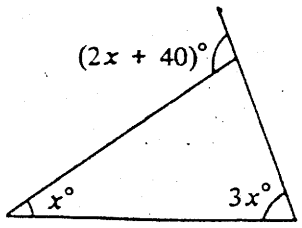
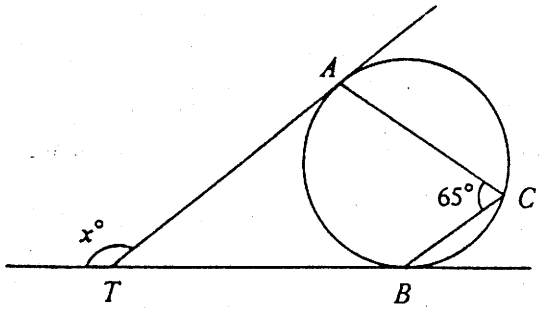
1(80) Find the value of x in Figure



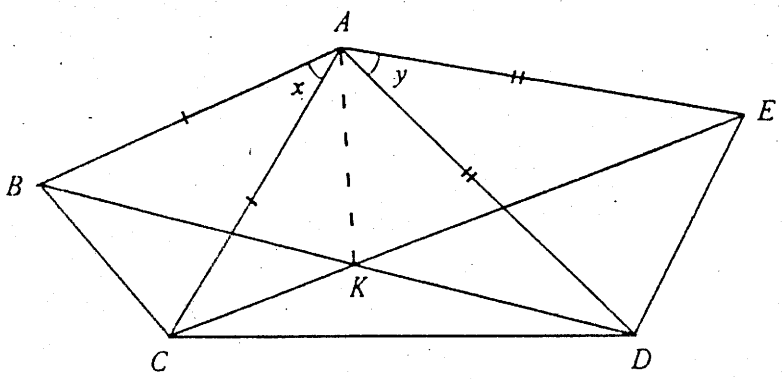
(80) In Figure TA and TB touch the circle at A and B respectively. $\angle ACB = 65°$. Find the value of x.

(5 marks)



Figure

3.(80)



Figure

In Figure , $AB = AC$, $AD = AE$, $\angle x = \angle y$. Straight lines BD and CE intersect at K.

(a) Prove that ΔABD and ΔACE are congruent.

(5 marks)

(b) Prove that $ABCK$ is a cyclic quadrilateral.

(3 marks)

(c) Besides the quadrilateral $ABCK$, there is another cyclic quadrilateral in the figure. Write it down (proof is not required).

(2 marks)

4.(80) In ΔABC (see Figure), $BD = \frac{1}{4} AB$, $CE = \frac{1}{3} AC$, BE intersects CD at P. $\angle x = \angle y$.

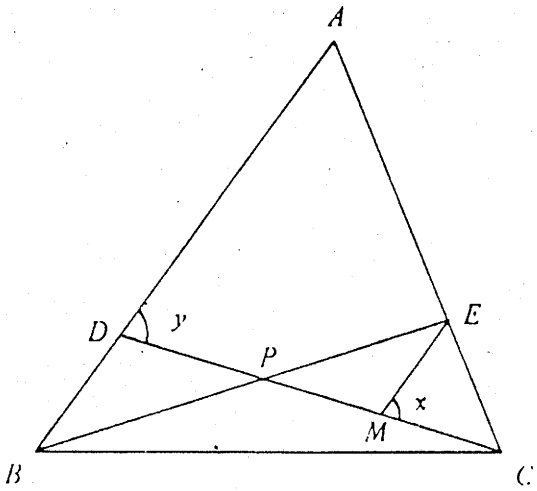
Prove that

(a) ΔEMC and ΔADC are similar and $EM = \frac{1}{4} AB$, (4 marks)

(b) ΔBDP and ΔEMP are congruent. (2 marks)

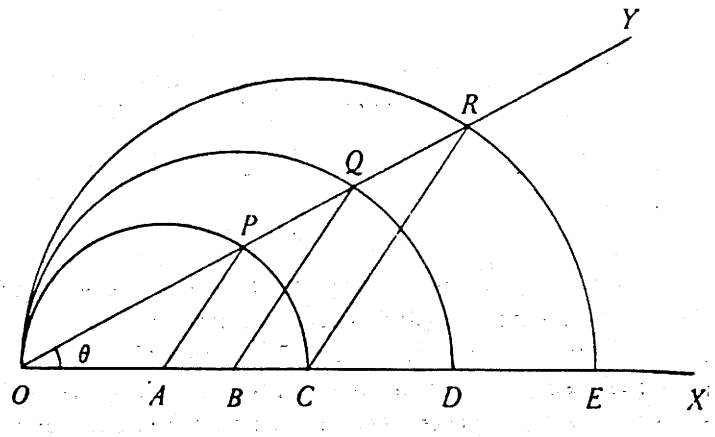
(c) $PM = CM$. (2 marks)

(d) area of triangle BDP is half the area of triangle PEC.



Figure

5(80)



Figure

A, B and C are three points on the line OX such that $OA = 2$, $OB = 3$ and $OC = 4$. With A, B, C as centres and OA, OB, OC as radii, three semi-circles are drawn as shown in Figure . A line OY cuts the three semi-circles at P, Q, R respectively.

(a) If $\angle YOX = \theta$, express $\angle PAX$, $\angle QBX$ and $\angle RCX$ in terms of θ.

(3 marks)

(b) Find the following ratios:

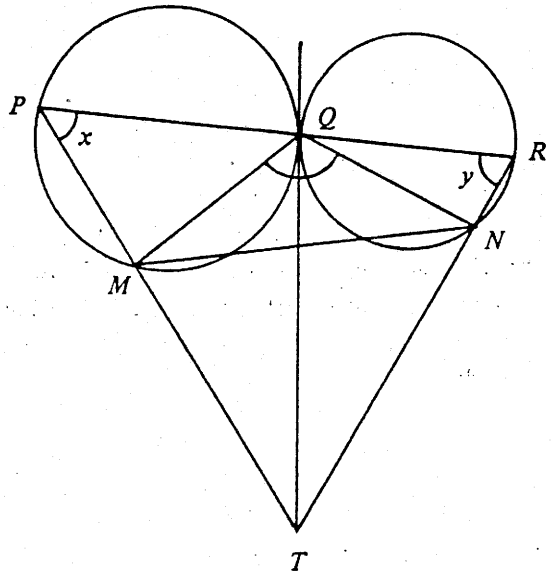
area of sector OAP: area of sector OBQ : area of sector OCR.

(4 marks)

(c) If $RD \perp OX$, calculate the angle θ.

(3 marks)

6(81) In Figure , circles PMQ and QNR touch each other at Q. QT is a common tangent. PQR is a straight line. TP and TR cut the circles at M and N respectively.



Figure

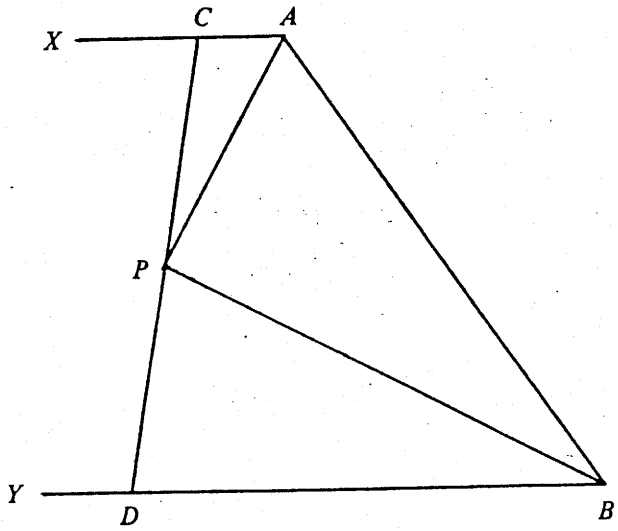
(a) If $\angle P = x$ and $\angle R = y$, express $\angle MQN$ in terms of x and y. (2 marks)

(b) Prove that Q, M, T and N are concyclic. (3 marks)

(c) Prove that P, M, N and R are concyclic. (4 marks)

(d) There are several pairs of similar triangles in the figure.
Name any two pairs (no proof is required). (3 marks)

7(81)



Figure

In Figure , $AX \parallel BY$. AP and BP bisect $\angle XAB$ and $\angle YBA$ respectively, and they meet at P. A straight line passing through P meets AX and BY at C and D respectively.

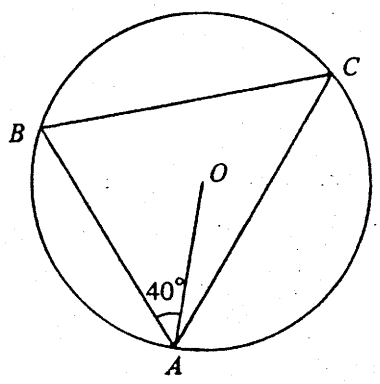
Prove that (a) $\angle APB = 90°$, (4 marks)

(b) $CP = DP$, (5 marks)

(c) $AC + BD = AB$. (3 marks)

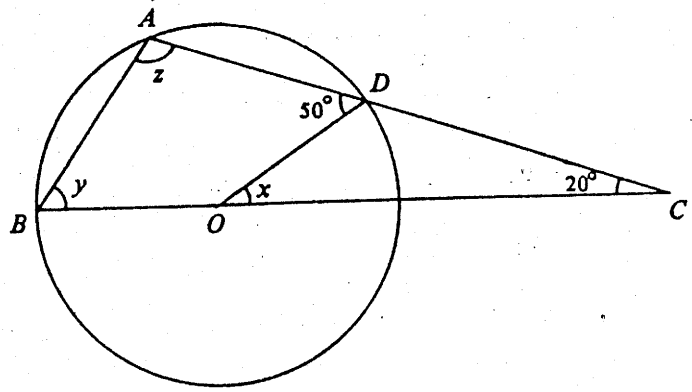
8(81) In Figure , O is the centre of circle ABC. $\angle OAB = 40°$.

Calculate $\angle BCA$.



9(82) In Figure , O is the centre of the circle BAD. BOC and ADC are straight lines. If $\angle ADO = 50°$ and $\angle ACB = 20°$, find x, y and z.

(6 marks)



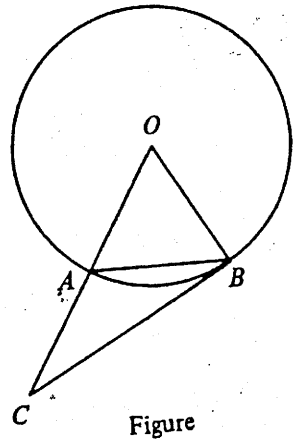
Figure

10(83) In Figure , O is the centre of the circle. A and B are two points on the circle such that OAB is an equilateral triangle. OA is produced to C such that $OA = AC$.

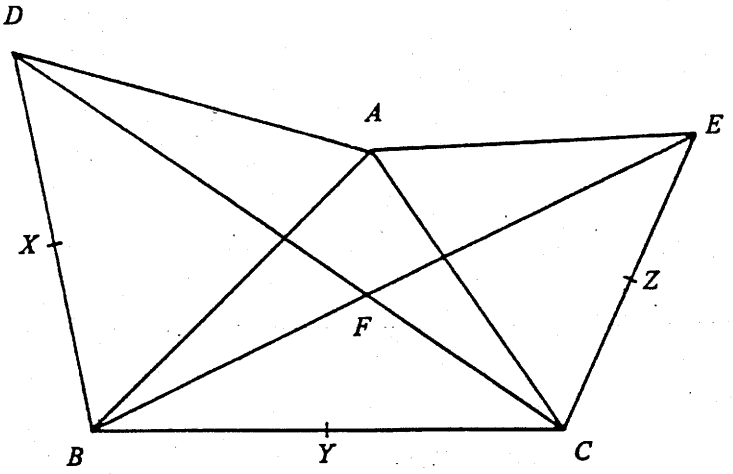
(a) Find $\angle ABC$.

(b) Is CB a tangent to the circle at B? Give a reason for your answer.

(5 marks)



Figure



Figure

In Figure , ΔADB and ΔACE are equilateral triangles. DC and BE intersect at F.

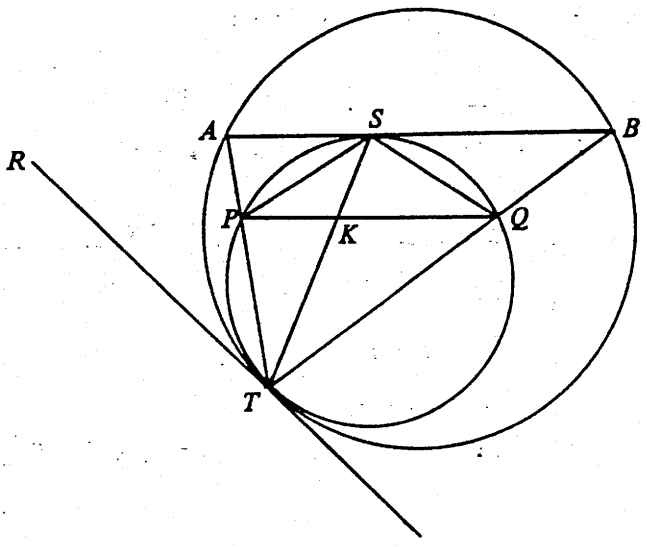
(a) Prove that $DC = BE$.
[Hint: Consider ΔADC and ΔABE.]
(4 marks)

(b) (i) Prove that A, D, B and F are concyclic.

(ii) Find $\angle BFD$.
(4 marks)

(c) Let the mid-points of DB, BC and CE be X, Y and Z respectively. Find the angles of ΔXYZ.
(4 marks)

12(82)



Figure

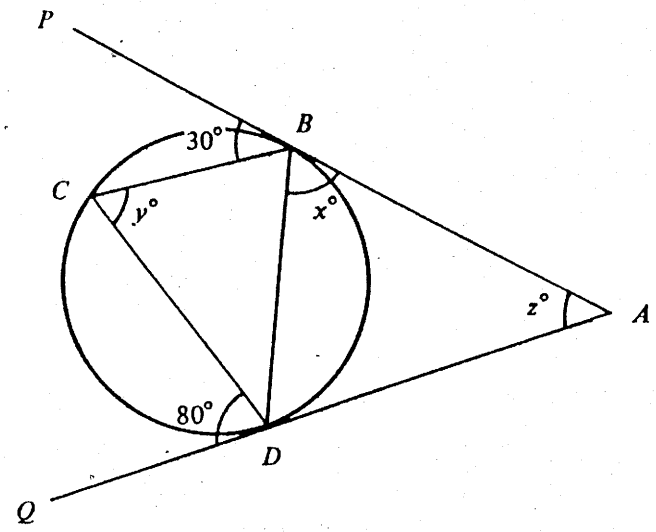
In Figure , two circles touch internally at T. TR is their common tangent. AB touches the smaller circle at S. AT and BT cut the smaller circle at P and Q respectively. PQ and ST intersect at K.

(a) Prove that $PQ \parallel AB$. (4 marks)

(b) Prove that ST bisects $\angle ATB$. (4 marks)

(c) ΔSTQ is similar to four other triangles in Figure 7.
Write down any three of them. (No proof is required.)
(4 marks)

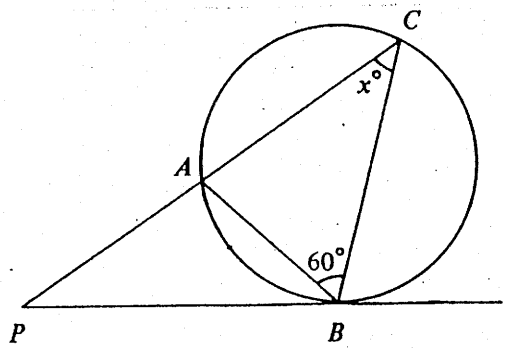
13(84)



Figure

In Figure , AP and AQ touch the circle BCD at B and D respectively. $\angle PBC = 30^\circ$ and $\angle CDQ = 80^\circ$. Find the values of x, y and z.

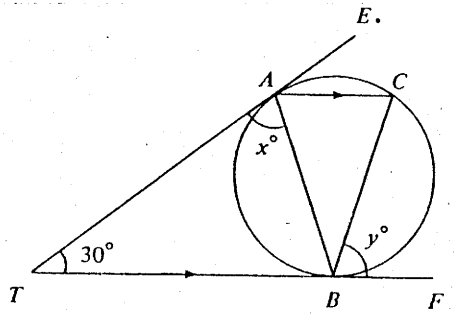
14(85)



Figure

In Figure , PB touches the circle ABC at B. PAC is a straight lin $\angle ABC = 60^\circ$. $AP = AB$. Find the value of x.

15(86)

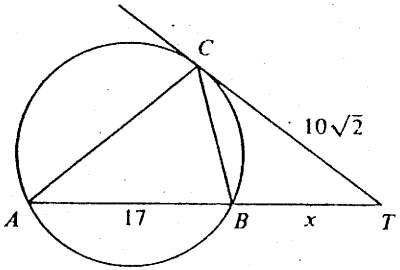


Figure

In Figure , TAE and TBF are tangents to the circle ABC. If $\angle ATB = 30^\circ$ and $AC \parallel TF$, find x and y.

(5 marks)

16(86)



Figure

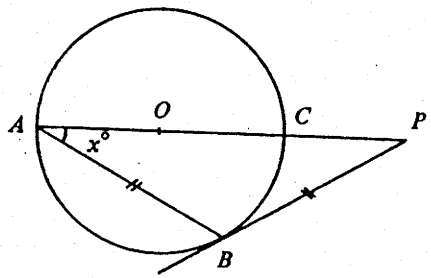
In Figure , A, B and C are three points on the circle. CT is a tangent and ABT is a straight line.

(a) Name a triangle which is similar to ΔBCT.

(b) Let $BT = x$, $AB = 17$ and $CT = 10\sqrt{2}$. Find x.

(6 marks)

17(87)



Figure

In Figure , O is the centre of the circle. $AOCP$ is a straight line, PB touches the circle at B, $BA = BP$ and $\angle PAB = x^\circ$. Find x.

(6 marks)

(88)(a) P is a point inside a square $ABCD$ such that PBC is an equilateral triangle. AP is produced to meet CD at Q.

(i) Draw a diagram to represent the above information.

(ii) Calculate $\angle PAB$ and $\angle PQC$.

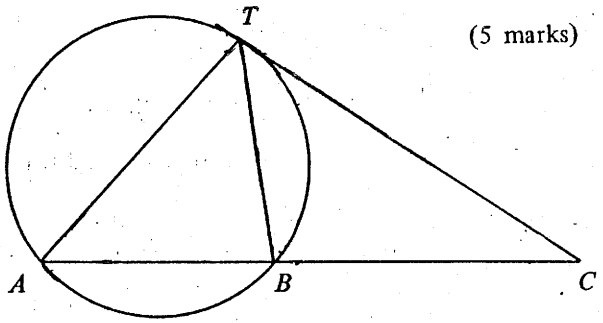
(7 marks)

(b) In Figure , CT is tangent to the circle ABT.

(i) Find a triangle similar to ΔACT and give reasons.

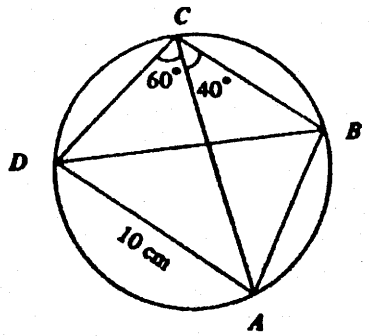
(ii) If $CT = 6$ and $BC = 5$, find AB

(5 marks)



Figure

19(89)



Figure

In Figure , $ABCD$ is a cyclic quadrilateral with $AD = 10$ cm, $\angle ACD = 60°$ and $\angle ACB = 40°$.

(a) Find $\angle ABD$ and $\angle BAD$.

(b) Find the length of BD in cm, correct to 2 decimal places.

marks)

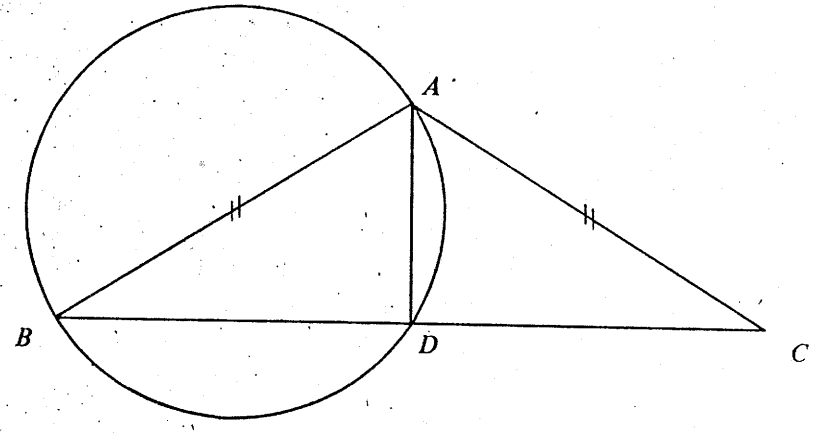
20(89) **AB is a diameter of a circle and M is a point on the circumference. C is a point on BM produced such that $BM = MC$.**

(a) Draw a diagram to represent the above information.

(b) Show that AM bisects $\angle BAC$

(6 marks)

21(90)



Figure

In Figure , AB is a diameter of the circle ADB and ABC is an isosceles triangle with $AB = AC$.

(a) Prove that ΔABD and ΔACD are congruent.

(3 marks)

(b) The tangent to the circle at D cuts AC at the point E. Prove that ΔABD and ΔADE are similar.

(2 marks)

(c) In (b), let $AB = 5$ and $BD = 4$.

(i) Find DE.

(ii) CA is produced to meet the circle at the point F. Find AF.

(7 marks)

22 (91)

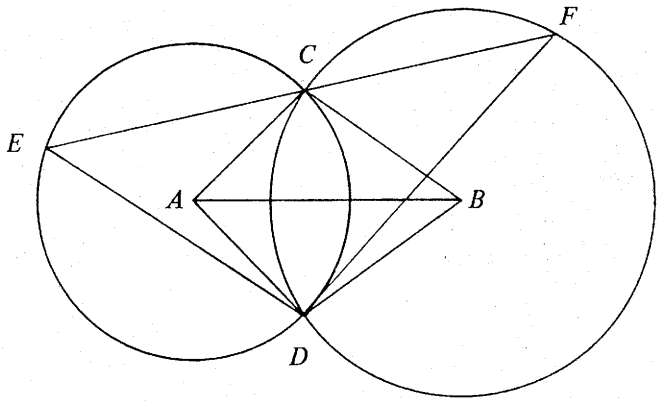


Figure 7

In Figure 7, A , B are the centres of the circles DEC and DFC respectively. ECF is a straight line.

(a) Prove that triangles ABC and ABD are congruent.

(3 marks)

(b) Let $\angle FED = 55°$, $\angle ACB = 95°$.

(i) Find $\angle CAB$ and $\angle EFD$.

(ii) A circle S is drawn through D to touch the line CF at F.

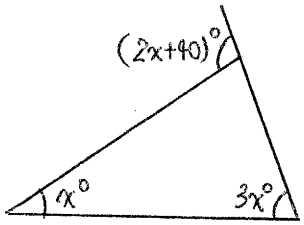
(1) Draw a labelled rough diagram to represent the above information.

(2) Show that the diameter of the circle S is $2DF$.

(9 marks)

HKCEE Problems (Plane Geometry)

1. $x^\circ + 3x^\circ = (2x+40)^\circ$ (ext. $\angle$s.)

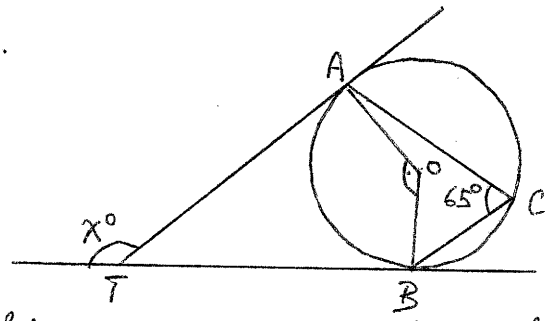


$x + 3x = 2x + 40$

$2x = 40$

$x = 20$.

2.



Let O be the centre of circle.

$\angle AOB = 2\angle ACB$

$= 2(65^\circ)$

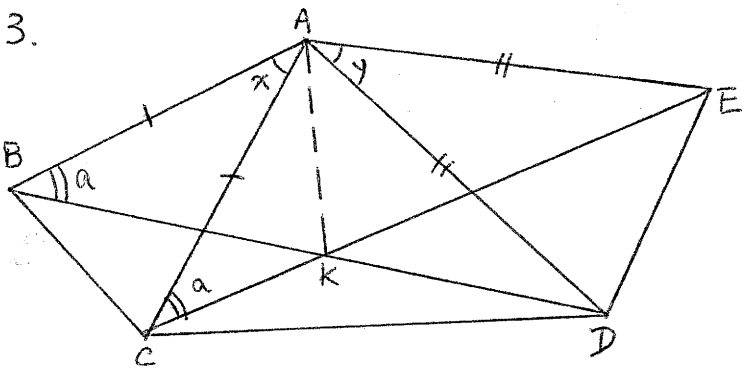
$= 130^\circ$.

AOBT is cyclic.

$\therefore X^\circ = \angle AOB$.

$x = 130$.

3.



a) In $\triangle ABD$ & $\triangle ACE$

$AB = AC$ (given)

$AD = AE$ (given)

$\angle BAD = \angle x + \angle CAD$.

$\angle CAE = \angle y + \angle CAD$

since $\angle x = \angle y$ (given).

$\therefore \angle BAD = \angle CAE$.

$\therefore \triangle ABD \cong \triangle ACE$. (S.A.S.)

b) Since $\triangle ABD \cong \triangle ACE$.

$\angle ABD = \angle ACD = \angle a$.

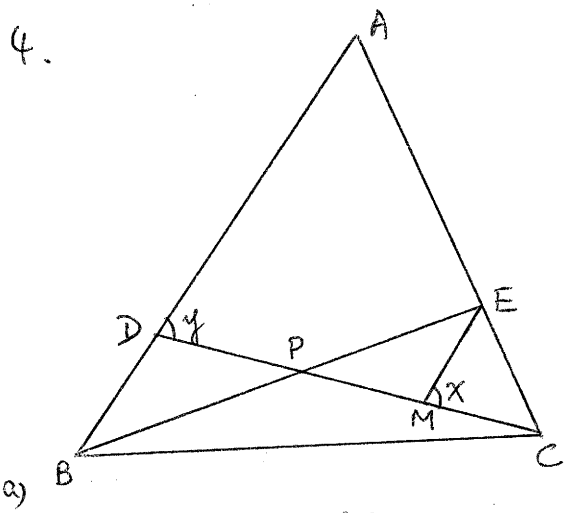
(equal angle, equal arc.)

$\therefore$ A, B, C & K are lie on a circle.

$\therefore$ ABCK is cyclic quadrilateral.

c) Besides ABCK, AEDK is also cyclic quadrilateral.

4.



a) Since $\angle x = \angle y$ (given)

$\angle ECM$ is common.

$\therefore \triangle EMC \sim \triangle ADC$. (A.A.A.)

$\therefore \frac{EM}{AD} = \frac{CE}{AC}$.

$\frac{EM}{AD} = \frac{1}{3}$.

Since $BD = \frac{1}{4}AB$.

$\therefore AD = \frac{3}{4}AB$.

$EM = (\frac{1}{3})(\frac{3}{4}AB)$

$EM = \frac{1}{4}AB$

b) In $\triangle BDP$ & $\triangle EMP$.

$EM = \frac{1}{4}AB = CE$.

$\angle BPD = \angle EPM$ (opp. $\angle$s.)

$\angle BDP = 180^\circ - \angle y$

$\angle EMP = 180^\circ - \angle x$.

$\therefore \angle BDP = \angle EMP$.

Hence $\triangle BDP \cong \triangle EMP$ (A.A.S.).