1. (83) If the lengths of the diagonals of a rhombus are 2 cm and 4 cm respectively, what is the area of the rhombus ?
   A. 2 cm^2 B. 4 cm^2 C. 8 cm^2 D. 16 cm^2 .
   E. It can not be determined.

2. (83) A hollow cylindrical metal pipe, 1 m long, has an external radius and an internal radius of 5 cm and 4 cm respectively. The volume of metal is
   A. 90π cm^3 B. 100π cm^3 C. 180π cm^3
   D. 900π cm^3 E. 1800π cm^3

3. (83) A solid sphere is cut into two hemisphere. The percentage increase in the total surface area is
   A. 25% B. $33^1/3$ % C. 50% D. 75% E. 100%

4. (84) A rectangle box, without a lid, is 40 cm long, 30 cm wide and 10 cm high. The area of the external area of the box is
   A. 2600 cm^2 B. 3400 cm^2 C. 3500 cm^2
   D. 3800 cm^2 E. 12,000 cm^2

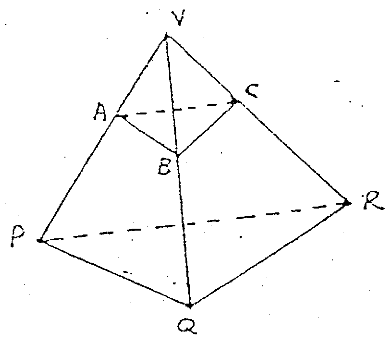
5. (84) The external and internal radii of a hollow metal sphere are 4 cm and 3 cm respectively.
   $$\frac{\text{Volume of metal}}{\text{Volume of the enclosed empty space}} =$$
   A. 1/27 B. 1/3 C. 4/3 D. 37/27 E. 64/27

6. (84) A solid metal sphere of volume 252 cm^3 is melted and recast into 3 smaller solid spheres whose radii are in the ratio 1 : 2 : 3. The volume of the smallest sphere is
   A. 5 cm^3 B. 7 cm^3 C. 14 cm^3 D. 18 cm^3 E. 28 cm^3

7. (84) The base radii of two right circular cylinders are in the ratio 2 : 3. If the two cylinders have the same height, what is the ratio of their curved surface area ?
   A. 2 : 3 B. 4 : 9 C. 8 : 27 D. $\sqrt{8}$: $\sqrt{27}$
   E. None of the above

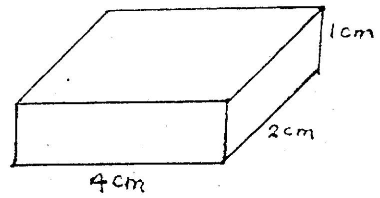
8. (85) A cone of base radius 2r cm and height h cm has a volume of 60 cm^3. The volume of a cylinder of base radius r cm and height 4h cm is
   A. 40 cm^3 B. 120 cm^3 C. 180 cm^3 D. 240 cm^3 E. 360 cm^3

9. (85) In the figure, the volume of the pyramids VABC and VPQR are 27 cm^3 and 64 cm^3 respectively. Planes ABC and PQR are parallel.
   area of $\triangle ABC$: area of $\triangle PQR$ =
   A. $\sqrt{27}$: $\sqrt{64}$ B. $\sqrt{37}$: $\sqrt{64}$
   C. 3 : 4 D. 9 : 16
   E. 27 : 64



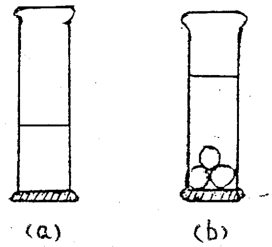
16. (87) A solid rectangular iron block, 4 cm x 2 cm x 1 cm is melted and recast into a cube. The decrease in the total surface area is

A. 1 cm^2
B. 2 cm^2
C. 3 cm^2
D. 4 cm^2
E. 5 cm^2



17. (87) Figure a shows a circular measuring cylinder 4 cm in diameter containing water. Three iron balls, each of diameter 2 cm, are dropped into the cylinder as shown in figure b. What is the rise in the water level ?

A. 1/4 cm
B. 1/3 cm
C. 1/2 cm
D. 1 cm
E. 2 cm



18. (88) A solid iron sphere of radius r is melted and recast into a circular cone and a circular cylinder. If both of them have the same height h and the same base radius r, find h in terms of r.

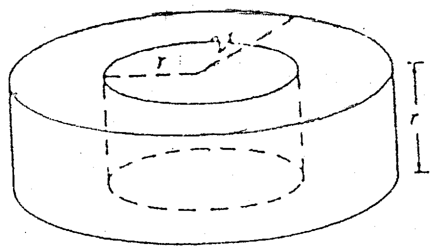
A. r/2 B. 9r/16 C. 2r/3 D. 3r/4 E. r

19. (88) The weight of a gold coin of a given thickness varies as the square of its diameter. If the weights of two such coins are in the ratio 1 : 4, then their diameters are in the ratio

A. 1 :2 B. 2 : 1 C. 1 : 4 D. 4 : 1 E. 1 : 16

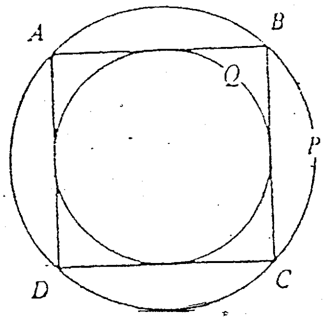
20. (88) A cylindrical hole of radius r is drilled through a solid cylinder, base radius 2r and height r, as shown in the figure. The percentage increase in the total surface area is

A. 0%
B. $16^2/3$ %
C. 20%
D. 25%
E. $33^1/3$ %



21. (88) The figure shows the circumscribed circle P and the inscribed circle Q of the square ABCD. Find area of P : area of Q.

A. $\sqrt{2}$: 1
B. 2 : 1
C. $2\sqrt{2}$: 1
D. π : 1
E. 4 : 1

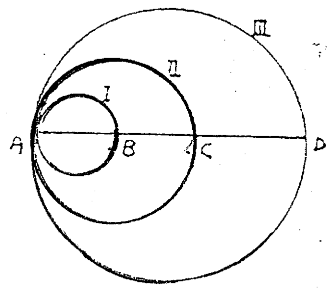


ANSWERS

1.B 2.D 3.C 4.A 5.D 6.B 7.A 8.C 9.D 10.C
11.A 12.E 13.D 14.D 15.A 16.D 17.D 18.E 19.A 20.A
21.B

10. (86) In the figure, ABCD is a straight line with AB=BC=CD. Three circles I, II and III are drawn respectively on AB, AC and AD as diameters. Area of circle I : Area of circle II : Area of circle III =

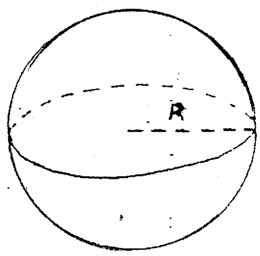
A. 1 : 2 : 3
B. 1 : 2 : 4
C. 1 : 4 : 9
D. 1 : 4 : 16
E. 1 : 8 : 27



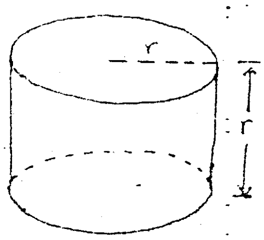
11. (86) In the figure, if $\frac{\text{Volume of the sphere}}{\text{Volume of the right circular cylinder}} = \frac{9}{2}$

then $\frac{R}{r} =$

A. $3/2$
B. $3/\sqrt{2}$
C. 3
D. $\sqrt[3]{9}/\sqrt[3]{2}$
E. $9/2$



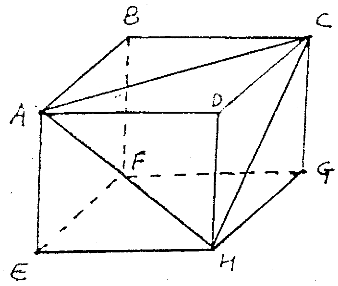
Sphere



Right circular cylinder

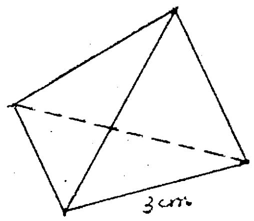
12. (86) ABCDEFGH is a cube of side 3 cm. A tetrahedron DACH is cut away along the plane ACH. The volume of the remaining solid is

A. 6 cm^3
B. 9 cm^3
C. 13.5 cm^3
D. 18 cm^3
E. 22.5 cm^3



13. (86) The total surface area of a regular tetrahedron of side 3 cm is

A. $9\sqrt{3}/4$ cm^2
B. 9 cm^2
C. $27\sqrt{3}/4$ cm^2
D. $9\sqrt{3}$ cm^2
E. $12\sqrt{3}$ cm^2

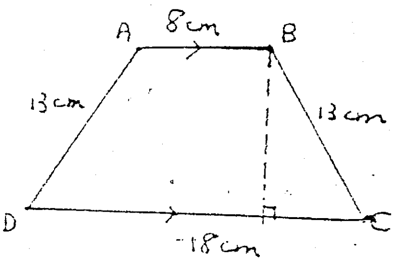


14. (87) The radii of two solid spheres made of the same material are in the ratio 2 : 3. If the smaller sphere weighs 16 kg, then the larger one weighs

A. 24 kg B. 36 kg C. 48 kg D. 54 kg E. 60 kg

15. (87) ABCD is a trapezium in which AB // DC, AB = 8 cm, DC = 18 cm, AD=BC=13 cm. Find the area of the trapezium.

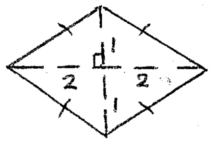
A. 156 cm^2
B. 169 cm^2
C. 216 cm^2
D. 312 cm^2
E. 338 cm^2



HKCEE M.C. (85~88).

Mensurations

1.



since the angle between the diagonals $= 90°$

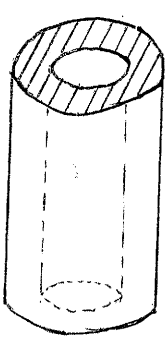
area of the rhombus.

$= 4 \times$ area of triangles

$= 4 \times \frac{1}{2}(1)(2)$

$= 4\ cm^2$. (B.)

2. the vol. of metal

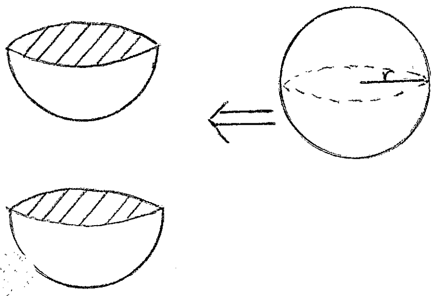


$[\pi(5)^2(100) - \pi(4)^2(100)]\ cm^3$

$= 900\pi\ cm^3$

(D.)

3. Let r be the radius.



original surface area.

$= 4\pi r^2$.

area increased

$= 2 \times \pi r^2$

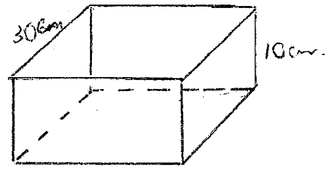
$= 2\pi r^2$.

% increase.

$= \frac{2\pi r^2}{4\pi r^2} \cdot 100\%$

$= 50\%$. (C.)

4.



the area of external area.

$= (30 \times 40 + 10 \times 40 \times 2 + 10 \times 30 \times 2)\ cm^2$

$= 2600\ cm^2$. (A.)

5. vol. of metal.

vol. of the enclosed empty space

$= \frac{\frac{4}{3}\pi(4)^3 - \frac{4}{3}\pi(3)^3}{\frac{4}{3}\pi(3)^3}$

$= \frac{64 - 27}{27} = \frac{37}{27}$. (D.)

6. Let r be the radius of the smallest sphere.

$\therefore \frac{4}{3}\pi r^3 + \frac{4}{3}\pi(2r)^3 + \frac{4}{3}\pi(3r)^3 = 252$

$36(\frac{4}{3}\pi r^3) = 252$

$\frac{4}{3}\pi r^3 = 7$.

$\therefore$ the vol. of smallest sphere

$= 7\ cm^3$. (B.)

7. Let the radii be. 2r & 3r. resp.

curved surface area

$= 2\pi r h$.

$\therefore$ the ratio. of curved surface area.

$= 2 : 3$. (A.)

8. $\frac{1}{3}\pi(2r)^2 h = 60$.

$\frac{4}{3}\pi r^2 h = 60$

$\pi r^2 h = 45\ cm^3$

the vol. of cylinder.

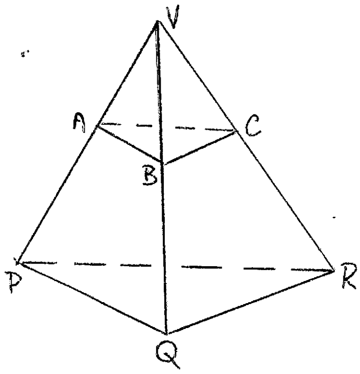
$= \pi r^2(4h)$

$= 4(\pi r^2 h)$

$= 4(45)\ cm^3$

$= 180\ cm^3$. (C.)

9.



since planes. ABC // PQR.

$\therefore \left(\frac{AB}{PQ}\right)^3 = \frac{\text{vol. of VABC}}{\text{vol. of VPQR}}$

$\left(\frac{AB}{PQ}\right)^3 = \frac{27}{64}$

$\frac{AB}{PQ} = \frac{3}{4}$.

$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \left(\frac{AB}{PQ}\right)^2$

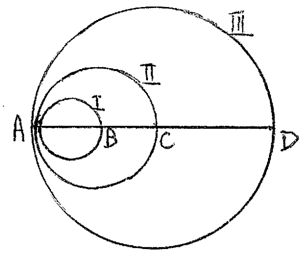
$= \left(\frac{3}{4}\right)^2$

$= 9/16$.

$\therefore$ area of ΔABC : area of ΔPQR

$= 9 : 16$. (D.)

10.



$AB = BC = CD$.

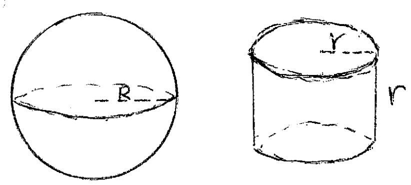
Let $AB = r$.

area of circles I : II : III

$= \pi\left(\frac{r}{2}\right)^2 : \pi r^2 : \pi\left(\frac{3r}{2}\right)^2$

$= \frac{\pi r^2}{4} : \pi r^2 : \frac{9}{4}\pi r^2$

$= 1 : 4 : 9$. (C.)



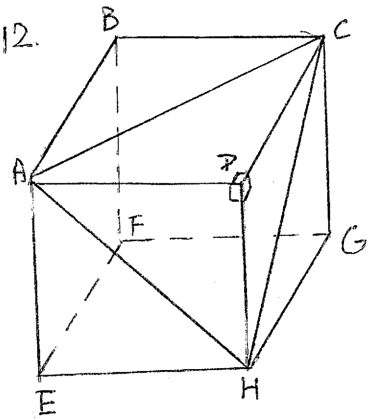
$\frac{\text{vol. of sphere}}{\text{vol. of cylinder}} = \frac{9}{2}$

$\frac{\frac{4}{3}\pi R^3}{\pi(r^2)(r)} = \frac{9}{2}$.

$\frac{R^3}{r^3} = \frac{27}{8}$

$\frac{R}{r} = \frac{3}{2}$. (A.)

12.



vol. of cube $= 3^3\ cm^3$

$= 27\ cm^3$

vol. of ACDH $= \frac{1}{3}\left(\frac{3\times3}{2}\right)\cdot 3\ cm^3$.

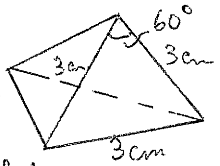
$= \frac{9}{2}\ cm^3$

vol of the remaining solid.

$= (27 - 4.5)\ cm^3$

$= 22.5\ cm^3$. (E.)

13.



total surface area

$= 4 \times$ area of triangles.

$= 4 \cdot \frac{1}{2}(3\times3 \sin 60°)$

$= 4 \cdot \frac{1}{2}(3 \cdot 3 \cdot \frac{1}{2})\ cm^2$

$= 9\ cm^2$ (B.)

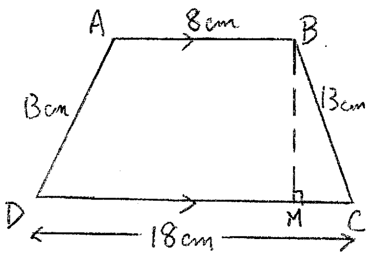
14. wt. $\propto r^3$. Let x be the weight.

$\therefore \left(\frac{2r}{3r}\right)^3 = \frac{16}{x}$.

$x = 16 \cdot \left(\frac{3}{2}\right)^3$

$= 54$ kg (D)

15.



$MC = \frac{DC - AB}{2}$

$MC = \frac{18-8}{2}\ cm = 5\ cm$

$\therefore BM = \sqrt{BC^2 - MC^2}$

$= \sqrt{13^2 - 5^2}$

$= 12$.

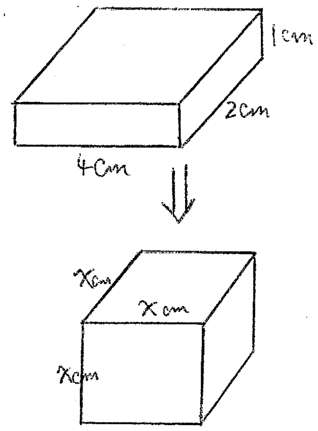
$\therefore$ the area of trapezium.

$= \frac{1}{2}(AB + DC)\cdot(BM)$

$= \frac{1}{2}(8 + 18)(12)\ cm^2$.

$= 156\ cm^2$ (A.)

16.



Let the side be x cm.

$\therefore x^3 = (4\cdot2\cdot1)\ cm^3$

$x^3 = 8\ cm^3$.

$x = 2\ cm$

total surface area of rectangular block.

$= 2(4\times2 + 1\times2 + 4\times1)\ cm^2$

$= 28\ cm^2$.

total surface area of cube

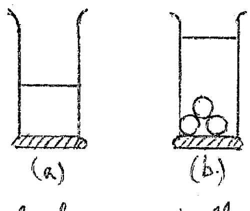
$= 6(2\times2)\ cm^2$.

$= 24\ cm^2$.

decrease in total surface area $= (28 - 24)\ cm^2$

$= 4\ cm^2$. (D.)

17.



the vol. of iron balls.

$= 3\cdot\left[\frac{4}{3}\pi\left(\frac{2}{2}\right)^3\right]$

$= 4\pi\ cm^3$.

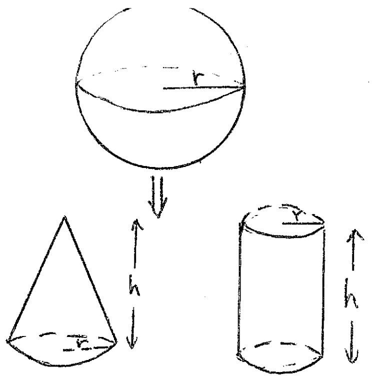
Let h be the rise in water.

$\pi\left(\frac{4}{2}\right)^2 h = 4\pi$.

$4\pi h = 4\pi$

$h = 1$ cm. (D.)

18.



$\therefore \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h + \pi r^2 h.$

$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^2 h.$

$r = h.$ (E.)

$\therefore r : h = 1 : 1$

19. $w \propto d^2$

$w = kd^2.$

$w_1 : w_2 = 1 : 4.$

$\therefore \frac{w_1}{w_2} = \frac{k d_1^2}{k d_2^2}.$

$\frac{1}{4} = \frac{d_1^2}{d_2^2}$

$\frac{d_1}{d_2} = \frac{1}{2}$

$\therefore d_1 : d_2 = 1 : 2$ (A.)

20. area increased after drilled

$= (2\pi r) r.$

$= 2\pi r^2$

area decreased after drilled

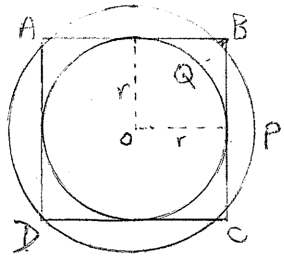
$= 2 \cdot (\pi r^2)$

$= 2\pi r^2$

Since there is no net change of area.

$\therefore$ % increase $= 0\%$

(A.)



Let the radius of Q be r.

$\therefore$ the radius of P.

$= \sqrt{r^2 + r^2}$

$= \sqrt{2} r.$

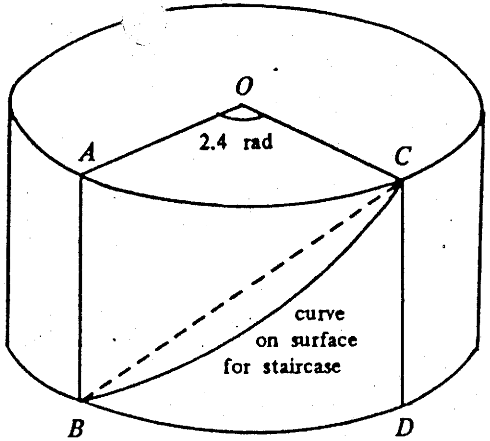
area of P : area of Q

$= \pi(\sqrt{2}r)^2 : \pi r^2$

$= 2 : 1$

HKCEE Problems. Mensuration

1(81) Figure shows a cylinder 10 metres high and 10 metres in radius used for storing coal-gas. AB and CD are two vertical lines on the curved surface of the cylinder. The arc AC subtends an angle of 2.4 radians at the point O, which is the centre of the top of the cylinder.



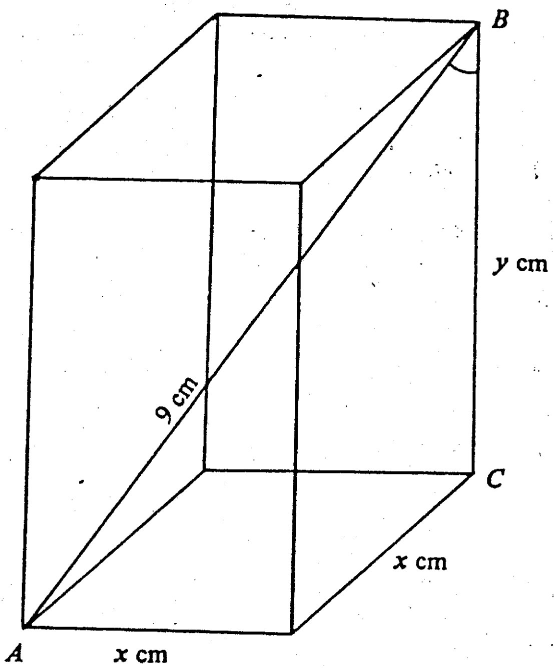
Figure

(a) Inside the cylinder, a straight pipe runs from B to C. Calculate the length of the pipe BC correct to 3 significant figures. (5 marks)

(b) Calculate the area of the curved surface $ABDC$ bounded by the minor arcs AC, BD and the lines AB, CD. (3 marks)

(c) A staircase from B to C is built along the shortest curve on the curved surface $ABDC$. Find the length of the curve. (4 marks)

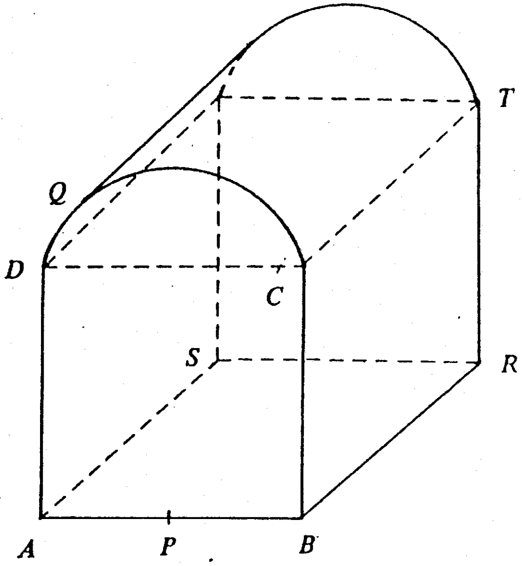
2(82) Figure represents the framework of a cuboid made of iron wire. It has a square base of side x cm and a height of y cm. The length of the diagonal AB is 9 cm. The total length of wire used for the framework (including the diagonal AB) is 69 cm.



(a) Find all the values of x and y. (10 marks)

(b) Hence calculate $\angle ABC$ to the nearest degree for the case in which $y > x$. (2 marks)

3(84)



Figure

In Figure , all vertical cross-sections of the solid that are parallel to $APBCQD$ are identical. $ABCD$, $BRTC$ and $ABRS$ are squares, each of side 20 cm. P is the mid-point of AB. CQD is a circular arc with centre P and radius PC.

(In this question, give your answers correct to 1 decimal place.)

(a) Find, in degrees, $\angle CPD$. (3 marks)

(b) Find, in cm, the length of the arc CQD. (3 marks)

(c) Find, in cm^2, the area of the cross-section $APBCQD$. (3 marks)

(d) Find, in cm^2, the total surface area of the solid. (3 marks)

4(83)

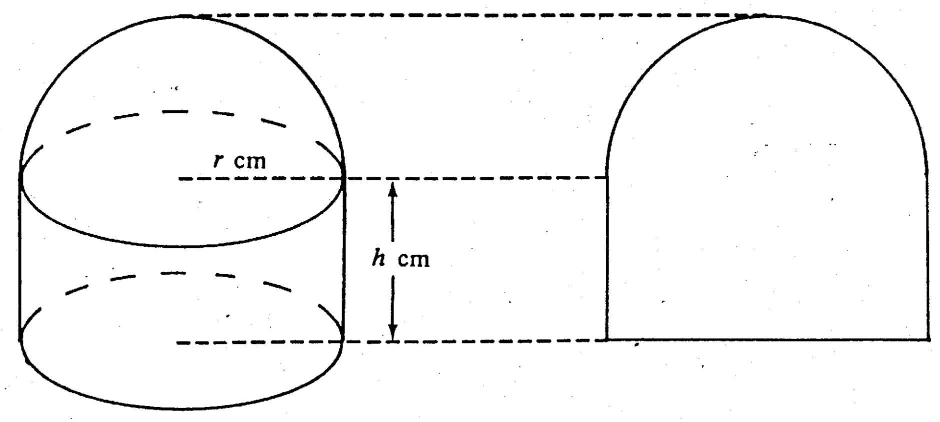


Figure (a)   Figure (b)

The solid in Figure (a) is made up of two parts. The lower part is a right circular cylinder of height h cm and radius r cm; the upper part is a hemisphere of the same radius r cm. The two parts are of the same volume.

(a) Find the ratio $r : h$. (3 marks)

(b) Figure (b) shows a section of the solid through the axis of the cylinder. The perimeter of this section is 136 cm.

(i) Calculate r to 2 significant figures.

(ii) Calculate the total external surface area (including the base) of the solid in cm^2 to 1 significant figure.

(9 marks)

5(85)

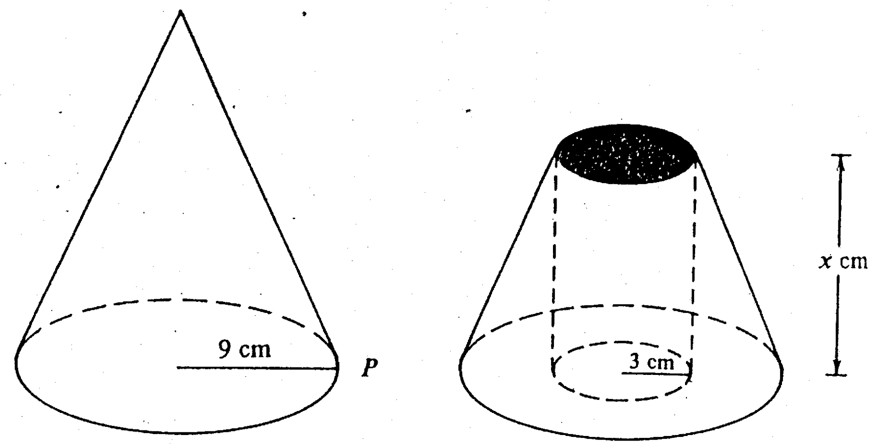


Figure (a)   Figure (b)

Figure (a) shows a solid right circular cone. O is the vertex and P is a point on the circumference of the base. The area of the curved surface is 135π cm^2 and the radius of the base is 9 cm.

(a) (i) Find the length of OP.

(ii) Find the height of the cone.

(5 marks)

(b) The cone in Figure (a) is cut into two portions by a plane parallel to its base. The upper portion is a cone of base radius 3 cm. The lower portion is a frustum of height x cm.

(i) Find the value of x.

(ii) A right cylindrical hole of radius 3 cm is drilled through the frustum (see Figure (b)). Find the volume of the solid which remains in the frustum. (Give your answer in terms of π.)

(7 marks)

6(86)

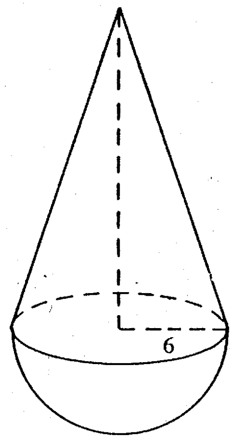


Figure a

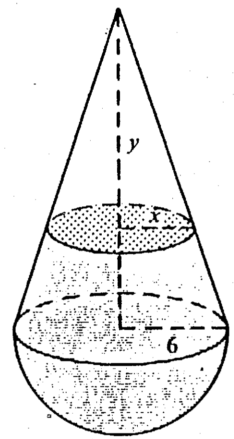


Figure b

Figure a shows a solid consisting of a right circular cone and a hemisphere with a common base which is a circle of radius 6. The volume of the cone is equal to $\frac{4}{3}$ of the volume of the hemisphere.

(a) (i) Find the height of the cone.

(ii) Find the volume of the solid. (Leave your answer in terms of π.)

(6 marks)

(b) (i) The solid is cut into two parts. The upper part is a right circular cone of height y and base radius x as shown in Figure b. Find $\frac{x}{y}$.

(ii) If the two parts in (b)(i) are equal in volume, find y, correct to 1 decimal place.

(6 marks)

7(87)

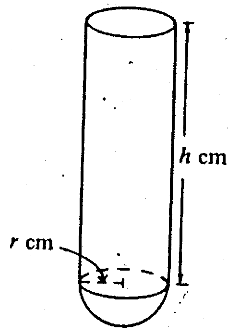


Figure a

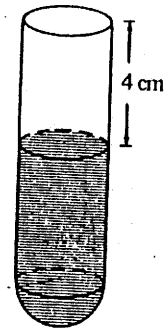


Figure b

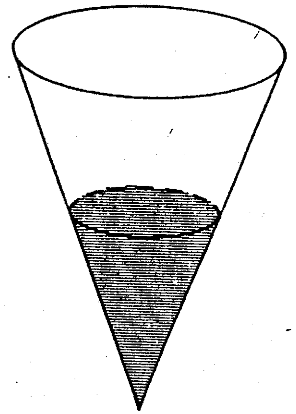


Figure c

Figure a shows a test-tube consisting of a hollow cylindrical tube joined to a hemispherical bowl of the same radius. The height of the cylindrical tube is h cm and its radius is r cm. The capacity of the test-tube is 108π cm^3. The capacity of the hemispherical part is $\frac{1}{6}$ of the whole test-tube.

(a) (i) Find r and h.

(ii) The test-tube is placed upright and water is poured into it until the water level is 4 cm beneath the rim as shown in Figure b. Find the volume of the water. (Leave your answer in terms of π.)

(9 marks)

(b) The water in the test-tube is poured into a right circular conical vessel placed upright as shown in Figure c. If the depth of water is half the height of the vessel, find the capacity of the vessel. (Leave your answer in terms of π.)

(3 marks)

8(89) Figure a shows a rectangular swimming pool 50 m long and 20 m wide. The floor of the pool is an inclined plane. The depth of water is 10 m at one end and 2 m at the other.

(a) Find the volume of water in the pool in m^3. (2 marks)

(b) Water in the pool is now pumped out through a pipe of internal radius 0.125 m. Water flows in the pipe at a constant speed of 3 m/s.

(i) Find the volume of water, in m^3, REMAINING in the pool when the depth of water is 8 m at the deeper end.

(ii) Find the volume of water pumped out in 8 hours, correct to the nearest m^3.

(iii) Let h metres be the depth of water at the deeper end after 8 hours (see Figure b). Find the value of h, correct to 1 decimal place. (10 marks)

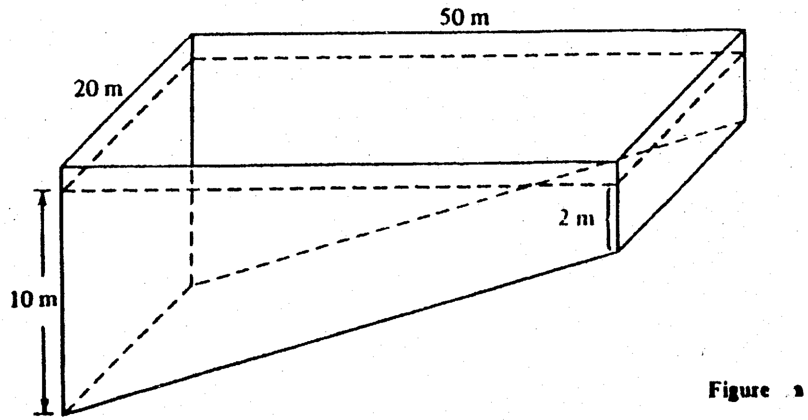


Figure a

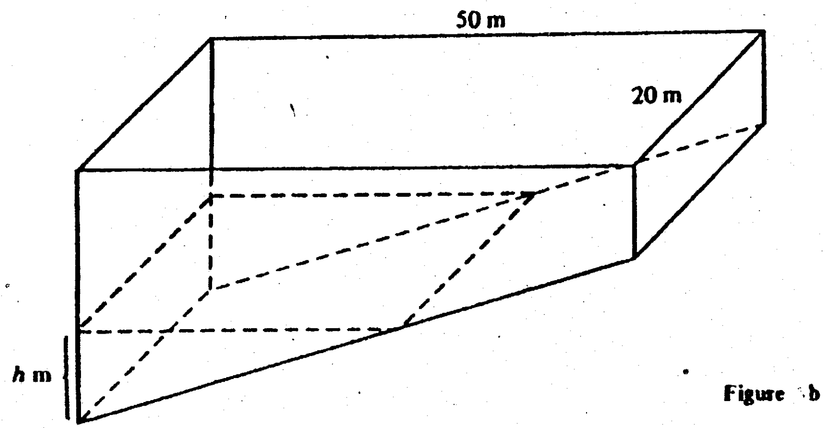


Figure b

9(91)

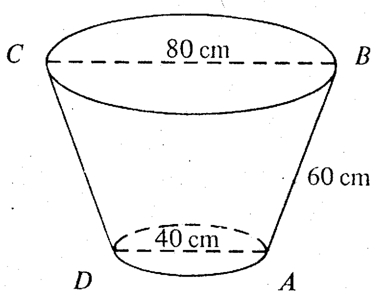


Figure 5a

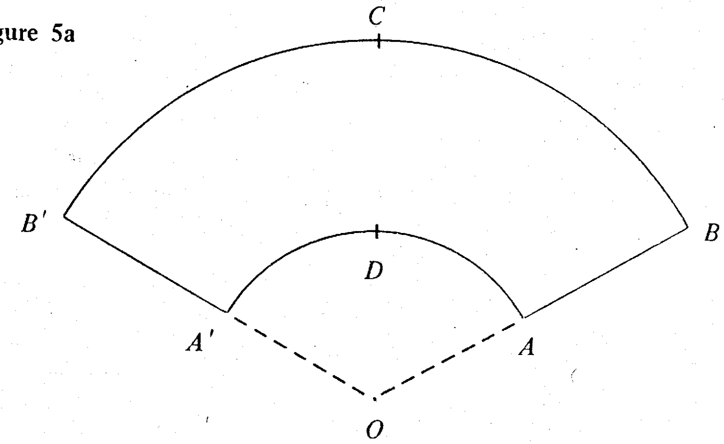


Figure 5b

Figure 5a shows a metal bucket. Its slant height AB is 60 cm. The diameter AD of the base is 40 cm and the diameter BC of the open top is 80 cm. The curved surface of the bucket is formed by the thin metal sheet $ABB'A'$ shown in Figure 5b, where $\widehat{ADA'}$ and $\widehat{BCB'}$ are arcs of concentric circles with centre O.

(a) Find OA and $\angle AOA'$. (5 marks)

(b) Find the area of the metal sheet $ABB'A'$, leaving your answer in terms of π. (3 marks)

(c) There is an ant at the point A on the outer curved surface of the bucket. Find the shortest distance for it to crawl along the outer **curved surface** of the bucket to reach the point C. (4 marks)