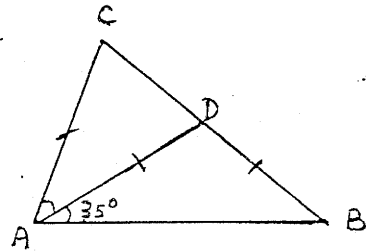
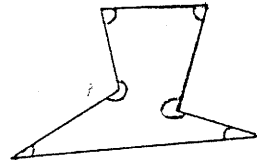


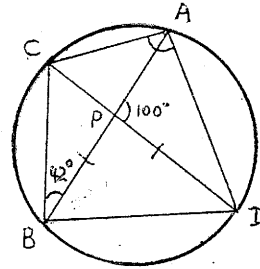
1. In the figure, D is a point on BC and $AC=AD=BD$. $\angle CAD =$
- 20°
 - 25°
 - 30°
 - 35°
 - 40°



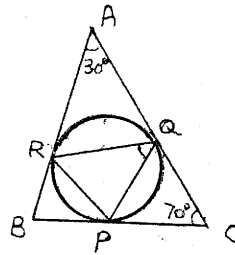
2. The sum of the six marked angles in the figure is
- 360°
 - 540°
 - 600°
 - 720°
 - 900°



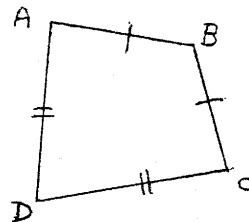
3. In the figure, chords AB and CD intersect at P. $BP=DP$. $\angle CAD =$
- 58°
 - 86°
 - 88°
 - 92°
 - 142°



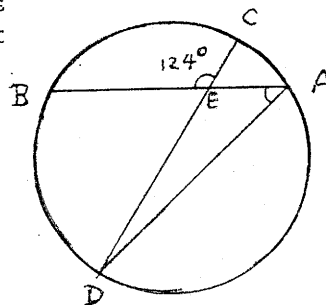
4. In the figure, the three sides of $\triangle ABC$ touch the circle at the points P, Q and R. $\angle PQR =$
- 30°
 - 50°
 - 55°
 - 70°
 - 75°



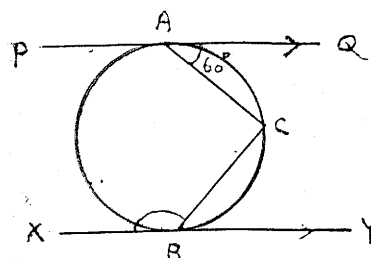
5. In the figure, ABCD is a quadrilateral with $AB=BC$ and $AD=DC$. Which of the following is/are true ?
- $\angle BAD = \angle BCD$
 - $AC \perp BD$
 - BD bisects AC
- (1) only
 - (1) and (2) only
 - (1) and (3) only
 - (2) and (3) only
 - (1), (2) and (3)



6. In the figure, chords AB and CD intersect at E. The length of the minor arc BD is three times the length of the minor arc AC. $\angle BAD =$
- 31°
 - 35°
 - 42°
 - 45°
 - 56°



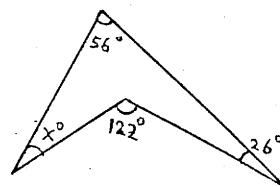
7. In the figure, PQ and XY touch the
 (83) circle at A and B respectively,
 $PQ \parallel XY$ and $\angle QAC = 60^\circ$. $\angle CBX =$
 A. 150°
 B. 135°
 C. 120°
 D. 110°
 E. 100°



8. The sum of the interior angles of a convex polygon is
 (84) greater than the sum of the exterior angles by 360° . How
 many sides has the polygon?
 A. 3 B. 4 C. 5 D. 6 E. 8

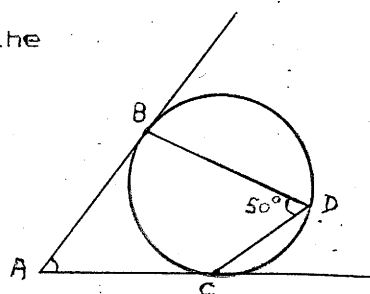
9. In the figure, $x = ?$

- (84) A. 31
 B. 34
 C. 40
 D. 48
 E. It cannot be determined.



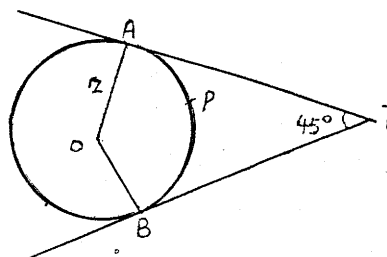
10. In the figure, AB and AC touch the
 (84) circle at B and C respectively.

- $\angle A =$
 A. 30°
 B. 40°
 C. 50°
 D. 80°
 E. 85°



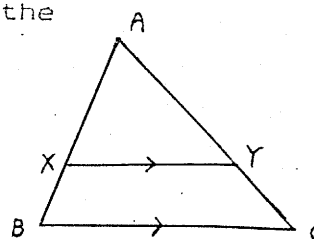
11. In the figure, O is the centre of
 (84) the circle. TA and TB touch the
 circle at A and B respectively.
 $OA = 2$. The length of the arc
 APB is

- A. $\pi/4$
 B. $\pi/2$
 C. $3\pi/4$
 D. $3\pi/2$
 E. 3π



12. In the figure, $XY \parallel BC$. $AX:XB=2:1$. If the
 (84) area of the trapezium BCYX = 20, then
 the area of $\triangle ABC =$

- A. 80
 B. 60
 C. 45
 D. 40
 E. 36

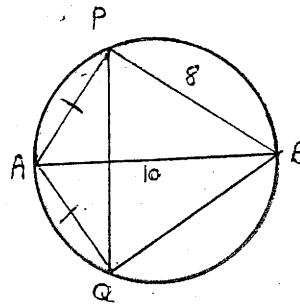


13. In $\triangle ABC$, $BC=a$, $AC=b$, $AB=c$ and $a > b > c$. Which of the
 (84) following must be true?

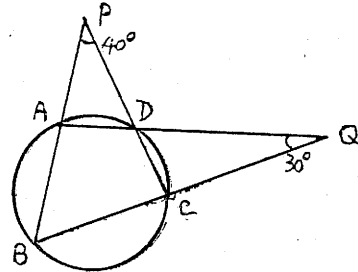
- (1) $\angle A > \angle B > \angle C$ (2) $b+c > a$ (3) $\angle B + \angle C > \angle A$

- A. (1) only B. (2) only C. (1) and (2) only
 D. (2) and (3) only E. (1), (2) and (3)

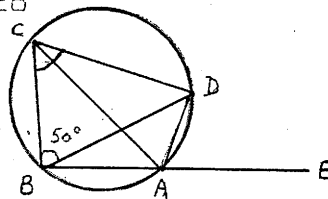
14. In the figure, AB is a diameter of (84) the circle. $AP = AQ$. $AB = 10$ and $BP = 8$. $PQ =$
- A. 5
 - B. 6
 - C. 6.4
 - D. 8
 - E. 9.6



15. In the figure, the chords BA and CD, (84) when produced, meet at P. The chords AD and BC, when produced, meet at Q. $\angle B =$
- A. 35°
 - B. 40°
 - C. 45°
 - D. 50°
 - E. 55°

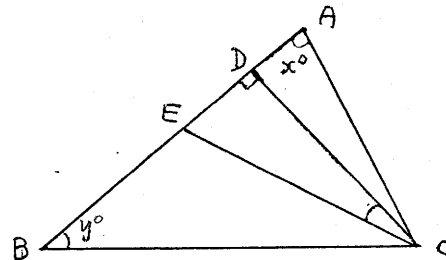


16. In the figure, ABCD is a cyclic (85) quadrilateral. BA is produced to E. DA bisects $\angle CAE$. $\angle BCD =$
- A. 40°
 - B. 45°
 - C. 50°
 - D. 55°
 - E. 65°

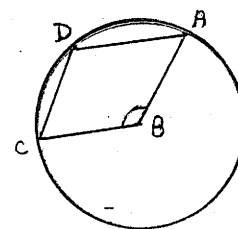


17. The exterior angles (interior) of a pentagon are x° , $2x^\circ$, $3x^\circ$, $4x^\circ$ and (85) $5x^\circ$. The smallest angle of the pentagon is
- A. 120°
 - B. 60°
 - C. 48°
 - D. 36°
 - E. 24°

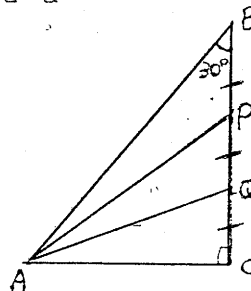
18. In the figure, A, D, E and B lie on (85) a straight line. CE bisects $\angle ACB$ and $CD \perp AB$. $\angle DCE =$
- A. $\frac{1}{2}(x^\circ - y^\circ)$
 - B. $\frac{1}{2}(x^\circ + y^\circ)$
 - C. $x^\circ - y^\circ$
 - D. $90^\circ - \frac{1}{2}(x^\circ + y^\circ)$
 - E. $90^\circ - (x^\circ - y^\circ)$



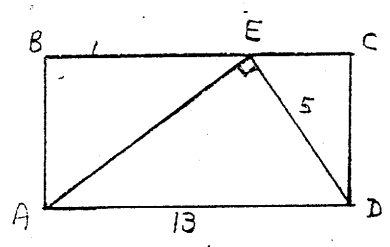
19. In the figure, ABCD is a rhombus. B is (85) the centre of the circle. $\angle ABC =$
- A. 105°
 - B. 120°
 - C. 130°
 - D. 135°
 - E. 150°



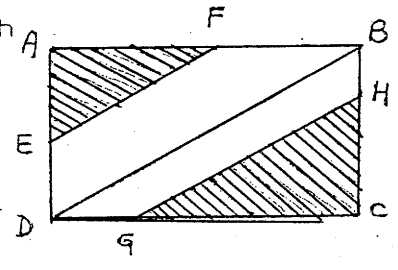
20. In the figure, $\angle C = 90^\circ$. P and Q (85) are points on BC such that $BP = PQ = QC$. $\angle CAQ =$
- A. 30°
 - B. 25°
 - C. 22°
 - D. 20°
 - E. 15°



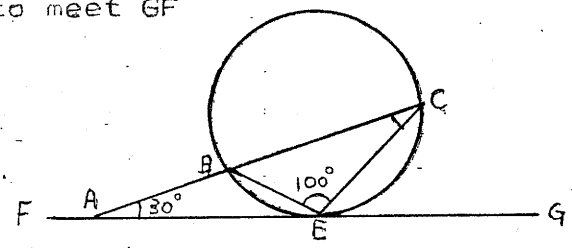
21. In the figure, ABCD is a rectangle. E (85) is a point on BC such that $\angle AED = 90^\circ$. AD = 13 and DE = 5. The area of ABCD =
- A. 30
 - B. 52
 - C. 60
 - D. 65
 - E. 120



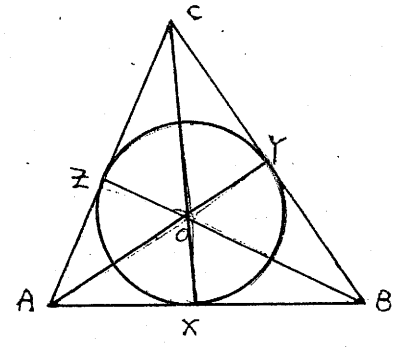
22. In the figure, ABCD is a rectangle. E, F, (85) G and H are points on the four sides such that $EF \parallel DB \parallel GH$. $AF = FB$ and $HC = 2BH$. What fraction of the area of ABCD is shaded?
- A. $13/36$
 - B. $3/12$
 - C. $25/36$
 - D. $25/72$
 - E. $47/72$



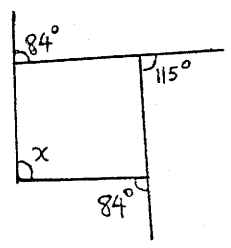
23. In the figure, FG touches the circle at (85) E. The chord CB is produced to meet GF at A. $\angle ACE =$
- A. 10°
 - B. 20°
 - C. 25°
 - D. 30°
 - E. 35°



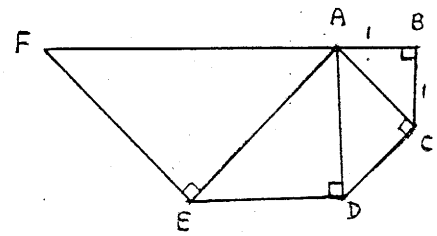
24. In the figure, the circle touches the (85) sides of $\triangle ABC$ at X, Y and Z. O is the centre of the circle. Which of the following must be true?
- I. OA bisects $\angle BAC$
 - II. A, X, O and Z are concyclic
 - III. $AX = AZ$
- A. III only
 - B. I and II only
 - C. I and III only
 - D. II and III only
 - E. I, II and III.



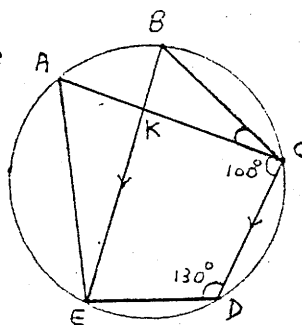
25. In the figure, x = (86)
- A. 77°
 - B. 84°
 - C. 96°
 - D. 103°
 - E. 115°



26. In the figure, ABC, ACD, ADE and (86) AEF are right-angled isosceles triangles. If $AB = BC = 1$, how long is AF?
- A. $2\sqrt{5}$
 - B. 4
 - C. $2\sqrt{3}$
 - D. 3
 - E. $\sqrt{5}$

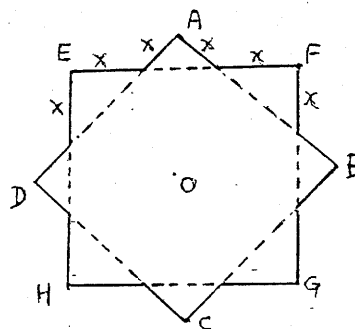


27. In the figure, A, B, C, D and E lie on a circle. AC intersects BE at K. $\angle ACD = 100^\circ$ and $\angle CDE = 130^\circ$. If $BE \parallel CD$, then $\angle ACB =$
- A. 25°
 - B. 30°
 - C. 36°
 - D. 40°
 - E. 42°



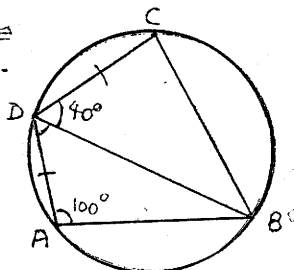
28. If the five interior angles of a convex pentagon form an A.P. with a common difference of 10° , then the smallest interior angle of the pentagon is
- A. 52°
 - B. 72°
 - C. 88°
 - D. 98°
 - E. 108°

29. In the figure, ABCD and EFGH are two squares of side 1. They are placed one upon the other with their centres both at O to form a star with 16 sides, each of length x. Find x.

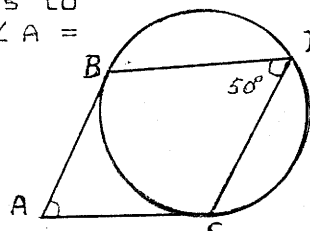


- A. $2/7$
- B. $1/3$
- C. $2/5$
- D. $1/(2 + \sqrt{2})$
- E. $1/(1 + \sqrt{2})$

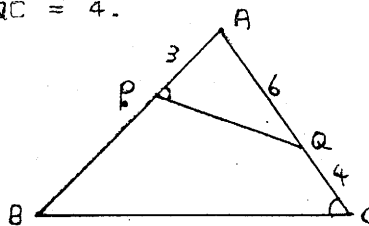
30. AD and DC are equal chords of the circle ABCD. $\angle CDB = 40^\circ$, $\angle DAB = 100^\circ$. $\angle ADB =$
- A. 20°
 - B. 25°
 - C. 30°
 - D. 35°
 - E. 40°



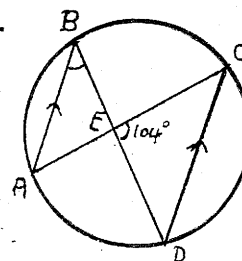
31. In the figure, AB and AC are tangents to the circle BCD. If $\angle BDC = 50^\circ$, then $\angle A =$
- A. 130°
 - B. 100°
 - C. 85°
 - D. 80°
 - E. 50°



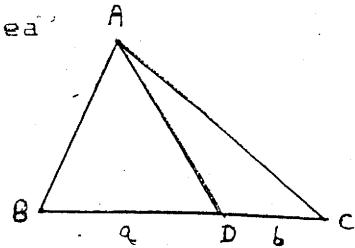
32. In $\triangle ABC$, $AP = 3$, $AQ = 6$ and $QC = 4$. If $\angle APQ = \angle ACB$, then $PB =$
- A. 7
 - B. 8
 - C. 10
 - D. 17
 - E. 20



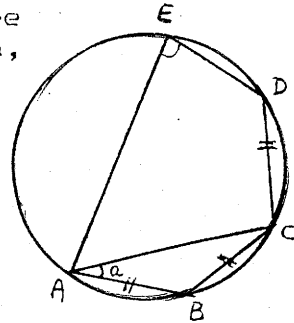
33. In the figure, chords AC and BD meet at E and $AB \parallel DC$. If $\angle CED = 104^\circ$, find $\angle ABD$.
- A. 76°
 - B. 52°
 - C. 38°
 - D. 14°
 - E. It cannot be determined.



34. In the figure, $BD=a$, $DC=b$ and the area (87) of $\triangle ABD=s$. Find the area of $\triangle ABC$.
- A. $s(a+b)/a$
 - B. $s(a+b)/b$
 - C. $s(a+b)^2/a^2$
 - D. $s(a+b)^2/b^2$
 - E. $s(a^2+b^2)/a^2$



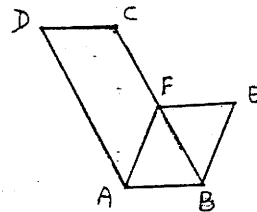
35. In the figure, AB, BC and CD are three (87) equal chords of a circle. If $\angle BAC = a$, then $\angle AED =$
- A. $2a$
 - B. $3a$
 - C. $90^\circ - a$
 - D. $180^\circ - 2a$
 - E. $180^\circ - 3a$



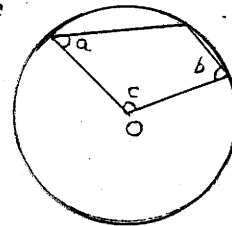
36. In the figure, ABCD and ABEF are (87) parallelograms. Area of ABCD

$$\frac{\text{Area of ABCD}}{\text{Area of ABEF}} =$$

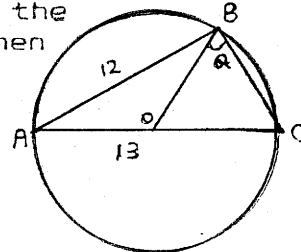
- A. AD/AF
- B. BC/BF
- C. BC/EF
- D. AD^2/AF^2
- E. BC^2/EF^2



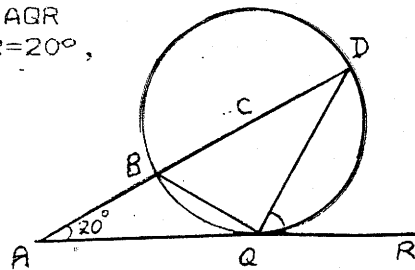
37. In the figure, O is the centre of the (87) circle. $a + b =$
- A. 180°
 - B. c
 - C. $c/2$
 - D. $180^\circ - c$
 - E. $180^\circ - c/2$



38. In the figure, O is the centre of the (87) circle. If $AB = 12$ and $AC = 13$, then $\cos \theta =$
- A. $5/12$
 - B. $5/13$
 - C. $12/13$
 - D. $12/25$
 - E. $13/25$

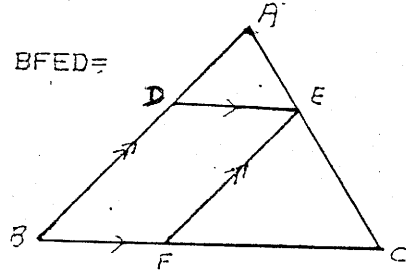


39. In the figure, C is the centre of the (87) circle. ABCD is a straight line. AGR touches the circle at Q. If $\angle DAR = 20^\circ$, then $\angle DQR =$
- A. 35°
 - B. 40°
 - C. 55°
 - D. 65°
 - E. 70°



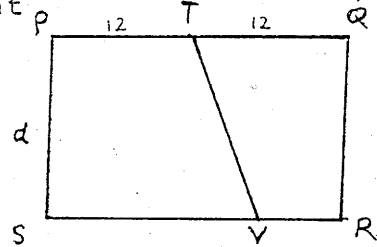
40. In the figure, $DE \parallel BC$ and $AB \parallel EF$.

- (87) If $AE : EC = 1 : 2$, then
 area of $\triangle ADE$: area of parallelogram $BFED =$
 A. 1 : 2
 B. 1 : 3
 C. 1 : 4
 D. 1 : 5
 E. 1 : 6



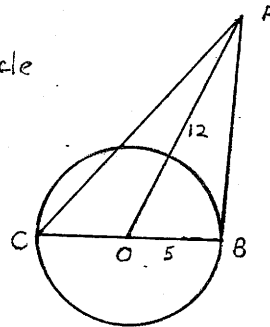
41. In the figure, PQRS is a rectangle with
 (88) $PQ = 24$ and $PS = d$. T is the mid-point
 of PQ. V is a point on SR and
 area PTVS

- = 2. $SV =$
 area TQRV
 A. 14
 B. 16
 C. 18
 D. 20
 E. 22



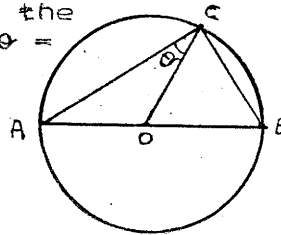
42. In the figure, O is the centre of the circle
 (88) of radius 5. AB is a tangent and
 $AO = 12$. $AC =$

- A. 13
 B. 17
 C. $\sqrt{219}$
 D. $\sqrt{244}$
 E. $\sqrt{269}$



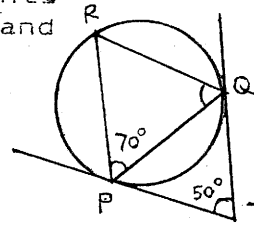
43. In the figure, O is the centre of the
 (88) circle of diameter 13. $AC = 12$. $\sin \theta =$

- A. $5/12$
 B. $5/13$
 C. $\sqrt{313}/13$
 D. $12/13$
 E. $13/12$



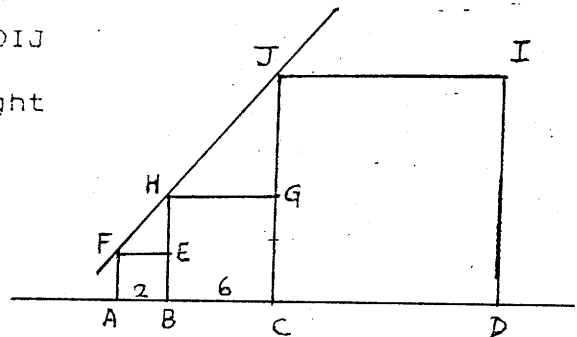
44. In the figure, TP and TQ are tangents
 (88) to the circle PQR. If $\angle RPQ = 70^\circ$ and
 $\angle PTQ = 50^\circ$, then $\angle RQP =$

- A. 20°
 B. 45°
 C. 50°
 D. 60°
 E. 70°

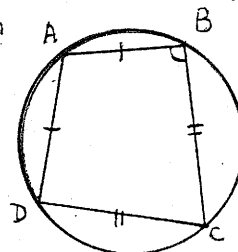


45. In the figure, ABEF, BCGH and CDIJ
 (88) are three squares. If $AB = 2$,
 $BC = 6$ and F, H, J lie on a straight
 line, then $CD =$

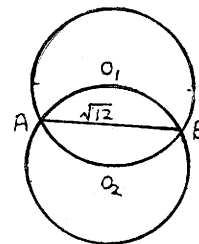
- A. 8
 B. 10
 C. 12
 D. 16
 E. 18



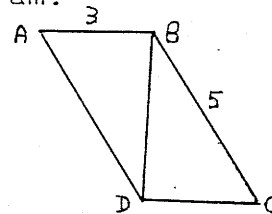
46. ABCD is a cyclic quadrilateral with
 (88) $AB = AD$ and $CB = CD$. Find $\angle ABC$.
 A. 75°
 B. 90°
 C. 105°
 D. 120°
 E. It cannot be found.



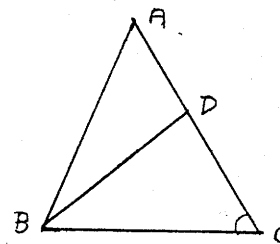
47. In the figure, O_1 and O_2 are the centres
 (88) of the two circles, each of radius r and
 $AB = \sqrt{12}$. Find r .
 A. $1/2$
 B. 2
 C. 4
 D. 6
 E. 8



48. In the figure, ABCD is a parallelogram.
 (88) $AB \perp BD$, $AB = 3$ and $BC = 5$. $AC =$
 A. 10
 B. 12
 C. $\sqrt{13}$
 D. $\sqrt{26}$
 E. $2\sqrt{13}$



49. In the figure, if $AB = AC$ and $AD = BD = BC$, then $\angle ACB =$
 (88) A. 30°
 B. 32°
 C. 36°
 D. 40°
 E. 72°

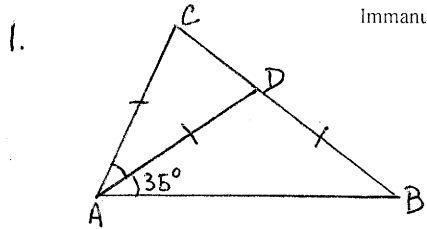


ANSWERS

1.E	2.D	3.C	4.B	5.E	6.C	7.A	8.D	9.C	10.D
11.D	12.E	13.C	14.E	15.E	16.C	17.B	18.A	19.B	20.A
21.C	22.D	23.C	24.E	25.D	26.B	27.B	28.C	29.D	30.A
31.D	32.D	33.C	34.A	35.B	36.B	37.E	38.B	39.C	40.C
41.D	42.C	43.B	44.B	45.E	46.B	47.B	48.E	49.E	

Plane Geometry.

Immanuel Lutheran College

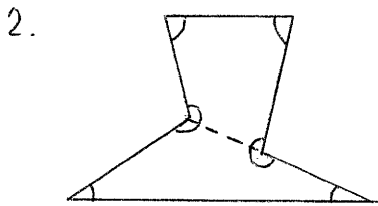


1. since $AD = DB$.
 $\therefore \angle DBA = \angle DAB = 35^\circ$.

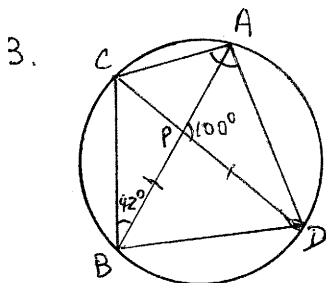
$\therefore \angle ADC = \angle DBA + \angle DAB = 35^\circ + 35^\circ = 70^\circ$

since $AC = AD$.
 $\angle ACD = \angle ADC = 70^\circ$

$\therefore \angle CAD = 180^\circ - 70^\circ - 70^\circ = 40^\circ$ (E.)

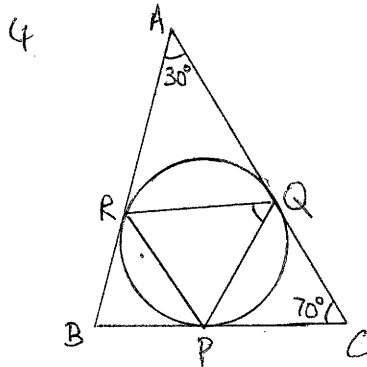


2. sum of six angle
 sum of Δ + sum of Δ + 180°
 $= 180^\circ + 360^\circ + 180^\circ = 720^\circ$ (D.)



3. since $BP = PD$.
 $\therefore \angle PBD = \angle PDB$.
 $\angle PBD + \angle PDB = 100^\circ$
 $\therefore \angle PDB = 50^\circ$

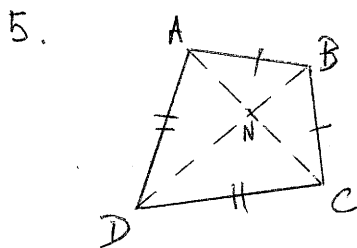
$\angle CDA - \angle CDB = 42^\circ$
 $\therefore \angle BAD = 180^\circ - 100^\circ - 42^\circ = 38^\circ$.
 $\angle CAB = \angle CDB = 50^\circ$
 $\angle CAD = \angle BAD + \angle CAB = 38^\circ + 50^\circ = 88^\circ$ (C.)



4. since AR, AQ are tangents.
 $\therefore AR = AQ$.
 $\therefore \angle AQR = \angle ARQ = \frac{180^\circ - 30^\circ}{2} = 75^\circ$.

QC & PC are tangents.
 $QC = PC$.
 $\therefore \angle CQP = \frac{180^\circ - 70^\circ}{2} = 55^\circ$.

$\angle PQR = 180^\circ - 75^\circ - 55^\circ = 50^\circ$ (B.)

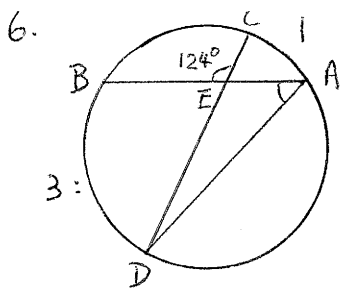


5.

(1) In ΔABC ,
 $AB = BC$.
 $\therefore \angle BAC = \angle BCA$.
 In ΔACD ,
 $AD = CD$.
 $\angle DAC = \angle ACD$.
 $\angle BAD = \angle BAC + \angle DAC = \angle BCA + \angle ACD = \angle BCD$.
 \therefore (1) is true.

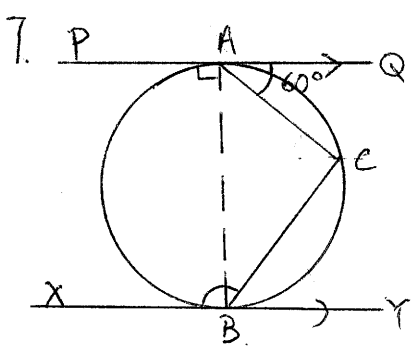
(2) the line segment is equidistant from the end points of the segment, the lines are perpendicular. $\therefore AC \perp BD$.

(3) $AB = BC$
 $AD = CD$
 and BD is common.
 $\therefore \Delta ABD \cong \Delta CBD$ (S.S.S.)
 $\therefore \angle ABD = \angle CBD$.
 $AB = BC$
 & BN is common.
 $\therefore \Delta ABN \cong \Delta CBN$ (S.A.S.)
 $\therefore BD$ bisects AC .
 \therefore (1), (2) and (3) are true. (E.)



6. since $\widehat{BD} : \widehat{AC} = 3:1$
 $\therefore \angle BAD = 3\angle ADC$

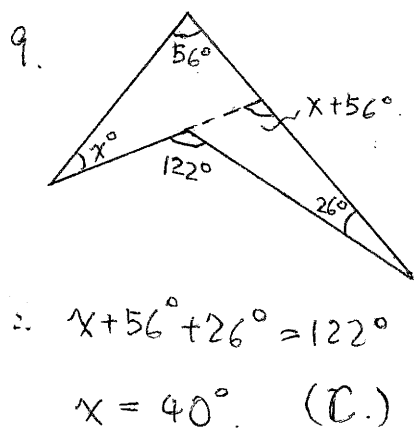
In $\triangle ADE$
 $180^\circ = 124^\circ + \angle BDA + \angle ADC$
 $\angle BDA + \frac{1}{3}\angle BDA = 56^\circ$
 $\angle BDA = 42^\circ$ (C)



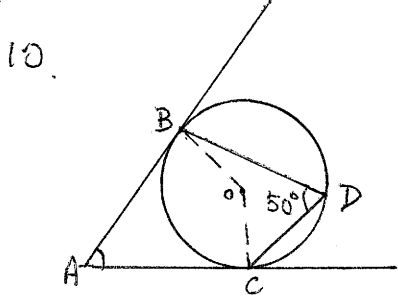
7. since PQ is a tangent.
 $\therefore \angle ABC = \angle QAC = 60^\circ$
 $\therefore Q \parallel XY$
 $\therefore \angle ABX = 90^\circ$
 $\therefore \angle CBX = \angle ABC + \angle ABX = 60^\circ + 90^\circ = 150^\circ$ (A.)

8. Let n be the side of polygon.
 interior angle.
 $= 180^\circ \times (n-2)$

exterior angle of all polygon $= 360^\circ$
 $\therefore 180^\circ(n-2) - 360^\circ = 360^\circ$
 $180^\circ(n-2) = 720^\circ$
 $n-2 = 4$
 $\therefore n = 6$ (D.)

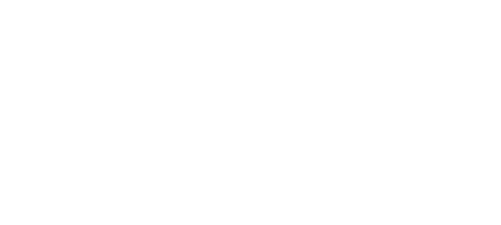
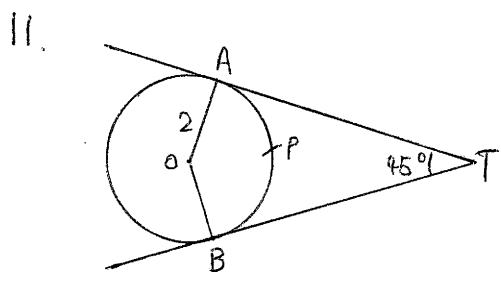


9. $x + 56^\circ + 26^\circ = 122^\circ$
 $x = 40^\circ$ (C.)

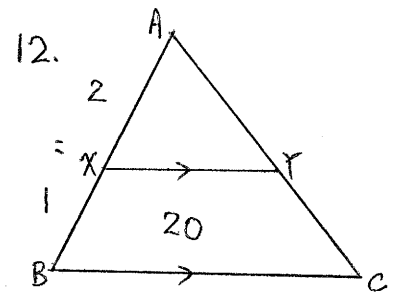


10. Let O be centre.
 $\angle BOC = 2\angle BDC = 2(50^\circ) = 100^\circ$

since $\angle BOC + \angle A = 180^\circ$
 $\therefore \angle A = 180^\circ - 100^\circ = 80^\circ$ (D.)



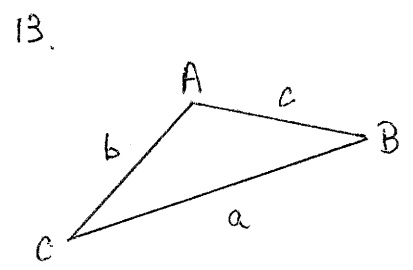
$\angle AOB + \angle ATB = 180^\circ$
 $\angle AOB + 45^\circ = 180^\circ$
 $\angle AOB = 135^\circ = \frac{3\pi}{4}$
 \therefore the length of the arc APB.
 $= r\theta^c$
 $= 2 \cdot (\frac{3\pi}{4})$
 $= \frac{3}{2}\pi$ (D.)



12. since $XY \parallel BC$
 $\triangle AXY \sim \triangle ABC$
 $AX : XB = 2:1$
 $\therefore AB : AX = 3:2$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle AXY} = \left(\frac{AB}{AX}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

\therefore area of $\triangle AXY = \frac{4}{9}$ area of $\triangle ABC$
 area of $\triangle ABC -$ area of $\triangle AXY = 20$
 $\therefore \frac{5}{9}$ area of $\triangle ABC = 20$
 area of $\triangle ABC = 36$ (E.)



$$1) \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{since } a > b > c$$

$$\sin A > \sin B > \sin C$$

$$\therefore A > B > C$$

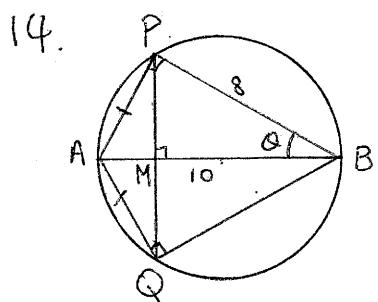
2) $b + c > a$ is true.

sum of any two sides is greater than the other side.

3) If $\angle A > 90^\circ$

$$\angle B + \angle C < \angle A$$

(1) & (2) only (C)



since AB is a diameter.

$$\therefore \angle APB = \angle AQB = 90^\circ$$

$$AP = AQ$$

$$\therefore AB \perp PQ$$

In $\triangle ABP$.

$$AP^2 = AB^2 - BP^2$$

$$AP = \sqrt{10^2 - 8^2}$$

$$AP = 6$$

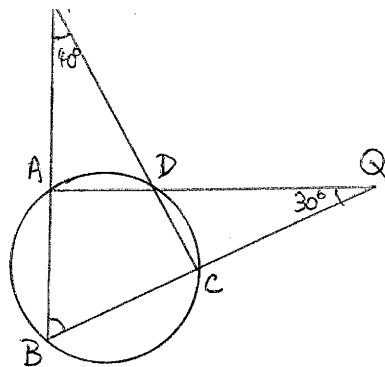
$$\sin \alpha = \frac{6}{10} = \frac{3}{5}$$

$$\frac{PM}{PB} = \sin \alpha$$

$$PM = 8 \left(\frac{3}{5} \right) = \frac{24}{5}$$

$$PQ = 2PM$$

$$= \frac{48}{5} = 9.6 \quad (E)$$



$$\angle B + \angle P = \angle C$$

$$\angle B + 40^\circ = \angle C$$

$$\angle C + 30^\circ = \angle D$$

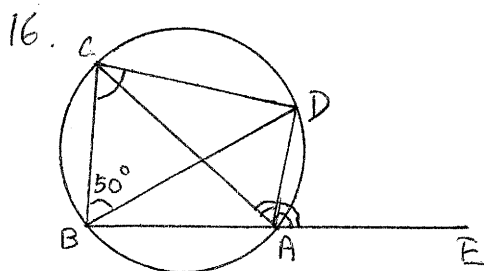
$$\angle B + 70^\circ = \angle D$$

since ABCD is cyclic quad.

$$\therefore \angle B + \angle D = 180^\circ$$

$$\therefore \angle B + 70^\circ = 180^\circ - \angle B$$

$$\angle B = 55^\circ \quad (E)$$



$$\angle CBD = \angle CAD \quad (\text{equal arc})$$

$$\angle CAD = 50^\circ$$

since DA bisects $\angle CAE$.

$$\therefore \angle DAE = \angle CAD = 50^\circ$$

ABCD is a cyclic quad.

$$\therefore \angle BCD = \angle DAE = 50^\circ \quad (E)$$

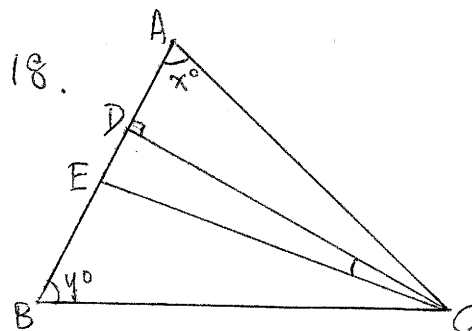
17. exterior angles = 360°

$$\therefore x^\circ + 2x^\circ + 3x^\circ + 4x^\circ + 5x^\circ = 360^\circ$$

$$15x = 360$$

$$x = 24$$

the smallest interior $\angle B$
 angle = $180^\circ -$ the largest
 exterior angle =
 $= 180^\circ - 5(24^\circ)$
 $= 60^\circ \quad (B)$



$$\angle ACB = 180^\circ - x^\circ - y^\circ$$

CE bisects $\angle ACB$.

$$\therefore \angle ACE = \frac{1}{2} \angle ACB$$

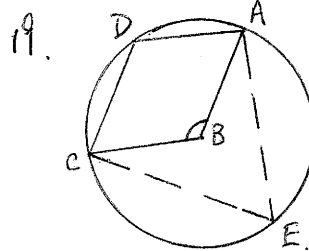
$$= \frac{1}{2} (180^\circ - x^\circ - y^\circ)$$

$$\angle ACD = 90^\circ - x^\circ$$

$$\therefore \angle DCE = \angle ACE - \angle ACD$$

$$= 90^\circ - \frac{1}{2}x^\circ - \frac{1}{2}y^\circ - 90^\circ + x^\circ$$

$$= \frac{1}{2}(x^\circ - y^\circ) \quad (A)$$



$$\angle AEC = \frac{1}{2} \angle ABC$$

$$\angle AEC + \angle ADC = 180^\circ$$

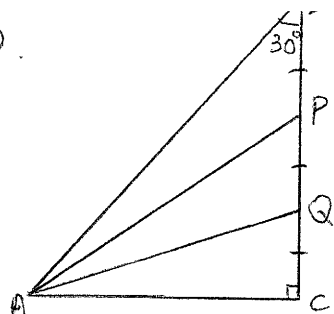
since ABCD is a rhombus.

$$\angle ADC = \angle ABC$$

$$\therefore \frac{1}{2} \angle ABC + \angle ABC = 180^\circ$$

$$\angle ABC = 120^\circ \quad (B)$$

20.



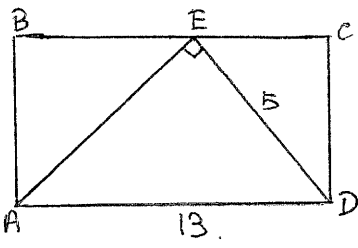
Let $QC = x$.
 $\therefore BC = 3x$.
 $\tan 60^\circ = \frac{3x}{AC}$
 $\sqrt{3} = \frac{3x}{AC}$
 $\frac{x}{AC} = \frac{\sqrt{3}}{3}$

In $\triangle ACQ$

$\tan \angle QAC = \frac{x}{AC}$
 $= \frac{\sqrt{3}}{3}$

$\angle QAC = 30^\circ$. (A.)

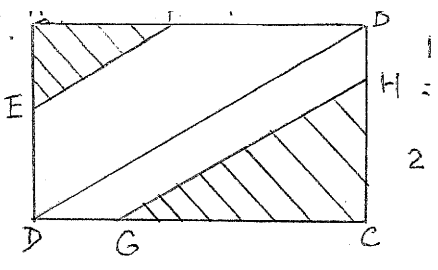
21.



$AE^2 = AD^2 - DE^2$
 $AE = \sqrt{13^2 - 5^2}$
 $AE = 12$

area of ABCD
 $= 2 \cdot \text{area of } \triangle ADE$
 $= 2 \cdot \left[\frac{1}{2} (5)(12) \right]$
 $= 60$. (C.)

22.



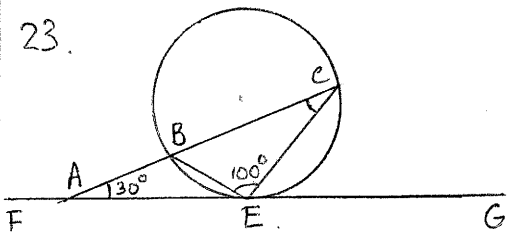
In $\triangle ABD$ & $\triangle AEF$.
 $EF \parallel BD$.
 $\therefore \triangle ABD \sim \triangle AEF$.
 $\frac{AF}{AB} = \frac{1}{2}$.
 $\frac{\text{area of } \triangle AEF}{\text{area of } \triangle ABD} = \left(\frac{1}{2}\right)^2$
 $= \frac{1}{4}$.

since $GH \parallel BD$.

$\therefore \triangle CBD \sim \triangle CHG$.
 $\therefore \frac{CH}{CB} = \frac{2}{3}$.
 $\frac{\text{area of } \triangle CHG}{\text{area of } \triangle CBD} = \left(\frac{2}{3}\right)^2$
 $= \frac{4}{9}$.

\therefore the fraction of shaded area
 $= \frac{1}{2} \left(\frac{1}{4}\right) + \frac{1}{2} \left(\frac{4}{9}\right)$
 $= \frac{25}{72}$. (D.)

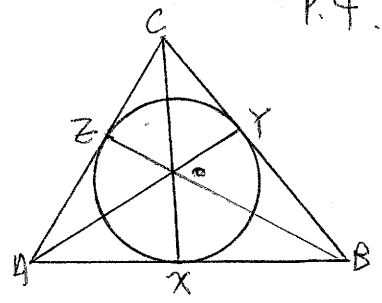
23.



since GF is a tangent.
 $\angle AEB = \angle ACE$.
 $\therefore 2\angle ACE + 30^\circ + 100^\circ = 180^\circ$
 $2\angle ACE = 50^\circ$
 $\angle ACE = 25^\circ$. (C.)

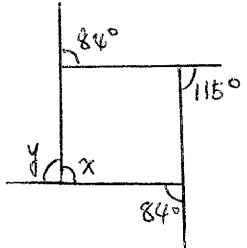
24.

P.4.

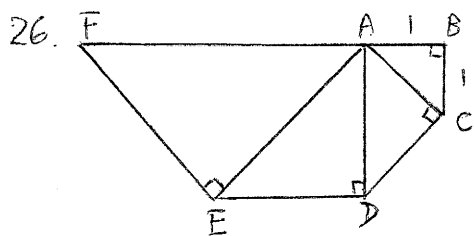


- (I) since AB & AC are tangent \therefore OA bisects $\angle BAC$.
- (II) $\angle OXA = \angle OZA = 90^\circ$.
 $\therefore \angle OXA + \angle OZA = 180^\circ$.
 \therefore A, X, O & Z are concyclic.
- (III) AB & AC are tangent.
 $\therefore AX = AZ$. (E.)

25.



since exterior angles = 360° .
 $\therefore 84^\circ + 115^\circ + 84^\circ + y = 360^\circ$
 $y = 77^\circ$
 $\therefore x = 180^\circ - 77^\circ$
 $= 103^\circ$. (D.)

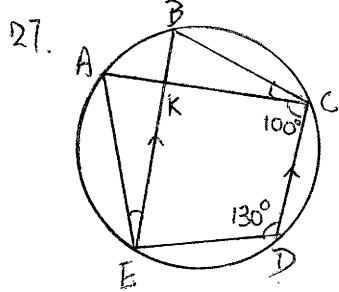


$$AC = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$AD = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$$

$$AE = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

$$AF = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4 \quad (B.)$$



Since $BE \parallel CD$.

$$\therefore \angle BED + \angle CDE = 180^\circ$$

$$\angle BED = 180^\circ - 130^\circ = 50^\circ$$

$$\angle BCA = \angle AEB$$

Since ABCD are concyclic.

$$\therefore \angle BCA + 50^\circ + 100^\circ = 180^\circ$$

$$\angle BCA = 30^\circ \quad (B.)$$

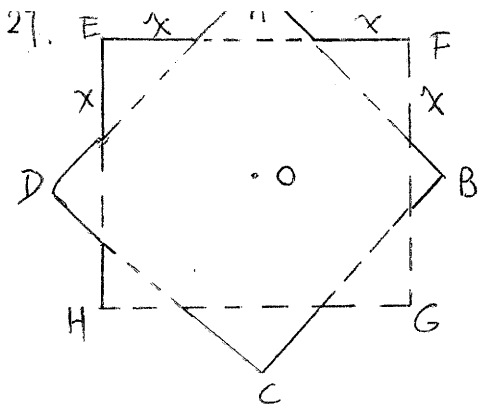
28, Let the smallest interior angle be x .

$$x, x+10^\circ, x+20^\circ, x+30^\circ, x+40^\circ$$

$$\therefore \frac{5}{2}(x + x + 40^\circ) = 540^\circ$$

$$2x + 40^\circ = 216^\circ$$

$$x = 88^\circ \quad (C)$$

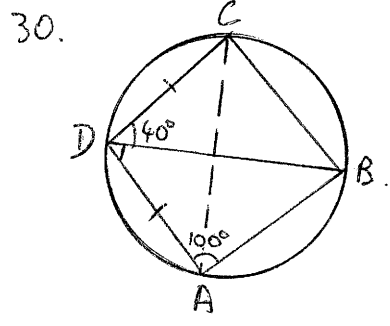


$$2x + \sqrt{x^2 + x^2} = 1$$

$$2x + \sqrt{2}x = 1$$

$$x(2 + \sqrt{2}) = 1$$

$$x = \frac{1}{2 + \sqrt{2}} \quad (D.)$$



$$\angle BAC = \angle BDC = 40^\circ$$

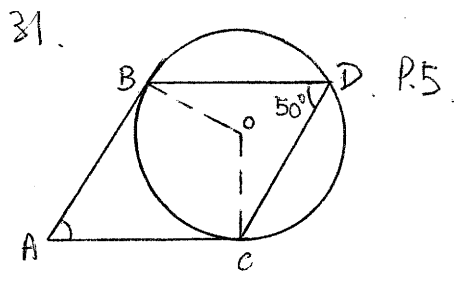
$$\therefore \angle CAD = \angle DAB - \angle BAC = 100^\circ - 40^\circ = 60^\circ$$

Since $AD = CD$

$$\therefore \angle DCA = \angle CAD = 60^\circ$$

$$\therefore \angle ADC = 180^\circ - 2 \times 60^\circ = 60^\circ$$

$$\angle ADB = \angle ADC - \angle BDC = 60^\circ - 40^\circ = 20^\circ \quad (A.)$$

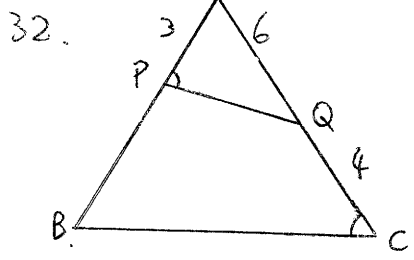


O is the centre.

$$\therefore \angle BOC = 2\angle BDC = 2(50^\circ) = 100^\circ$$

$$\angle A + \angle BOC = 180^\circ$$

$$\therefore \angle A = 80^\circ \quad (D.)$$



Since $LP = LC$, LA is a median.

$$\therefore \triangle ABC \sim \triangle AQP$$

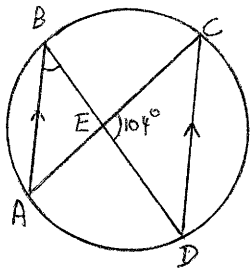
$$\therefore \frac{AB}{AQ} = \frac{AC}{AP}$$

$$\frac{AB}{6} = \frac{10}{3}$$

$$AB = 20$$

$$\therefore PB = AB - AP = 20 - 3 = 17 \quad (D)$$

33.



Let $\angle ABD = x$.

$$\angle ACD = \angle ABD \text{ (equal arc.)} = x$$

Since $AB \parallel CD$.

$$\angle BAC = \angle ACD = x$$

$$\angle CDB = \angle BAC = x$$

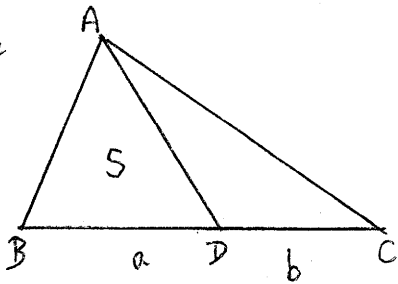
In $\triangle CDE$.

$$\angle ACD + \angle CDB + \angle CED = 180^\circ$$

$$2x + 104^\circ = 180^\circ$$

$$x = 38^\circ \text{ (C.)}$$

34.

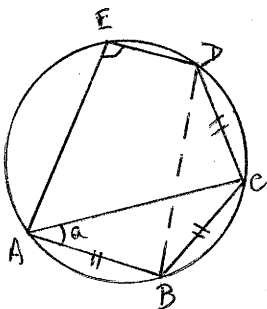


$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle ABD} = \frac{\frac{1}{2}(AB)(BC)\sin B}{\frac{1}{2}(AB)(BD)\sin B}$$

$$\frac{\text{area of } \triangle ABC}{S} = \frac{a+b}{a}$$

$$\text{area of } \triangle ABC = \left(\frac{a+b}{a}\right)S \text{ (A.)}$$

35.



$$\angle BDC = \angle BAC = a \text{ (equal arc.)}$$

since $AB = BC$.

$$\angle BCA = \angle BAC = a$$

$$\therefore BC = CD$$

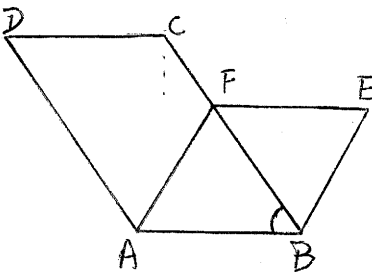
$$\therefore \angle CBD = \angle BDC = a$$

$$\begin{aligned} \therefore \angle ACD &= 180^\circ - \angle BCA - \angle CBD - \angle BDC \\ &= 180^\circ - 3a \end{aligned}$$

$$\text{Since } \angle AED + \angle ACD = 180^\circ$$

$$\begin{aligned} \therefore \angle AED &= 180^\circ - (180^\circ - 3a) \\ &= 3a \text{ (B.)} \end{aligned}$$

36.



$$\text{area of } ABCD = (AB)(BC)\sin \angle CBA$$

$$\text{area of } ABEF = 2 \cdot \text{area of } \triangle ABF$$

$$= 2 \cdot \left(\frac{1}{2}\right)(AB)(BF)\sin \angle CBA$$

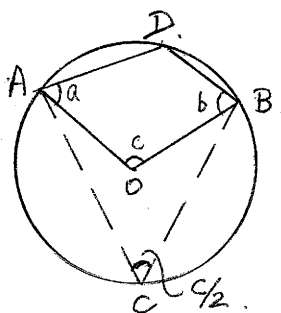
$$= (AB)(BF)\sin \angle CBA$$

$$\therefore \frac{\text{area of } ABCD}{\text{area of } ABEF}$$

$$= \frac{(AB)(BC)\sin \angle CBA}{(AB)(BF)\sin \angle CBA}$$

$$= \frac{BC}{BF} \text{ (B.)}$$

37.



$$\begin{aligned} \angle ACB &= \frac{1}{2}\angle AOB \\ &= \frac{\alpha}{2} \end{aligned} \text{ P.6.}$$

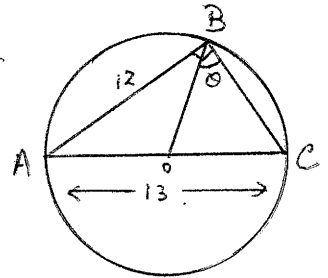
$$\begin{aligned} \angle D &= 180^\circ - \angle ACB \\ &= 180^\circ - \frac{\alpha}{2} \end{aligned}$$

In $\triangle ABD$.

$$a + b + c + \left(180^\circ - \frac{\alpha}{2}\right) = 360^\circ$$

$$\begin{aligned} a + b &= 360^\circ - 180^\circ - \frac{\alpha}{2} \\ &= 180^\circ - \frac{\alpha}{2} \text{ (E.)} \end{aligned}$$

38.



Since AC is a diameter.

$$\therefore \angle ABC = 90^\circ$$

$$\begin{aligned} \therefore BC &= \sqrt{AC^2 - AB^2} \\ &= \sqrt{13^2 - 12^2} \\ &= 5 \end{aligned}$$

$$OB = OC = \text{radius} = \frac{13}{2}$$

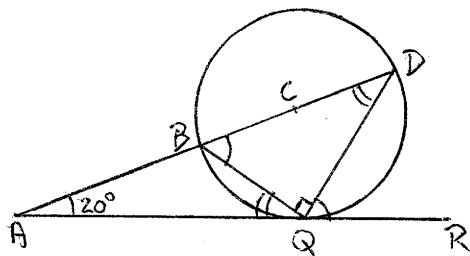
\therefore By cosine rule,

$$OC^2 = OB^2 + BC^2 - 2(OB)(BC)\cos \theta$$

$$\cos \theta = \frac{(6.5)^2 + (5)^2 - (6.5)^2}{2(5)(6.5)}$$

$$= \frac{5}{13} \text{ (B.)}$$

31.



Since BD is a diameter,

$$\angle BQD = 90^\circ$$

$$\angle DBQ = \angle DQR$$

$$\angle BQD = \angle BQA$$

$$\angle DBQ = 20^\circ + \angle BQA$$

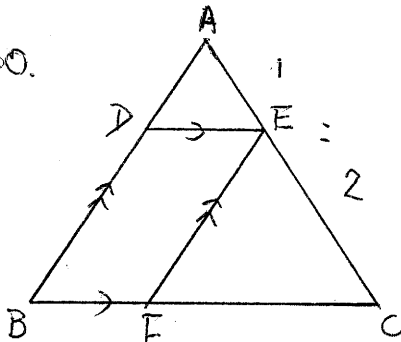
$$\angle DBQ + \angle BQD = 90^\circ$$

$$\angle DBQ + \angle DBQ - 20^\circ = 90^\circ$$

$$\therefore \angle DBQ = 55^\circ$$

$$\angle DQR = 55^\circ \quad (C.)$$

40.



Since $DE \parallel BF$.

$$\therefore \triangle ADE \sim \triangle ABC \sim \triangle EFC$$

$$\therefore \frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \left(\frac{AE}{AC}\right)^2 = \left(\frac{1}{3}\right)^2$$

$$\text{area of } \triangle ADE = \frac{1}{9} \text{ area of } \triangle ABC$$

$$\frac{\text{area of } \triangle EFC}{\text{area of } \triangle ABC} = \left(\frac{CE}{AC}\right)^2 = \left(\frac{2}{3}\right)^2$$

$$\text{area of } \triangle EFC = \frac{4}{9} \text{ area of } \triangle ABC$$

Area of $\triangle EFC$

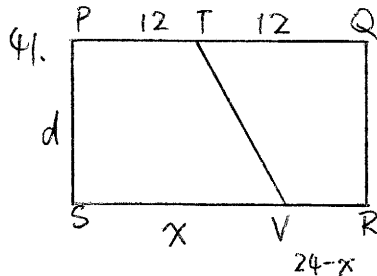
$$= \text{area of } \triangle ABC - \text{area of } \triangle ADE$$

$$- \text{area of } \triangle EFC$$

$$= \frac{4}{9} \text{ area of } \triangle ABC$$

$$\therefore \text{area of } \triangle ADE : \text{area of } \triangle EFC$$

$$= 1 : 4 \quad (C.)$$



Let SV be x .

$$\text{area of } PTVS = \frac{(12+x)d}{2}$$

$$\text{area of } TQRV = \frac{[(24-x)+12]d}{2}$$

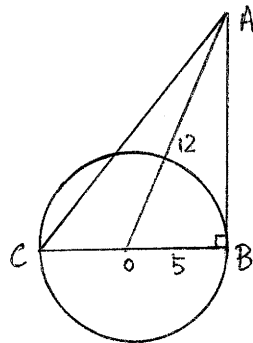
$$\therefore \frac{(12+x)\frac{d}{2}}{(36-x)\frac{d}{2}} = 2$$

$$12+x = 72 - 2x$$

$$3x = 60$$

$$x = 20 \quad (D.)$$

42.



Since AB is a tangent.

$$\therefore \angle CBA = 90^\circ$$

$$\therefore AB^2 = OA^2 - OB^2$$

$$AB = \sqrt{12^2 - 5^2} = \sqrt{119}$$

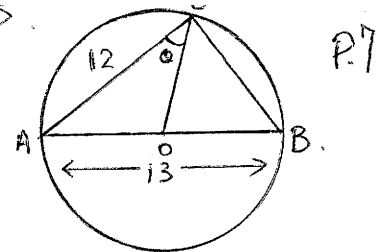
$$OC = OB = 5$$

$$\therefore AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{119 + 10^2}$$

$$= \sqrt{219} \quad (C.)$$

43.



Since AB is a diameter,

$$\therefore \angle ACB = 90^\circ$$

$$BC = \sqrt{AB^2 - AC^2} = \sqrt{13^2 - 12^2} = 5$$

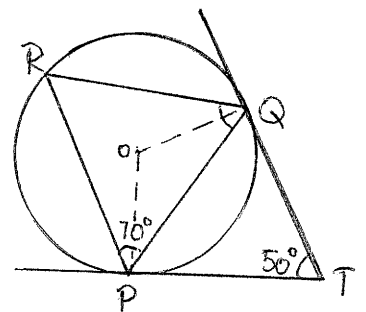
Since $OA = OC$.

$$\therefore \angle OAC = \angle ACO = \theta$$

$$\therefore \sin \theta = \frac{BC}{AB}$$

$$= \frac{5}{13} \quad (B.)$$

44.



Since QT & PT are tangents,

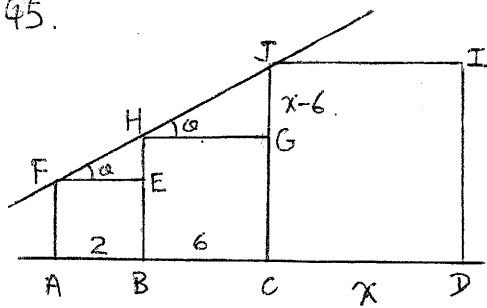
$$\angle QOP + \angle QTP = 180^\circ$$

$$\angle QOP = 180^\circ - 50^\circ = 130^\circ$$

$$\angle PRQ = \frac{1}{2} \angle QOP = 65^\circ$$

$$\therefore \angle RQP = 180^\circ - 70^\circ - 65^\circ = 45^\circ \quad (B.)$$

45.



Let CD be x .

In $\triangle EFH$.

$$\tan \alpha = \frac{6-2}{2} = 2.$$

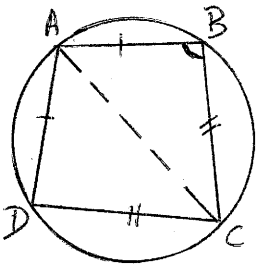
In $\triangle GHJ$.

$$\tan \alpha = \frac{x-6}{6}.$$

$$\frac{x-6}{6} = 2.$$

$$x = 18. \quad (E.)$$

46.



$$AB = AD.$$

$$CD = BC.$$

and AC is common.

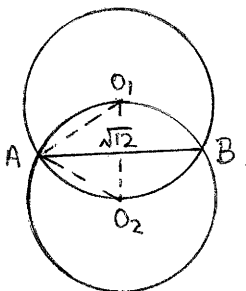
$$\therefore \triangle ACD \cong \triangle ACB$$

$$\therefore \angle ABC = \angle ADC.$$

$$\therefore \angle ABC + \angle ADC = 180^\circ$$

$$\therefore \angle ABC = 90^\circ. \quad (B.)$$

47.



In $\triangle O_1O_2A$.

$$O_1A = O_2A = O_1O_2 = r.$$

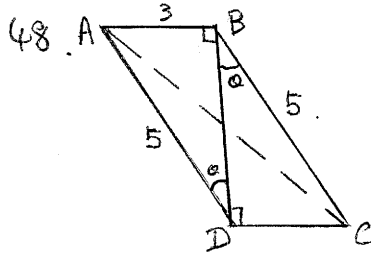
$$\therefore \angle O_1AO_2 = 60^\circ$$

$$\angle O_2AB = \frac{60^\circ}{2} = 30^\circ.$$

$$\therefore \cos 30^\circ = \frac{\sqrt{2}/2}{r}$$

$$r = \frac{\sqrt{2}/2}{\sqrt{3}/2}$$

$$= 2. \quad (B.)$$



$$\sin \theta = \frac{3}{5}.$$

In $\triangle ABC$.

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos(90^\circ + \theta)$$

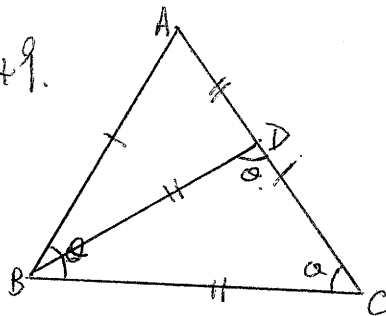
$$AC^2 = 3^2 + 5^2 - 2(3)(5)(-\sin \theta)$$

$$AC^2 = 9 + 25 + 30\left(\frac{3}{5}\right)$$

$$AC = \sqrt{52}$$

$$= 2\sqrt{13}. \quad (E.)$$

49.



Let $\angle ACB = \theta$

Since $AB = AC$

$$BC = BD$$

$$\therefore \angle BDC = \angle ABC = \angle ACB = \theta.$$

$$\therefore \angle A = 180^\circ - 2\theta.$$

$$\angle ABD = \angle A = 180^\circ - 2\theta.$$

$$\therefore (AD = BD).$$

$$\therefore \angle A + \angle ABD = \angle BDC \quad P.8.$$

$$2(180^\circ - 2\theta) = \theta.$$

$$360^\circ = 5\theta$$

$$\theta = 72^\circ \quad (E.)$$