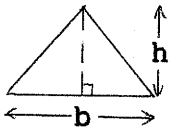
FORMULA (Revision)

A formula is a relation connecting two or more variable or letters in a definite way.

Example The area A of a triangle is given by the formula.

area A =



Example The volume V of a circular cylinder is given by

volume V =

Subject of a formula

In a formula, $V = \pi.r^2.h$

When one letter 'V' in a formula is expressed in terms of the other letter(s) 'r', 'h', the letter 'V' is called the subject of the formula.

Example 1 Make h as the subject of

$$A = 2\pi r.h + 2\pi r^2$$

Example 2 (88) If $x = \frac{1 + y}{1 - y}$, then express y in terms of x.

Example 3(84) If $a = \frac{2b(2y - x)}{x - 3y}$, express y in terms of a, b and x.

Example 4(85) If $\frac{ab}{ka + b} = \frac{1}{k}$, express b in terms of a and k.

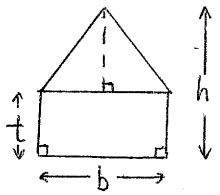
Example 5(86) If $1 - \frac{x + y}{y - x} = a$, where $a \neq 0$, express y in terms of x and a.

Example 6(87) If $a = \frac{b + 3cd}{b - 3cd}$, express c in terms of a, b and d.

Example 7 The figure is composited of a rectangle and a triangle.

a) Find the total area A of the figure in terms of b, h and t.

b) Change the subject of the formula from A to t.



1. (83) If $x = \dfrac{y^2}{\sqrt{a^2 + bz}}$, then $z =$

A. $\dfrac{1}{b}\left(\dfrac{y^4}{x^2} - a^2\right)$ B. $\dfrac{1}{b}\left(\dfrac{x^2}{y^4} - a^2\right)$ C. $\dfrac{1}{b}\left(a^2 - \dfrac{x^2}{y^4}\right)$

D. $\dfrac{1}{b}\left(a^2 - \dfrac{y^4}{x^2}\right)$ E. $\dfrac{1}{b}\left(a^2 - \dfrac{x^2}{y^2}\right)$

2. (83) A function f(x) is called an even function if $f(x) = f(-x)$
Which of the followings is/are even function(s) ?
(1) $f_1(x) = 1/x$ (2) $f_2(x) = x^2$ (3) $f_3(x) = x^3$
A. (1) only B. (2) only C. (3) only
D. (1) and (2) only E. (2) and (3) only

3. (84) If $a = \dfrac{2b(2y - x)}{x - 3y}$, then $y =$

A. $\dfrac{a + 2b}{3a + 4b} x$ B. $\dfrac{a - 2b}{-3a + 4b} x$ C. $-\dfrac{a + 2b}{3a + 4b} x$

D. $\dfrac{3a + 4b}{a + 2b} x$ E. $\dfrac{-3a + 4b}{a - 2b} x$

4. (84) If $f(x) = \log_{10} 2x - x$, then $f(x + 1) - f(x) =$

A. $\log_{10} 2 - 1$ B. $\log_{10}[(x + 1)/x]$

C. $\log_{10} \dfrac{10(x + 1)}{x}$ D $\log_{10} \dfrac{x + 1}{10x}$ E. $\log_{10} \dfrac{x + 1}{x} - 2x$

5. (85) If $\dfrac{ab}{ka + b} = \dfrac{1}{k}$, then $b =$

A. $\dfrac{a}{a - k}$ B. $\dfrac{ka}{ka - 1}$ C. $\dfrac{ka}{1 - ka}$ D. $\dfrac{k^2a}{a - k}$

E. $\dfrac{k^2a}{k - a}$

6. (85) If $a - \sqrt{b^2 + c^2} = d$, then $c =$

A. $d - a + b$ B. $a - b - d$ C. $\pm\sqrt{d^2 - a^2 + b^2}$

D. $\pm\sqrt{a^2 - d^2 - b^2}$ E. $\pm\sqrt{(a - d)^2 - b^2}$

7. If $f(2x) = 8x^3 + 4x$, then $f(3a) =$
(85) A. $9a^3 + 6a$ B. $12a^3 + 6a$ C. $27a^3 + 6a$ D. $108a^3 + 6a$
E. $216a^3 + 12a$

8. If $r = \sqrt[3]{h^3 - 7r^3}$, then the ratio $r : h$ is
(86) A. $1 : 8$ B. $1 : 2\sqrt{2}$ C. $1 : 2$ D. $1 : \sqrt{2}$ E. $1 : \sqrt[3]{2}$

9. (86) If $1 - \dfrac{x + y}{y - x} = a$ $(a \neq 0)$, then $y =$

A. x B. $x(a - 2)/a$ C. $x(a - 1)/a$ D. $x(2 - a)/a$
E. $x(1 - a)/a$

10. (87) If $a = \dfrac{b + 3cd}{b - 3cd}$, then $c =$

A. $\dfrac{a}{6d}$ B. $\dfrac{b}{3d}$ C. $\dfrac{b(a-1)}{6d}$ D. $\dfrac{b(a+1)}{a-1}$ E. $\dfrac{b(a-1)}{3d(a+1)}$

11. If $f(x) = x^2 + 1$, then $f(x - 1) =$
(87) A. x^2 B. x^2-1 C. x^2+2 D. x^2-2x E. x^2-2x+2

12. (88) If $x = \dfrac{1 + y}{1 - y}$, then $y =$

A. $\dfrac{x - 1}{x}$ B. $\dfrac{1 + x}{1 - x}$ C. $\dfrac{x + 1}{x - 1}$ D. $\dfrac{x - 1}{x + 1}$ E. $\dfrac{1 - x}{1 + x}$

13. If $f(x) = 3 + 2^n$, then $f(2x) - f(x) =$
(88) A. 2^n B. 2^{3n} C. $3 + 2^n$ D. $2^n(2^n + 1)$ E. $2^n(2^n - 1)$

ANSWERS
1.A 2.B 3.A 4.D 5.B 6.E 7.C 8.C 9.B 10.E
11.E 12.D 13.E

Formulae and Functions

1. $x = \dfrac{y^2}{\sqrt{a^2+bz}}$

$\sqrt{a^2+bz} = \dfrac{y^2}{x}$

$a^2+bz = \left(\dfrac{y^2}{x}\right)^2$.

$bz = \dfrac{y^4}{x^2} - a^2$.

$z = \dfrac{1}{b}\left(\dfrac{y^4}{x^2} - a^2\right)$ (A.)

2. even function:
$f(x) = f(-x)$.

(1) $f_2(x) = \dfrac{1}{x}$

$f_2(-x) = \dfrac{1}{-x} \neq f_2(x)$

∴ it is not an even function

(2). $f_2(x) = x^2$

$f_2(-x) = (-x)^2 = x^2 = f_2(x)$

∴ it is an even function.

(3). $f_3(x) = x^3$

$f_3(-x) = (-x)^3 = -x^3 \neq f_3(x)$

∴ it is not an even function.

∴ (2) only (B.)

3. $a = \dfrac{2b(2y-x)}{x-3y}$

$a(x-3y) = 4by - 2bx$.

$3ay + 4by = ax + 2bx$

$y(3a+4b) = x(a+2b)$

$y = \left(\dfrac{a+2b}{3a+4b}\right)x$ (A.)

4. $f(x) = \log_{10} 2x - x$.

$f(x+1) - f(x)$.

$= \log_{10} 2(x+1) - (x+1)$
$- \log_{10} 2x - x$.

$= \log_{10}\left(\dfrac{2x+2}{2x}\right) - 1$.

$= \log_{10}\left(\dfrac{x+1}{x}\right) - \log_{10} 10$

$= \log_{10}\left(\dfrac{x+1}{10x}\right)$ (D.)

5. $\dfrac{ab}{ka+b} = \dfrac{1}{k}$

$abk = ka + b$.

$b(ak-1) = ka$

$b = \dfrac{ka}{ka-1}$ (B.)

6. $a - \sqrt{b^2+c^2} = d$.

$a - d = \sqrt{b^2+c^2}$

$(a-d)^2 = b^2 + c^2$

$c^2 = (a-d)^2 - b^2$

$c = \pm\sqrt{(a-d)^2 - b^2}$ (E.)

7. $f(2x) = 8x^3 + 4x$

$f(x) = 8\left(\dfrac{x}{2}\right)^3 + 4\left(\dfrac{x}{2}\right)$

$= x^3 + 2x$

$\therefore f(3a) = (3a)^3 + 2(3a)$

$= 27a^3 + 6a$. (C.)

8. $r = \sqrt[3]{h^3 - 7r^3}$

$r^3 = h^3 - 7r^3$

$8r^3 = h^3$.

$2r = h$.

$\therefore r : h = 1 : 2$ (C.)

9. $y - x = a$, $x \neq 0$

$(y-x) - (x+y) = a(y-x)$

$y - x - x - y = ay - ax$

$ax - 2x = ay$

$y = \dfrac{x(a-2)}{a}$ (B.)

10. $a = \dfrac{b+3cd}{b-3cd}$

$ab - 3acd = b + 3cd$.

$ab - b = 3cd + 3acd$.

$3cd(1+a) = b(a-1)$

$c = \dfrac{b(a-1)}{3d(a+1)}$ (E.)

11. $f(x) = x^2 + 1$

$f(x-1) = (x-1)^2 + 1$

$= x^2 - 2x + 1 + 1$

$= x^2 - 2x + 2$ (E.)

12. $x = \dfrac{1+y}{1-y}$.

$x(1-y) = 1+y$.

$x - xy = 1 + y$

$y + xy = x - 1$

$y = \dfrac{x-1}{x+1}$ (D.)

13. $f(x) = 3 + 2^n$.

$f(2x) - f(x)$

$= (3 + 2^{2n}) - (3 + 2^n)$

$= 2^{2n} - 2^n$.

$= 2^n(2^n - 1)$. (E.)