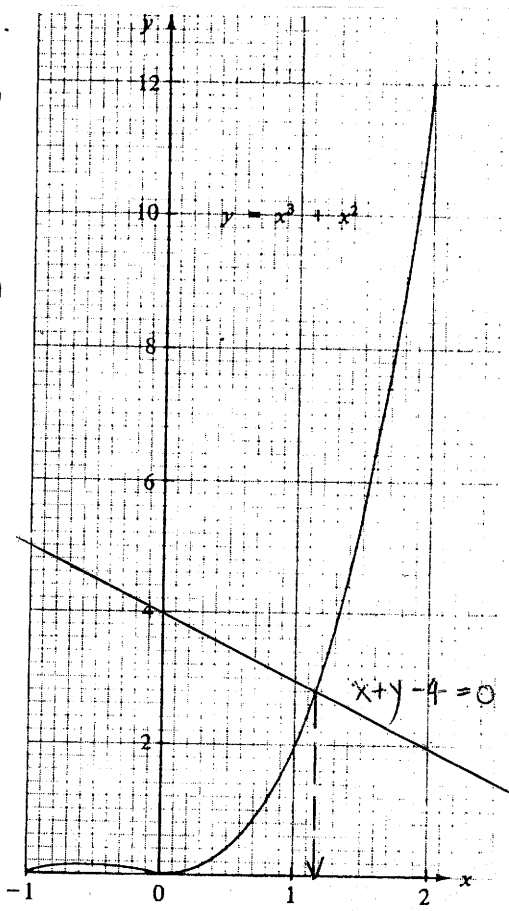
5(34).a)  $\begin{cases} y = x^3 + x^2 \\ x^3 + x^2 + x - 4 = 0 \end{cases}$

$\begin{cases} y = x^3 + x^2 & \text{--- (1)} \\ 0 = x^3 + x^2 + x - 4 & \text{--- (2)} \end{cases}$

① - ②  $y = -x + 4$

$\therefore x + y - 4 = 0$  is the line added on the graph.

$\therefore$  the root of the eqn. is 1.2 (1 dec. pl.)



Let  $f(x) = x^3 + x^2 + x - 4$

$f(1.1) = -0.354 < 0$

$f(1.2) = 0.368 > 0$

interval	mid-value	sign of $f(x)$
$1.1 \leq x \leq 1.2$	1.15	-
$1.15 \leq x \leq 1.2$	1.175	+
$1.15 \leq x \leq 1.175$	1.1625	+
$1.15 \leq x \leq 1.1625$	1.15625	+
$1.15 \leq x \leq 1.15625$	1.153125	+
$1.15 \leq x \leq 1.153125$		

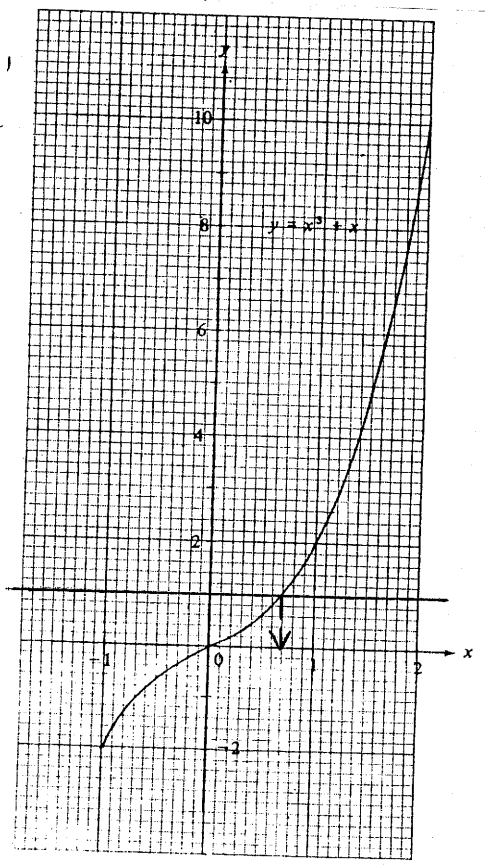
By method of bisection, the root is 1.15 (2 dec. pl.)

6(35).a.i)  $\begin{cases} y = x^3 + x \\ x^3 + x - 1 = 0 \end{cases}$

$\begin{cases} y = x^3 + x & \text{--- (1)} \\ 0 = x^3 + x - 1 & \text{--- (2)} \end{cases}$

① - ②  $y = 1$  is the line added on the graph.

From the graph, the root of the eqn. is 0.7 (1 dec. pl.)



Let  $f(x) = x^3 + x - 1 = 0$

$f(0.6) = -0.184 < 0$

$f(0.7) = 0.043 > 0$

interval	mid-value	sign of $f(x)$
$0.6 \leq x \leq 0.7$	0.65	-
$0.65 \leq x \leq 0.7$	0.675	-
$0.675 \leq x \leq 0.7$	0.6875	+
$0.675 \leq x \leq 0.6875$	0.68125	-
$0.68125 \leq x \leq 0.6875$	0.684375	+
$0.68125 \leq x \leq 0.684375$		

By method of bisection, the root of the eqn. is 0.68 (2 dec. pl.)

6(85) cont.

b) i).  $(x+1)^4 - (x-1)^4$

$$= [(x+1)^2 + (x-1)^2][(x+1)^2 - (x-1)^2]$$

$$= [x^2 + 2x + 1 + x^2 - 2x + 1][x^2 + 2x + 1 - x^2 + 2x - 1]$$

$$= 2(x^2 + 1)(4x)$$

$$= 8x^3 + 8x$$

ii). the ext.

$$(x+1)^4 - (x-1)^4 = 8$$

$$\therefore 8x^3 + 8x = 8$$

$$x^3 + x = 1$$

$$x^3 + x - 1 = 0$$

the ext. in a ii)

$\therefore$  the root of ext is also 0.68 (2 dec. pl.)

7(86).  $\begin{cases} y = x^4 + x \\ x^4 - x - 1 = 0 \end{cases}$

a)  $\begin{cases} y = x^4 + x & \text{--- (1)} \\ 0 = x^4 - x - 1 & \text{--- (2)} \end{cases}$

$\text{--- (1) - (2)}$

$$y = 2x + 1$$

$\therefore 2x - y + 1 = 0$  is the line added on

the graph.

from the graph, the root of the ext is 1.2

(1 dec. pl.)

b) Let  $f(x) = x^4 - x - 1$

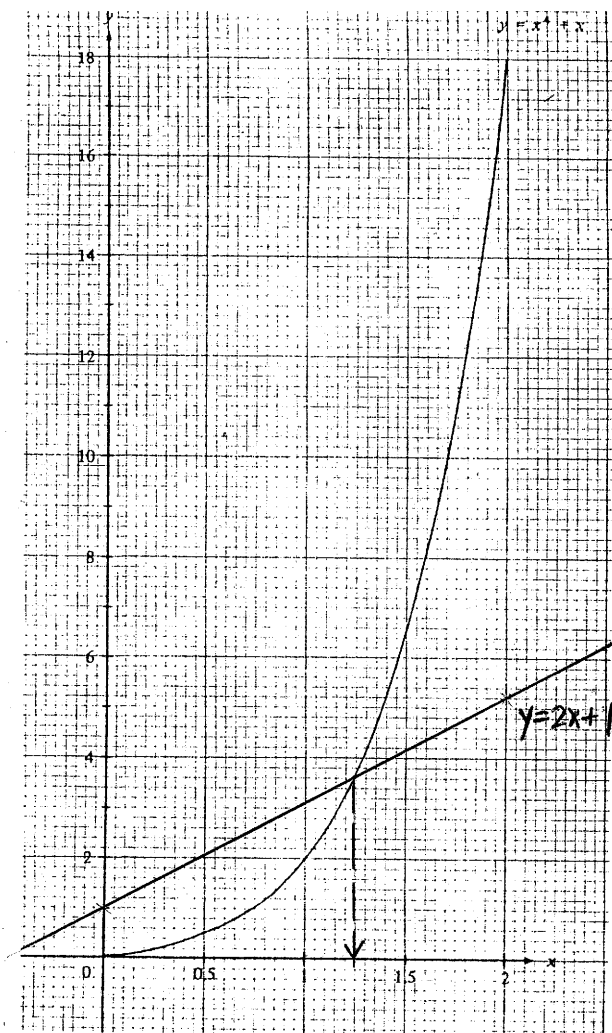
$$f(1.2) = -0.1264 < 0$$

$$f(1.3) = 0.5561 > 0$$

interval	mid-value	sign of $f(x)$
$1.2 \leq x \leq 1.3$	1.25	+
$1.2 \leq x \leq 1.25$	1.225	+
$1.2 \leq x \leq 1.225$	1.2125	-
$1.2125 \leq x \leq 1.225$	1.21875	-
$1.21875 \leq x \leq 1.225$		

By method of bisection,

the approximate root is 1.22 (2 dec. pl.)



c) the ext.

$$(x-1)^4 = x$$

put.  $x = y + 1$

$$\therefore (y+1-1)^4 = y+1$$

$$\therefore y^4 = y+1$$

$$y^4 - y - 1 = 0$$

is the same ext in (a),

By using part b)

the approximate value is 1.22 (2 dec. pl.)

3(87) a)  $y = x^3 - 6x^2 + 9x$

i)  $x^3 - 6x^2 + 9x - 1 = 0$

$\therefore \begin{cases} y = x^3 - 6x^2 + 9x & \text{--- ①} \\ 0 = x^3 - 6x^2 + 9x - 1 & \text{--- ②} \end{cases}$

①-②  $y = 1$  is the line added on the graph.

From the graph, the real roots are 0.2, 2.3 or 3.5. (1 dec. pl.)

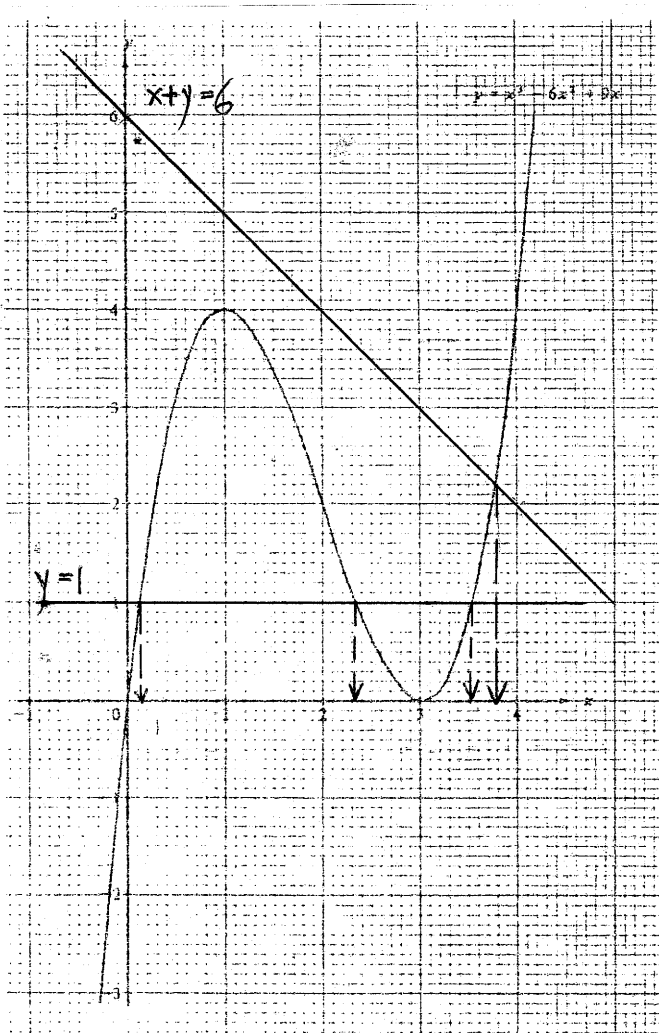
ii)  $x^3 - 6x^2 + 10x - 6 = 0$

$\begin{cases} y = x^3 - 6x^2 + 9x & \text{--- ①} \\ 0 = x^3 - 6x^2 + 10x - 6 & \text{--- ③} \end{cases}$

①-③  $y = -x + 6$

$\therefore x + y = 6$  is the line added on the graph.

From the graph, the real root is 3.8 (1 dec. pl.)



b) let  $f(x) = x^3 - 6x^2 + 10x - 6$

P. 6

$f(3.7) = -0.487 < 0$

$f(3.8) = 0.232 > 0$

interval	mid-value	sign of $f(x)$
$3.7 \leq x \leq 3.8$	3.75	-
$3.75 \leq x \leq 3.8$	3.775	+
$3.75 \leq x \leq 3.775$	3.7625	-
$3.7625 \leq x \leq 3.775$	3.76875	-
$3.76875 \leq x \leq 3.775$	3.771875	+
$3.76875 \leq x \leq 3.771875$		

By method of bisection,

the real root is 3.77 (2 dec. pl.)

c) From the graph,

the range of values of  $k$

is  $0 < k \leq 4$ .

4(88) a)  $y = x^3$

i)  $x^3 - \frac{4}{3}x - 6 = 0$

$\begin{cases} y = x^3 & \text{--- ①} \\ 0 = x^3 - \frac{4}{3}x - 6 & \text{--- ②} \end{cases}$

①-②  $y = \frac{4}{3}x + 6$

$3y = 4x + 18$

$4x - 3y + 18 = 0$  is the line added on the graph.

$\therefore$  The interval of  $r$  is

$2 \leq r \leq 2.1$

let  $f(x) = x^3 - \frac{4}{3}x - 6$

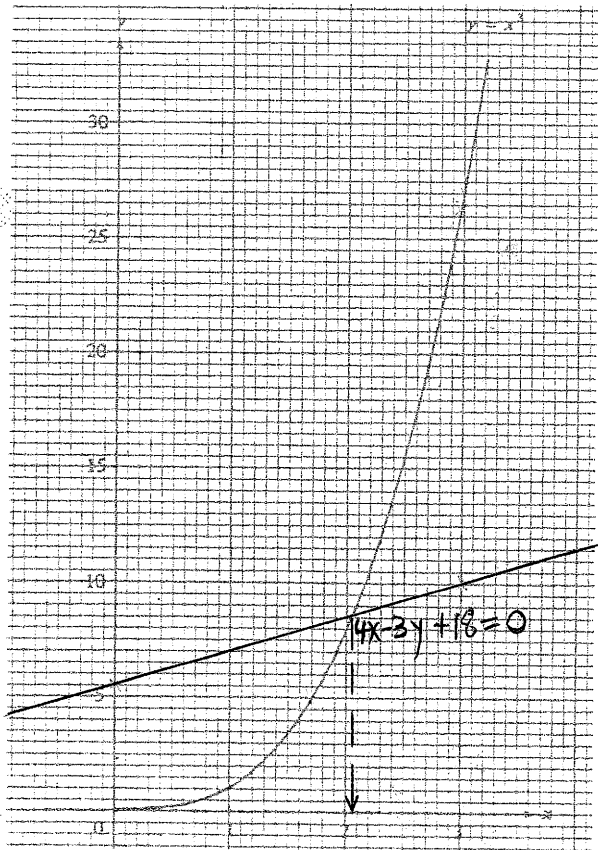
$f(2) = -0.67 < 0$

$f(2.1) = 0.461 > 0$

a) By method of bisection.

interval	mid-value	sign of f(x)
$2 \leq x \leq 2.1$	2.05	-
$2.05 \leq x \leq 2.1$	2.075	+
$2.05 \leq x \leq 2.075$	2.0625	+
$2.05 \leq x \leq 2.0625$	2.05625	-

$\therefore$  the value of  $r$  is 2.06 (2 dec. pl.)



The eqn.

$$3(t+1)^3 - 4(t+1) - 18 = 0$$

Let  $x = t+1$ .

$$\therefore 3x^3 - 4x - 18 = 0$$

$$x^3 - \frac{4}{3}x - 6 = 0 \text{ is the eqn}$$

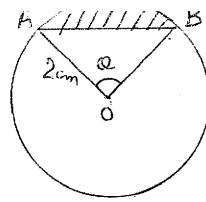
same as (a).

$\therefore$  the soln. of eqn.

$$x = 2.06$$

$$\therefore t+1 = 2.06$$

$$\Rightarrow t = 1.06 \text{ (2 dec. pl.)}$$



a) (i) area of  $\Delta OAB$

$$= \frac{1}{2} (OA)(OB) \sin \angle AOB$$

$$= \frac{1}{2} (2)(2) \sin \theta$$

$$= 2 \sin \theta$$

(ii) the area of  $\Delta OAB$  is greatest,

when  $\theta = \frac{\pi}{2}$ , since  $\sin \theta$  is max.

when  $\theta = \frac{\pi}{2}$ .

b) area of sector OAB

$$= \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (2)^2 \theta$$

$$= 2\theta$$

area of shaded segment

$$= \text{area of sector OAB} - \text{area of } \Delta OAB$$

$$\therefore 2 = 2\theta - 2 \sin \theta$$

$$\therefore \theta - \sin \theta - 1 = 0$$

c) Let  $f(\theta) = \theta - \sin \theta - 1$

$$f(0) = 0 - \sin 0 - 1 = -1 < 0$$

$$f(3) = 3 - \sin 3 - 1 = 1.859 > 0$$

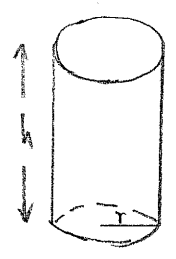
$\therefore \alpha$  lies between 0 and 3.

interval	mid-value	sign of f(x)
$0 < \theta \leq 3$	1.5	-
$1.5 \leq \theta \leq 3$	2.25	+
$1.5 \leq \theta \leq 2.25$	1.875	-
$1.875 \leq \theta \leq 2.25$	2.0625	+
$1.875 \leq \theta \leq 2.0625$	1.96875	+
$1.875 \leq \theta \leq 1.96875$	1.921875	-
$1.921875 \leq \theta \leq 1.96875$	1.9453125	+
$1.921875 \leq \theta \leq 1.9453125$		

$\therefore$  the value of  $\alpha = 1.9$  (1 dec. pl.)

11.(90). a)

$\left\{ \begin{array}{l} V - \text{volume.} \\ S - \text{total surface area.} \end{array} \right.$



$S = \text{curve surface area} + \text{base area.}$

$$S = 2\pi r h + 2\pi r^2$$

$$V = \pi r^2 h \implies h = \frac{V}{\pi r^2}$$

$$\therefore S = 2\pi r \left( \frac{V}{\pi r^2} \right) + 2\pi r^2$$

$$S = 2\pi r^2 + \frac{2V}{r}$$

b) Given that  $V = 2\pi$   
 $S = 6\pi$

$$\therefore 6\pi = 2\pi r^2 + \frac{2(2\pi)}{r}$$

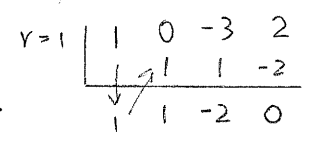
$$3 = r^2 + \frac{2}{r}$$

$$\therefore 3r = r^3 + 2$$

$$r^3 - 3r + 2 = 0$$

$$(r-1)(r^2+r-2) = 0$$

$$(r-1)^2(r+2) = 0$$



c)  $V = 3\pi, S = 10\pi$

$$10\pi = 2\pi r^2 + \frac{2(3\pi)}{r}$$

$$5 = r^2 + \frac{3}{r}$$

$$\therefore r^3 - 5r + 3 = 0$$

Let  $f(r) = r^3 - 5r + 3$

$$f(1) = -1 < 0$$

$$f(2) = 1 > 0$$

interval	mid-value	sig of f(x)
$1 \leq r \leq 2$	1.5	-
$1.5 \leq r \leq 2$	1.75	-
$1.75 \leq r \leq 2$	1.875	+
$1.75 \leq r \leq 1.875$	1.8125	-
$1.8125 \leq r \leq 1.875$	1.84375	+
$1.8125 \leq r \leq 1.84375$		

By method of bisection,  $r = 1.8$  (1 dec. pl.)