1. (83) $(x^2y^{-1}) \div (x^{1/2}y^{-1})^2 =$

A. xy B. xy^{-1} C. xy^{-3} D. $x^2y^{1/2}$ E. $x^{-1/2}y^{-2}$

2. (83) If a and b are positive numbers, which of the following is/are true ?

(1) $\log_{10}(a + b) = \log_{10}a + \log_{10}b$
(2) $\log_{10}(a/b) = \log_{10}a - \log_{10}b$
(3) $\dfrac{\log_{10}a}{\log_{10}b} = \dfrac{a}{b}$

A. (1) only B. (2) only C. (3) only
D. (1) and (2) only E. (1),(2) and (3)

3. (84) $(2^{n+1})^2 \times (2^{-2n-1}) \div 4^n =$

A. 1 B. 2^{2n-1} C. 2^{n^2+2n} D. 2^{n^2-2n} E. 2^{-2n+1}

4. (84) If $(\sqrt{3} - \sqrt{2})x = 1$, then

A. $\sqrt{3} + \sqrt{2}$ B. $\dfrac{1}{\sqrt{3} + \sqrt{2}}$ C. $\dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{2}}$ D. $\dfrac{1}{\sqrt{3}} - \dfrac{1}{\sqrt{2}}$ E. $\dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

5. (85) $\log_{10}(a^2 - b^2) =$

A. $\log_{10}a/\log_{10}b$ B. $2\log_{10}(a - b)$ C. $2\log_{10}a - 2\log_{10}b$
D. $\log_{10}(a+b)+\log_{10}(a-b)$ E. $(\log_{10}a+\log_{10}b)(\log_{10}a-\log_{10}b)$

6. (86) If $(10^x)^y = (2\ )(5\ )$, then which of the following must be true ?

A. $xy = z$ B. $xy = 2z$ C. $xy = z^2$ D. $x^y = z$ E. $x^y = 2z$

7. (86) $\left(\sqrt{\dfrac{x}{y}} + \sqrt{\dfrac{y}{x}}\right)^2 =$

A. $\dfrac{(x+y)^2}{xy}$ B. $\dfrac{x^2+y^2}{xy}$ C. $\dfrac{x+y+2}{xy}$ D. $\dfrac{x+y}{xy}$ E. 1

8. (86) If $\log x^2 + \log y^2 = \log z^2$, where x, y and z are positive numbers, which of the following must be true ?

(1) $x^2 + y^2 = z^2$ (2) $\log x + \log y = \log z$ (3) $x^2y^2 = z^2$

A. (1) only B. (2) only C. (3) only
D. (1) and (2) only E. (2) and (3) only

9. (87) If $x + 1/x = 1 + \sqrt{2}$, then $x^2 + 1/x^2 =$

A. 1 B. 3 C. $1 + 2\sqrt{2}$ D. $2 + 2\sqrt{2}$ E. $3 + 2\sqrt{3}$

10. (87) If $3^{2k+1} = 3^{2k} + 6$, then $k =$

A. $-1/4$ B. $-1/2$ C. $1/4$ D. $1/2$ E. 3

11. (87) If $\log_{10}x$, $\log_{10}y$, $\log_{10}z$ are in A.P., then

A. $y = 10^{(x+z)/2}$ B. $y = (x+z)/2$ C. $y^2 = x+z$
D. $y^2 = xz$ E. $y = 10^{xz}$

12. Solve the inequality $x\log_{10}0.1 > \log_{10}10$
(87) A. $x > -1$ B. $x > 1$ C. $x > 100$ D. $x < 1$ E. $x < -1$

13. Let n be a positive integer. Which of the following numbers
(87) is/are odd ?
(1) 2^{2n+1} (2) $2^n + 1$ (3) $3(2^n)$
A. (1) only B. (2) only C. (3) only
D. (2) and (3) only E. (1), (2) and (3)

14.
(88) Simplify $\dfrac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$
A. 7/8 B. 7/4 C. $1 - 2^{n+1}$ D. $2^{n+4} - 1/8$ E. 2^{n+1}

15. If $\log a > 0$ and $\log b < 0$, which of the following is/are
(88) true ?
(1) $\log(a/b) > 0$ (2) $\log b^2 > 0$ (3) $\log(1/a) > 0$
A. (1) only B. (2) only C. (3) only
D. (1) and (2) only E. (2) and (3) only

ANSWERS
1.A 2.B 3.E 4.A 5.D 6.A 7.A 8.E 9.C 10.D
11.D 12.E 13.B 14.A 15.A

Indices, Logarithms and Surds

1. $(x^2y^{-1}) \div (x^{\frac{1}{2}}y^{-1})^2$

$= \dfrac{x^2}{y} \div \left(\dfrac{x^{\frac{1}{2}}}{y}\right)^2$

$= \dfrac{x^2}{y} \cdot \dfrac{y^2}{x}$

$= x \cdot y$ (A.)

2. $a, b > 0$.

1) $\log_{10}a + \log_{10}b = \log_{10}(a \cdot b)$

$\neq \log_{10}(a+b)$.

2). $\log_{10}\left(\frac{a}{b}\right) = \log_{10}a - \log_{10}b$.

?) $\dfrac{\log_{10}a}{\log_{10}b} \neq \dfrac{a}{b}$.

(2) only (B.)

3. $(2^{n+1})^2 \times (2^{-2n-1}) \div 4n$.

$= \dfrac{2^{2n+2} \cdot 2^{-2n-1}}{4^n}$

$= \dfrac{2}{2^{2n}}$

$= 2^{-2n+1}$ (E.)

4. $(\sqrt{3} - \sqrt{2})x = 1$

$x = \dfrac{1}{\sqrt{3}-\sqrt{2}}$.

$= \dfrac{1}{\sqrt{3}-\sqrt{2}} \cdot \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \dfrac{\sqrt{3}+\sqrt{2}}{3-2}$

$= \sqrt{3}+\sqrt{2}$ (A.)

5, $\log_{10}(a^2-b^2)$

$= \log_{10}[(a-b)(a+b)]$

$= \log_{10}(a+b) + \log_{10}(a-b)$

(D.)

6. $(10^x)^y = (2^z)(5^z)$

$10^{x \cdot y} = (2 \cdot 5)^z$

$10^{x \cdot y} = 10^z$

$\therefore x \cdot y = z$ (A.)

7. $\left(\sqrt{\dfrac{x}{y}} + \sqrt{\dfrac{y}{x}}\right)^2$

$= \dfrac{x}{y} + 2\sqrt{\dfrac{x}{y}} \cdot \sqrt{\dfrac{y}{x}} + \dfrac{y}{x}$

$= \dfrac{x}{y} + 2 + \dfrac{y}{x}$.

$= \dfrac{x^2+2xy+y^2}{xy}$

$= \dfrac{(x+y)^2}{x \cdot y}$ (A.)

8. $\log x^2 + \log y^2 = \log z^2$.

$2\log x + 2\log y = 2\log z$

$\log x + \log y = \log z$.

$\therefore$ (2) is true.

$\log x^2 + \log y^2 = \log z^2$

$\log(x^2 \cdot y^2) = \log z^2$.

$x^2 \cdot y^2 = z^2$.

(3) is also true.

$\log x^2 + \log y^2 = \log z^2$

$\not\Rightarrow$ $x^2 + y^2 = z^2$.

$\therefore$ (2) & (3) only (E.)

9. $x + \dfrac{1}{x} = 1 + \sqrt{2}$.

$\therefore \left(x + \dfrac{1}{x}\right)^2 = (1+\sqrt{2})^2$

$x^2 + 2x\left(\dfrac{1}{x}\right) + \dfrac{1}{x^2} = 1 + 2\sqrt{2} + 2$

$x^2 + 2 + \dfrac{1}{x^2} = 3 + 2\sqrt{2}$

$x^2 + \dfrac{1}{x^2} = 1 + 2\sqrt{2}$ (C.)

10. $3^{2k+1} = 3^{2k} + 6$. P.1

$\therefore 3 \cdot 3^{2k} = 3^{2k} + 6$.

$2 \cdot 3^{2k} = 6$.

$3^{2k} = 3$.

$\therefore 2k = 1$

$k = \frac{1}{2}$. (D.)

11. if. $\log_{10}x, \log_{10}y, \log_{10}z$. in A.P.

$\log_{10}x + \log_{10}z = 2\log_{10}y$

$\log_{10}(x \cdot z) = \log_{10}y^2$

$x \cdot z = y^2$. (D.)

12. $x\log_{10}0.1 > \log_{10}10$

$x(-1) > 1$.

$x < -1$. (E.)

13. n be a positive integer.

(1) 2^{2n+1}

$2n+1$ = odd.

But the base is 2.

$\therefore 2^{2n+1} = \underbrace{2 \cdot 2 \cdots 2}_{2n+1 \text{ terms}}$.

$\therefore$ it is even.

(2). $2^n + 1$

Clearly, 2^n is even.

$\therefore 2^n + 1$ is odd.

(3). $3(2^n)$

2^n is even.

$\therefore 3 \cdot (2^n)$ is even.

(2) only. (B)

14. $\dfrac{2^{n+4}-2(2^n)}{2(2^{n+3})}$

$= \dfrac{2^{n+4}-2^{n+1}}{2^{n+4}}$

$= \dfrac{2^{n+1}[2^3-1]}{2^{n+4}}$

$= \dfrac{2^3-1}{2^3}$

$= \dfrac{7}{8}$ (A.)

15. If $\log a > 0$,

$\log b < 0$.

(1) $\log(\frac{a}{b})$

$= \log a - \log b$

$> 0 - \log b$

> 0.

(2) $\log b^2$

$= 2\log b$.

$< 2(0)$

$= 0$.

(3). $\log(\frac{1}{a})$

$= -\log a$.

< 0.

$\therefore$ (1) is true only. (A.)