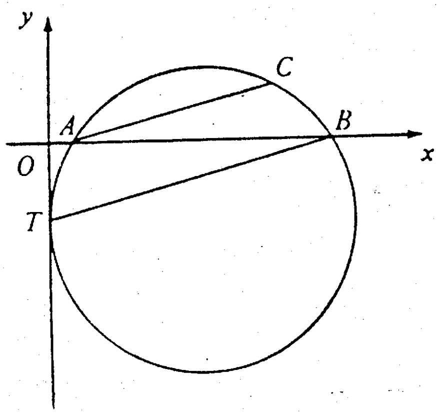
1\. (80) The circle $x^2 + y^2 - 10x + 8y + 16 = 0$ cuts the x-axis at A and B and touches the y-axis at T as shown in figure



a) Find the coordinate of A, B and T.

b) C is a point on the circle such that AC // TB.

i) Find the equation of AC.

ii) Find the coordinates of C by solving simultaneously the equation of AC and the equation of the given circle.

2\. (81) In figure (a) shows a circle of radius 15 with centre at the origin O. The line TP, of slope $\frac{3}{4}$ ($=\tan\theta$), touches the circle at T and cuts the x-axis at P.

a) Find the equation of the circle.

b) Calculate the length of OP.

c) Find the equation of the line TP.

Another circle, with centre C and radius 15 is drawn to touch TP at P. { see figure (b) }.

d) Find the equation of the line OC.

e) Find the equation of the circle with centre C.

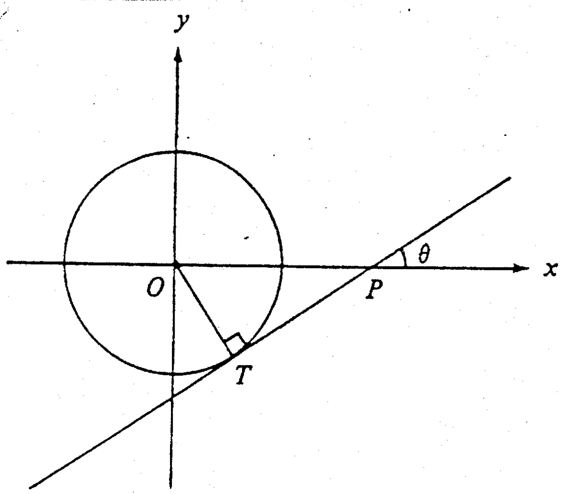


Figure (a)

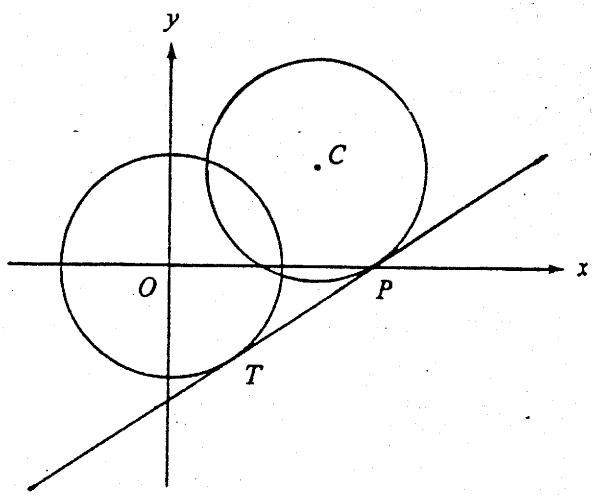


Figure (b)

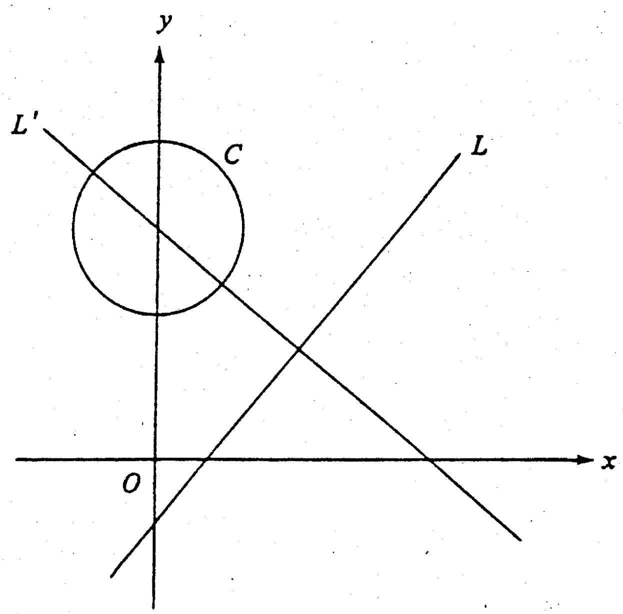
3. (82) In figure, C is the circle $x^2 + y^2 - 14y + 40 = 0$ and L is the line $4x - 3y - 4 = 0$

a) Find the radius and the coordinates of the centre of the circle C.

b) The line L′ passes through the centre of the circle C and is perpendicular to the given line L.
Find the equation of the line L′.

c) Find the coordinates of the point of intersection of the line L and the line L′.

d) Hence, or otherwise, find the shortest distance between the circle C and the line L.

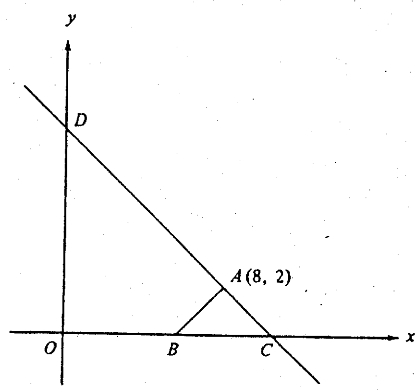


4. (83) In figure, O is the origin and A is the point (8,2).

a) B is a point on the x-axis such that the slope of AB is 1. Find the coordinates of B.

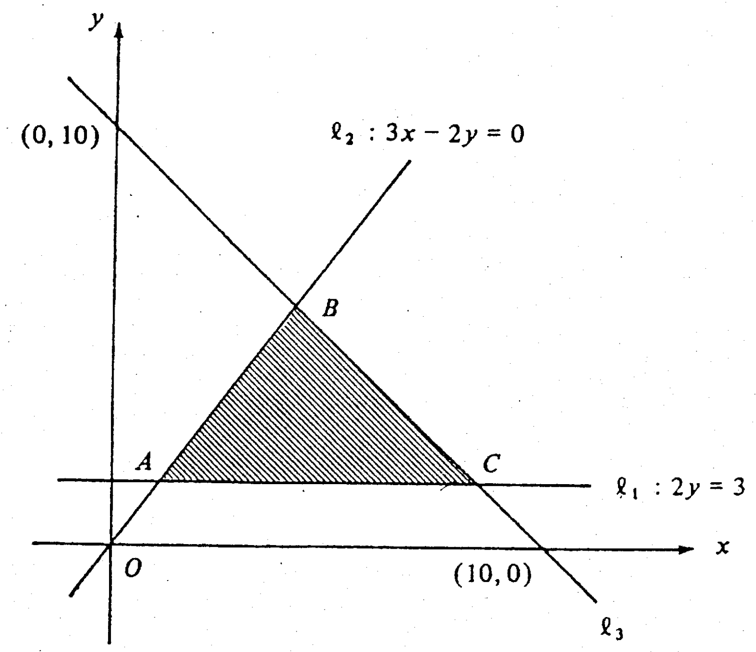
b) C is another point on the x-axis such that AB=AC. Find the coordinate of C.

c) Find the equation of the straight line AC. If the line AC cuts the y-axis at D, find the coordinates of D.



d) Find the equation of the circle passing through the points O, B and D, show that this circle passes through A.

5. (84)



In figure, $l_1 : 2y = 3$

$l_2 : 3x - 2y = 0$

The line l_3 passes through (0,10) and (10,0).

a) Find the equation of l_3.

b) Find the coordinates of points A,B and C.

6. (84) Let L be the line $y = k - x$ (k being a constant) and C be the circle $x^2 + y^2 = 4$.

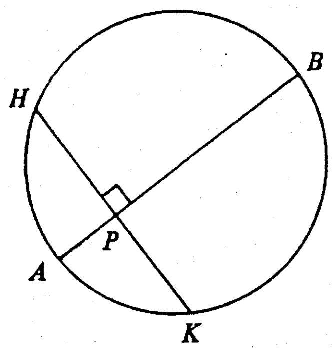
a) If L meets C at exactly one point, find the two values of k.

b) If L intersects C at the points A(2,0) and B,

i) find the value of k and the coordinates of B,

ii) find the equation of the circle with AB as diameter.

7. (85) In figure, A(2,Ø) and B(7,5) are the end-points of a diameter of the circle. P is a point an AB such that $\frac{AP}{PB} = \frac{1}{4}$.



a) Find the equation of the circle.

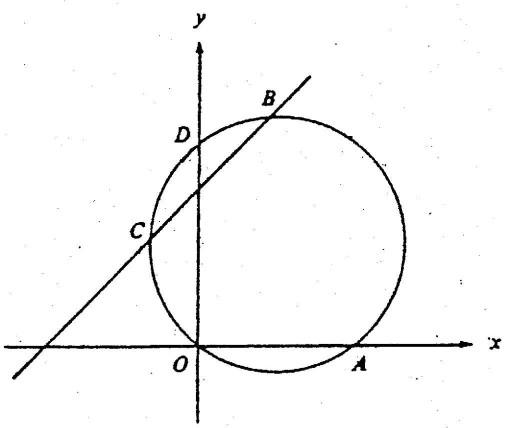
b) Find the coordinates of P.

c) The chord HPK is perpendicular to AB.

   i) Find the equation of HPK.

   ii) Find the coordinates of H and K.

8. (86)



The line $y - x - 6 = Ø$ cut the circle $x^2 + y^2 - 6x - 8y = Ø$ at the points B and C as shown in above figure. The circle cuts the x-axis at the origin O and the point A, it also cuts the y-axis at D.

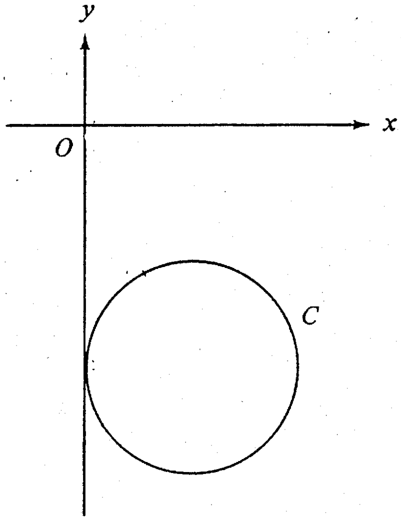
a) Find the coordinates of B and C.

b) Find the coordinates of A and D.

c) Find ∠ADO, ∠ABO and ∠ACO, correct to the nearest degree.

d) Find the area of △ ACO.

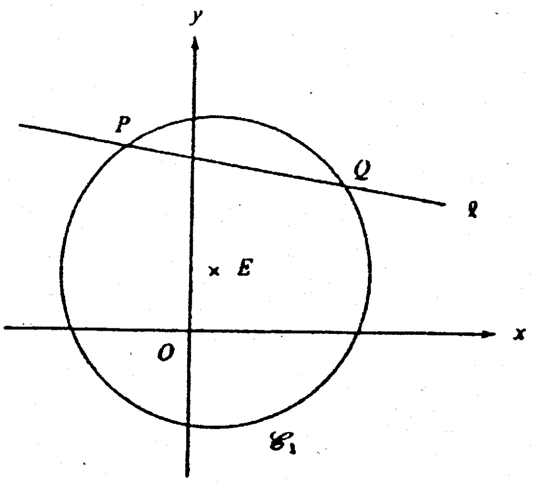
9. (88) In figure, the circle C has equation $x^2 + y^2 - 4x + 10\ y + k = Ø$, where k is a constant.



a) Find the coordinates of the centre of C.

b) If C touches the y-axis, find the radius of C and the value of k.

10. (89)



Let E be the centre of the circle

$C_1 : x^2 + y^2 - 2x - 4y - 20 = 0$.

The line l : $x + 7y - 40 = 0$ cuts C_1 at the points P and Q as shown in figure.

a) Find the coordinates of E.

b) Find the coordinates of P and Q.

c) Find the equation of the circle C_2 with PQ as diameter.

d) Show that C_2 passes through E. Hence or otherwise, find $\angle$EPQ.

11. (90) Let (C_1) be the circle $x^2 + y^2 - 2x + 6y + 1 = 0$ and A be the point (5,0).

a) Find the coordinates of the centre and the radius of (C_1).

b) Find the distance between the centre of (C_1) and A. Hence determine whether A lies inside, outside or on (C_1).

c) Let s be the shortest distance from A to (C_1).

   i) Find s

   ii) Another circle (C_2) has centre A and radius s. Find its equation.

d) A line touches the above two circles (C_1) and (C_2) at two distinct points E and F respectively. Draw a rough diagram to show this information. Find the length of EF.

12(87)

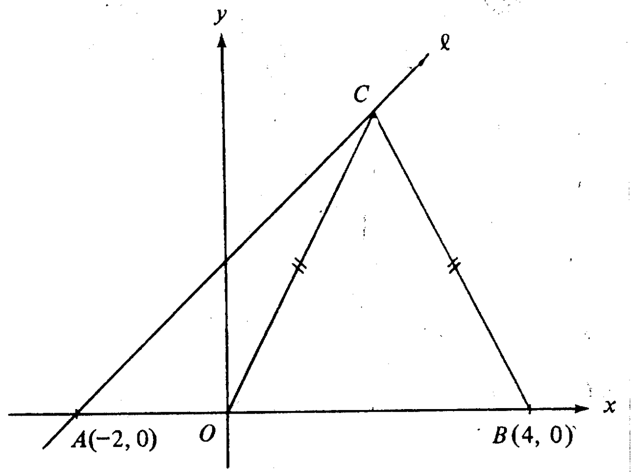


Figure 3

In Figure 3, O is the origin. A and B are the points $(-2, 0)$ and $(4, 0)$ respectively. ℓ is a straight line through A with slope 1. C is a point on ℓ such that $CO = CB$.

(a) Find the equation of ℓ.

(2 marks)

(b) Find the coordinates of C.

(3 marks)

(c) Find the equation of the circle passing through O, B and C.

(4 marks)

(d) If the circle OBC cuts ℓ again at D, find the coordinates of D.

(3 marks)

13 (91)

P.6

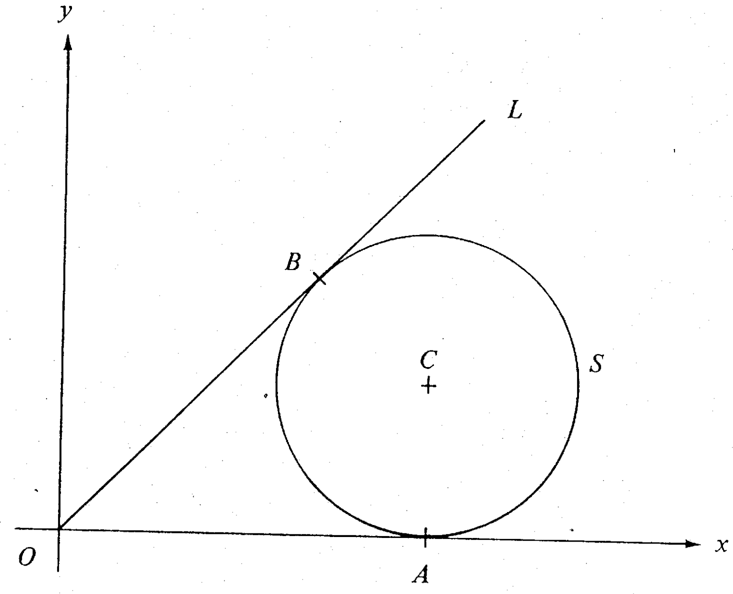


Figure 4

In Figure 4, the circle $S : x^2 + y^2 - 4x - 2y + 4 = 0$ with centre C touches the x-axis at A. The line $L : y = mx$, where m is a non-zero constant, passes through the origin O and touches S at B.

(a) Find the coordinates of C and A.

(2 marks)

(b) Show that $m = \dfrac{4}{3}$.

(5 marks)

(c) (i) Explain why the four points O, A, C, B are concyclic.

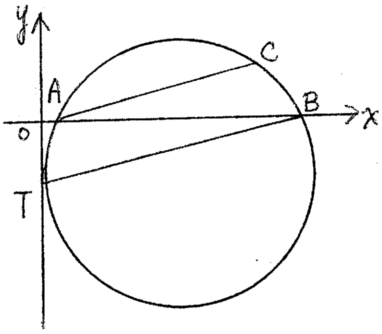
(ii) Find the equation of the circle passing through these four points.

(5 marks)

Coordinate Geometry

1.

$x^2+y^2-10x+8y+16=0$.



a) For $y=0$.

$\therefore x^2-10x+16=0$

$(x-8)(x-2)=0$

$\therefore x=2$ or 8.

$\therefore A=(2,0)$; $B=(8,0)$.

For $x=0$,

$y^2+8y+16=0$

$(y+4)^2=0$

$y=-4$.

$\therefore T=(0,-4)$.

b) i) the slope of AC

= the slope of TB.

$= \dfrac{-4-0}{0-8} = \dfrac{1}{2}$

$\therefore$ the eqt. of AC. (pt-slope form.)

$(y-0)=\frac{1}{2}(x-2)$

$2y = x-2$

$x-2y-2=0$.

ii)
$$\begin{cases} x^2+y^2-10x+8y+16=0 & \text{——①} \\ x-2y-2=0 & \text{——②} \end{cases}$$

since A & C are the intersection of the circle & the lines.

From ②, $x=2y+2$ ——③.

sub. into ①.

$(2y+2)^2+y^2-10(2y+2)+8y+16=0$.

$4y^2+8y+4+y^2-20y-20+8y+16=0$

$5y^2-4y=0$.

$y(5y-4)=0$

$\therefore y=\frac{4}{5}$ or 0.

For the coordinates of C.

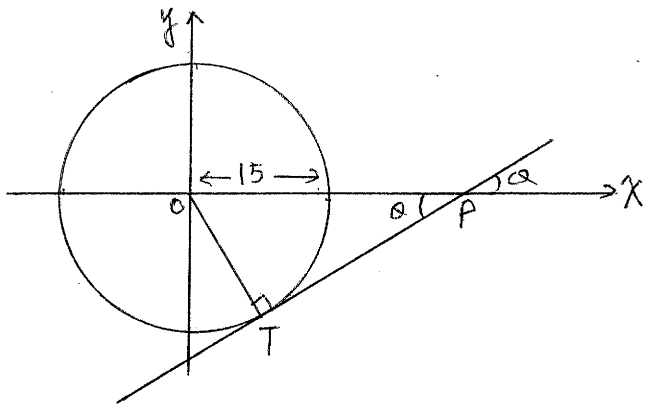
$y=4/5$.

$\therefore x = 2(\frac{4}{5})+2$

$= \frac{18}{5}$.

$\therefore C=(\frac{18}{5}, 4/5)$.

2. a)

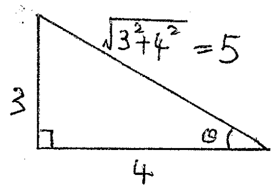


the eqt. of circle.

$(x-0)^2+(y-0)^2=15^2$

$x^2+y^2-225=0$.

b) In $\triangle OPT$.



since. $\tan\theta=\frac{3}{4}$.

$\therefore \sin\theta=\frac{3}{5}$.

$\frac{OT}{OP}=\sin\theta$.

$OP = \dfrac{15}{3/5} = 25$.

c) since $OP=25$, $\therefore P(25,0)$.

the slope $=\tan\theta = 3/4$.

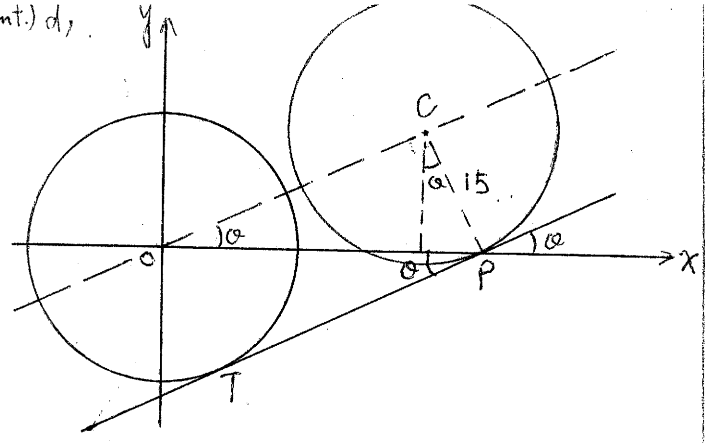
the eqt. of TP. (pt-slope form.)

$(y-0) = \frac{3}{4}(x-25)$.

$4y = 3x-75$

$3x-4y-75=0$.

2(cont.) d)



(since the circle is slided from P to T)

∴ the slope of OC

= the slope of TP

= $3/4$.

∴ the eqt. of OC. (pt-slope form)

$(y-0) = \frac{3}{4}(x-0)$

$4y = 3x$

$3x-4y=0$.

Let C be (x, y).

$\frac{y}{15} = \cos\theta$

$y = 15\left(\frac{4}{5}\right)$

$=12$.

From the eqt. of OC.

$3x-4(12)=0$

$x = 16$.

∴ $C=(16,12)$.

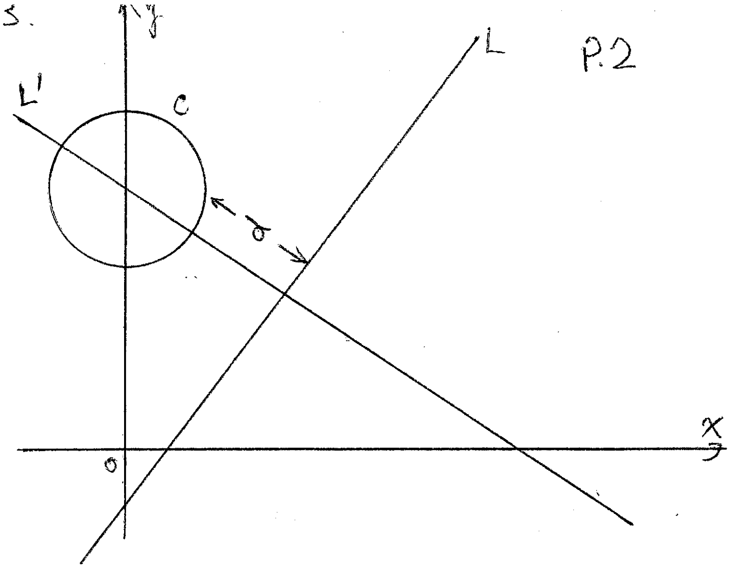
the eqt. of circle with centre C.

$(x-16)^2+(y-12)^2=15^2$.

$x^2-32x+16^2+y^2-24y+144=225$.

$x^2+y^2-32x-24y+175=0$.

5.



C: $x^2+y^2-14y+40=0$.

∴ the centre $= \left(0, \frac{-(-14)}{2}\right)$

$=(0,7)$.

the radius $= \sqrt{0^2+\left(\frac{-14}{2}\right)^2-40}$

$=3$.

b) Let the slope of L' be m.

∴ L: $4x-3y-4=0$.

$3y = 4x-4$

$y = \frac{4}{3}x - \frac{4}{3}$.

∴ the slope of $L = 4/3$.

∴ $m \cdot \left(\frac{4}{3}\right) = -1$ (since they are ⊥ar)

$m = -3/4$.

∴ the eqt. of L'.

$(y-7) = -\frac{3}{4}(x-0)$

$4y-28 = -3x$

$3x+4y-28=0$.

c) $\begin{cases} 4x-3y-4=0 & \text{—①} \\ 3x+4y-28=0 & \text{—②} \end{cases}$

①×4 + ②×3,

$16x+9x-16-84=0$

$25x = 100$

$x = 4$.

3(cont.) c, $x = 4$,

$3(4) + 4(y) - 28 = 0$

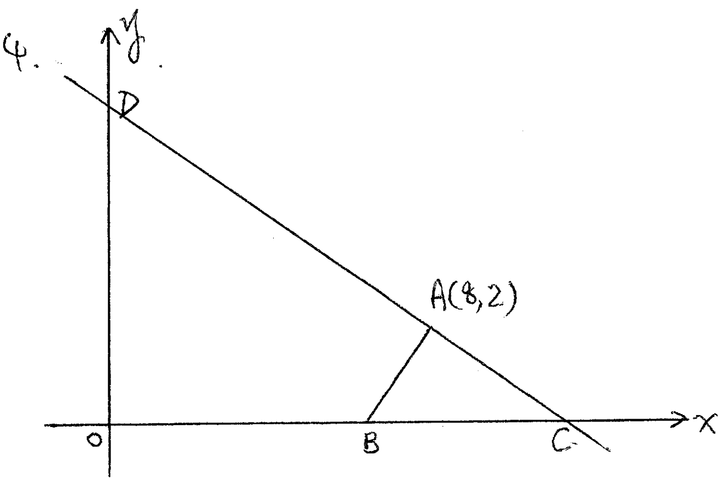
$y = 4$

∴ the pt. of intersection $= (4, 4)$.

d, Let the shortest distance between the circle C and the line L be d.

$$\therefore \quad d = \sqrt{(4-0)^2+(4-7)^2} - \text{the radius}$$
$$= \sqrt{4^2+3^2} - 3$$
$$= 5 - 3 = 2 .$$

4.



a) Since B on x-axis.

∴ Let $B = (x, 0)$.

the slope of $AB = 1$.

$$\therefore \quad \frac{2-0}{8-x} = 1.$$

$\therefore \quad 2 = 8 - x$

$x = 6$.

$\therefore B = (6, 0)$.

b, since C is also on x-axis.

∴ Let C be $(x_1, 0)$.

$AB = AC$.

$(8-6)^2 + (2-0)^2 = (8-x_1)^2 + (2-0)^2$. P.3

$4 + 4 = (8-x_1)^2 + 4$.

$64 - 16x_1 + x^2 = 4$

$x^2 - 16x_1 + 60 = 0$

$(x_1 - 6)(x_1 - 10) = 0$

$\therefore x_1 = 10$ or 6 (coordinate of B.)

$\therefore C = (10, 0)$

c, the eqt. of AC (two-pts form.)

$$\left(\frac{y-0}{x-10}\right) = \frac{2-0}{8-10}$$

$y = -1(x-10)$

$x + y - 10 = 0$.

since D on y-axis.

∴ put. $x = 0$,

$y = 10$.

$\therefore D = (0, 10)$.

d, Let the eqt. of circle be

$x^2 + y^2 + 2gx + 2fy + C = 0$.

∴ For $(0,0)$.

$0^2 + 0^2 + 2g(0) + 2f(0) + C = 0$

$\therefore C = 0$.

For $B = (6, 0)$.

$6^2 + 0^2 + 2g(6) + 2f(0) = 0$.

$\therefore 12g = -36$

$g = -3$.

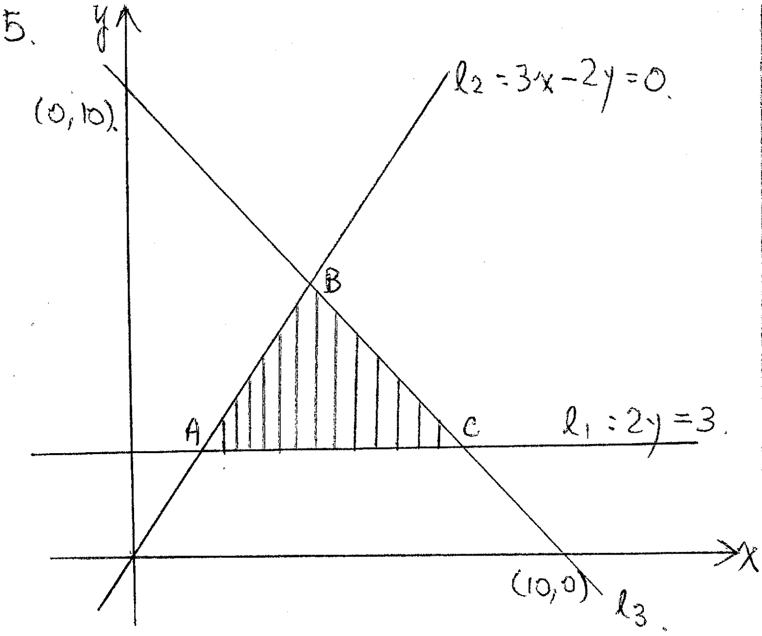
For $D = (0, 10)$

$\therefore 0^2 + 10^2 + 2g(0) + 2f(10) = 0$.

$\therefore f = -5$.

the eqt. is $x^2 + y^2 - 6x - 10y = 0$.

5.



a) the eqt. of l_3. (two-pts. form.)

$$\frac{y-0}{x-10} = \frac{10-0}{0-10}$$

$$\frac{y}{x-10} = -1$$

$$y = -x+10$$

$$x+y-10=0$$

b) pt. A is the intersection of l_1 & l_2.

$$\begin{cases} 2y=3 & \text{——} ① \\ 3x-2y=0 & \text{——} ② \end{cases}$$

From ① $y = 3/2$.

sub into ②.

$\therefore$ $3x - 2(3/2) = 0$

$\therefore$ $x = 1$

$A = (1, 3/2)$.

pt. B is the intersection of l_2 & l_3.

$$\begin{cases} 3x-2y=0 & \text{——} ③ \\ x+y-10=0 & \text{——} ④ \end{cases}$$

③ + ④ × 2.

$$5x - 20 = 0$$

$$x = 4.$$

$\therefore$ $y = 6$. $\therefore$ $B = (4,6)$.

pt. C is the intersection of l_3 & l_1.

P.4.

$$\begin{cases} 2y=3 & \text{——} ⑤ \\ x+y-10=0 & \text{——} ⑥ \end{cases}$$

From ⑤ $y = 3/2$.

$\therefore$ $x + \frac{3}{2} - 10 = 0$.

$$x = \frac{17}{2}.$$

$\therefore$ $C = (\frac{17}{2}, 3/2)$.

6. $$\begin{cases} L: \; y = k - x. & \text{——} ① \\ C: \; x^2+y^2=4 & \text{——} ② \end{cases}$$

a) sub ① into ②.

$\therefore$ $x^2 + (k-x)^2 = 4$

$$x^2 + x^2 - 2kx + k^2 = 4$$

$$2x^2 - 2kx + k^2 - 4 = 0.$$

since. L meets C at exactly one pt.

$\therefore$ $\Delta = 0$

$$(-2k)^2 - 4(2)(k^2-4) = 0$$

$$4k^2 - 8k^2 + 32 = 0$$

$$4k^2 = 32$$

$$k^2 = 8$$

$$k = \pm 2\sqrt{2}.$$

b) i) L intersects C at A(2,0) & B.

$\therefore$ A satisfies L : $y = k - x$.

put (2,0).

$\therefore$ $0 = k - 2$.

$\therefore$ $k = 2$.

$$\begin{cases} y = 2-x \\ x^2+y^2=4 \end{cases}$$

$$x^2 + (2-x)^2 = 4.$$

$$x^2 + 4 - 4x + x^2 = 4$$

$$2x^2 - 4x = 0$$

$\therefore$ $x(x-2) = 0$

6 (cont.). $x = 0$ or 2

For $x = 0$,

$$y = 2 - 0$$
$$= 2.$$

$\therefore B = (0, 2)$

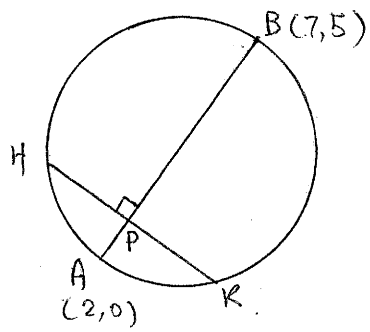
b ii) if AB is a diameter.

the centre $= \left(\frac{2+0}{2}, \frac{0+2}{2}\right)$
$= (1, 1)$

$\therefore$ the eqt. of circle.

$$(x-1)^2+(y-1)^2 = (2-1)^2+(1-2)^2$$
$$x^2-2x+1+y^2-2y+1 = 1+1$$
$$x^2+y^2-2x-2y = 0$$

7. a).



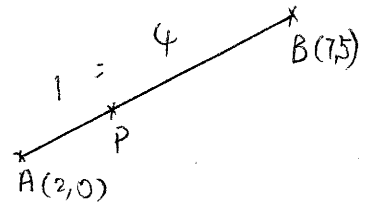
As AB is a diameter.

$\therefore$ the centre $= \left(\frac{2+7}{2}, \frac{0+5}{2}\right)$
$= \left(\frac{9}{2}, \frac{5}{2}\right)$

the eqt. of circle.

$$\left(x-\frac{9}{2}\right)^2+\left(y-\frac{5}{2}\right)^2 = \left(2-\frac{9}{2}\right)^2+\left(0-\frac{5}{2}\right)^2$$
$$x^2-9x+\frac{81}{4}+y^2-5y+\frac{25}{4} = \frac{25}{4}+\frac{25}{4}$$
$$x^2+y^2-9x-5y+14 = 0.$$

b) Let the coordinate of P be (x, y)



$$\therefore x = \frac{2(4)+7(1)}{1+4}$$
$$= 3.$$
$$y = \frac{0(4)+5(1)}{1+4}$$
$$= 1.$$

$\therefore P = (3, 1).$

c i) the slope of AB.

$$= \frac{5-0}{7-2} = 1.$$

Let the slope of HPK be m

since $HK \perp AB$.

$$\therefore m \cdot (1) = -1$$
$$m = -1.$$

the eqt. of HPK. (pt-slope form.)

$$(y-1) = -1(x-3)$$
$$y-1 = -x+3$$
$$x+y-4 = 0.$$

ii) the coordinates of H & K are the intersection of circle & line.

$$\begin{cases} x^2+y^2-9x-5y+14=0 & \text{①} \\ x+y-4=0. & \text{②} \end{cases}$$

From ② $\quad x = -y+4.$

sub into ①.

$$(-y+4)^2+y^2-9(-y+4)-5y+14 = 0.$$
$$y^2-8y+16+y^2+9y-36-5y+14 = 0$$
$$2y^2-4y-6 = 0$$
$$y-2y-3 = 0$$
$$(y-3)(y+1) = 0$$
$$\therefore y = -1 \text{ or } 3.$$

when $y = -1$

From ②. $x - 1 - 4 = 0$

$x = 5$,

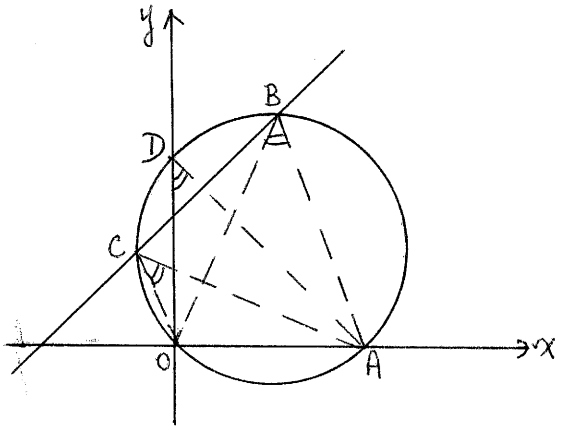
when $y = 3$

$x + 3 - 4 = 0$

$x = 1$.

$\therefore$ $H = (3, 1)$ & $K = (5, -1)$.

8.



a) B and C are the intersection of.

$$\begin{cases} x^2 + y^2 - 6x - 8y = 0 & \text{①} \\ y - x - 6 = 0 & \text{②} \end{cases}$$

From ② $y = x + 6$.

sub into ①.

$x^2 + (x+6)^2 - 6x - 8(x+6) = 0$.

$x^2 + x^2 + 12x + 36 - 6x - 8x - 48 = 0$

$2x^2 - 2x - 12 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$.

$\therefore$ $x = -2$ or 3.

when $x = -2$.

$y = -2 + 6 = 4$

when $x = 3$

$y = 3 + 6 = 9$

$\therefore$ $B = (3, 9)$ & $C = (-2, 4)$

b) For A,

$y = 0$,

$\therefore$ $x^2 - 6x = 0$.

$\therefore x(x-6) = 0$

$x = 6$ or 0 (coordinate of O)

$\therefore$ $A = (6, 0)$

For D,

$x = 0$,

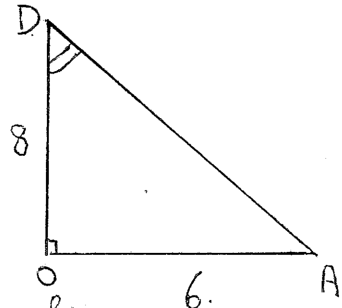
$\therefore$ $y^2 - 8y = 0$

$y(y-8) = 0$

$y = 8$ or 0.

$\therefore$ $D = (0, 8)$.

c,



$\tan \angle ADO = \dfrac{OA}{OD}$

$= \dfrac{6}{8}$.

$\angle ADO = 37°$ (nearest degree.)

since. $\angle ADO$, $\angle ABO$ & $\angle ACO$.

subtended an equal arc.

$\therefore$ $\angle ADO = \angle ABO = \angle ACO = 37°$.

d, the area of $\triangle ACO$

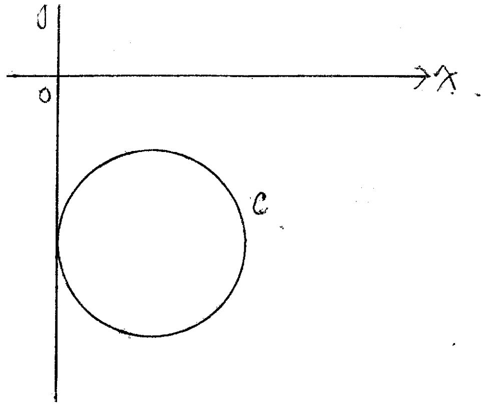
$= \frac{1}{2}(OA)(OC) \sin \angle ACO$.

$= \frac{1}{2}(6)(\sqrt{2^2 + 4^2}) . \sin 37°$.

$= \frac{1}{\cancel{2}}(6) . \cancel{2}\sqrt{5} \left(\frac{3}{5}\right)$

$= \dfrac{18\sqrt{5}}{5}$. sq. unit.

9.



$x^2+y^2-4x+10y+k=0$.

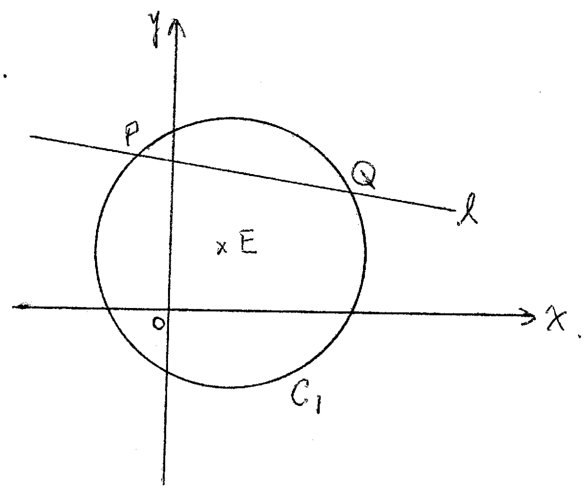
a) the centre $= \left(\frac{-(-4)}{2}, \frac{-(10)}{2}\right)$
$= (2,-5)$.

b) (C) touches y-axis.
$\therefore$ put $x=0$.
$\therefore y^2+10y+k=0$.
since (C) touches y-axis.
$\Delta = 0$
$10^2-4(1)(k)=0$
$\therefore k=25$.

$\therefore x^2+y^2-4x+10y+25=0$

the radius $= \sqrt{\left(\frac{-4}{2}\right)^2+\left(\frac{10}{2}\right)^2-25}$
$= \sqrt{4+25-25}$
$= 2$.

10.



a) $x^2+y^2-2x-4y-20=0$.
$\therefore$ the centre $E = \left(\frac{-(-2)}{2}, \frac{-(-4)}{2}\right)$
$= (1,2)$.

b) P and Q are the intersection of l and C_1.

$$\begin{cases} x^2+y^2-2x-4y-20=0 & \text{—①} \\ x+7y-40=0 & \text{—②} \end{cases}$$

From ② $x = -7y+40$

$(-7y+40)^2+y^2-2(-7y+40)-4y-20=0$
$49y^2-560y+1600+y^2+14y-80-4y-20=0$.
$50y^2-550y+1500=0$.
$y^2-11y+30=0$.
$(y-6)(y-5)=0$.
$\therefore y=5$ or 6.

when $y=5$,
$x=-7(5)+40$
$=5$,

when $y=6$
$x=-7(6)+40$
$=-2$.

$\therefore P(-2,6)$ & $Q(5,5)$.

c) ∵ PQ as a diameter.
$\therefore$ the centre $= \left(\frac{-2+5}{2}, \frac{6+5}{2}\right)$
$= \left(\frac{3}{2}, \frac{11}{2}\right)$

the eqt. of C_2.
$(x-\frac{3}{2})^2+(y-\frac{11}{2}) = (\frac{3}{2}-5)^2+(\frac{11}{2}-5)^2$
$x^2-3x+\frac{9}{4}+y^2-11y+\frac{121}{4} = \frac{49}{4}+\frac{1}{4}$.
$x^2+y^2-3x-11y+20=0$

10 (cont.) a).

$x^2+y^2-3x-11y+20=0$.

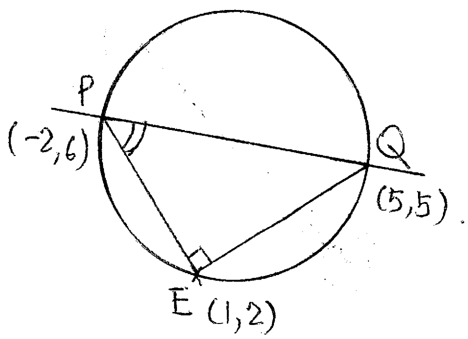
put $E=(1,2)$.

$\therefore$ L.H.S. $=1^2+2^2-3(1)-11(2)+20$

$=1+4-3-22+20$

$=0$ = R.H.S.

$\therefore$ C_2 passes through E.



$\sin\angle EPQ = \dfrac{QE}{PQ}$

$= \dfrac{\sqrt{(5-2)^2+(5-1)^2}}{\sqrt{(-2-5)^2+(6-5)^2}}$

$= \dfrac{5}{5\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}}$.

$\therefore \angle EPQ = 45^\circ$.

11. (C_1) $x^2+y^2-2x+6y+1=0$.

a) the centre $= \left(-\frac{(-2)}{2}, -\frac{6}{2}\right)$

$=(1,-3)$

the radius $=\sqrt{\left(\frac{-2}{2}\right)^2+\left(\frac{6}{2}\right)^2-1}$

$=\sqrt{1^2+3^2-1}$

$=3$.

b) the distance of Centre & A

$=\sqrt{(5-1)^2+[0-(-3)]^2}$

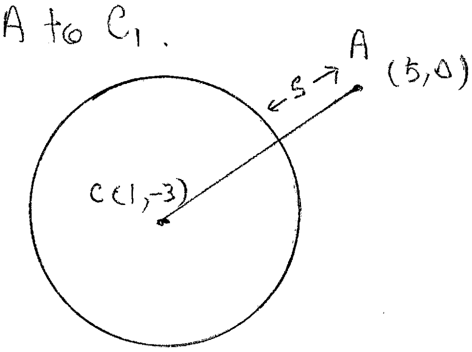
$=\sqrt{4^2+3^2}=5$.

since the distance of centre & A

$=5>$ the radius of $C_1=3$.

$\therefore$ A lies outside (C_1).

c) i) s be shortest distance from A to C_1.



s = the distance of centre & A − the radius.

$=5-3=2$.

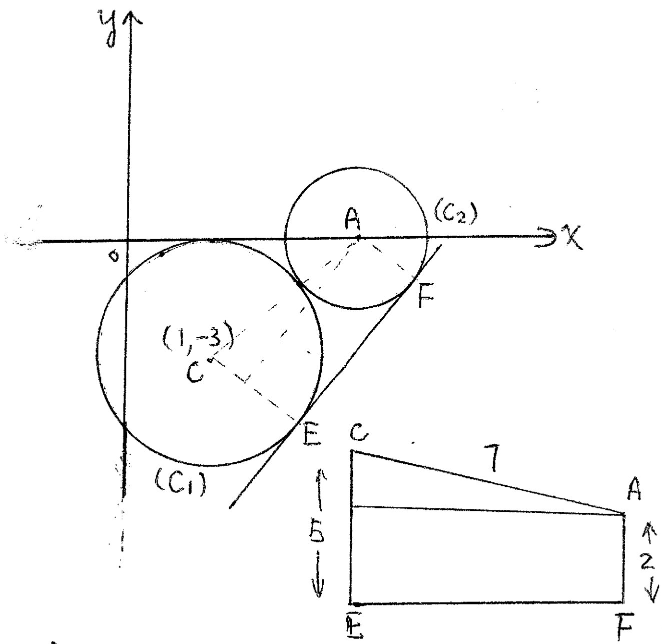
ii) the eqt. of (C_2)

$(x-5)^2+(y-0)^2=2^2$

$x^2-10x+25+y^2=4$.

$x^2+y^2-10x+21=0$.

d).



$\therefore EF^2=7^2-3^2$

$EF=\sqrt{40}$

$=2\sqrt{10}$.