HKCEE Problems.

Function - Polynomials

1. (80) Factorize

a) $a(3b - c) + c - 3b$

b) $x^4 - 1$

2. (80) It is given that $f(x) = 2x^2 + ax + b$

i) If $f(x)$ is divided by $(x - 1)$, the remainder is -5.
If $f(x)$ is divided by $(x + 2)$, the remainder is 4.
Find the values if a and b.

ii) If $f(x) = 0$, find the value of x.

3. (81) Let $f(x) = (x + 2)(x - 3) + 3$

When $f(x)$ is divided by $(x - k)$, the remainder is k.
Find k.

4. (81) Factorize $(1 + x)^4 - (1 - x^2)^2$

5. (83) Factorize $(x^2 + 4x + 4) - (y - 1)^2$

6. (84) If $3x^2 - kx - 2$ is divisible by $x - k$, where k is a constant, find the two values of k.

7. (84) Factorize

a) $x^2y + 2xy + y$

b) $x^2y + 2xy + y - y^3$

8. (85) a) Factorize $\alpha^4 - 16$ and $\alpha^3 - 8$

b) Find the L.C.M. of $\alpha^4 - 16$ and $\alpha^3 - 8$.

9. (85) Given $f(x) = ax^2 + bx - 1$, where a and b are constants, $f(x)$ is divisible by $(x - 1)$. When divided by $(x + 1)$, $f(x)$ leaves a remainder of 4. Find the values of a and b.

10. (86) Factorize

a) $x^2 - 2x - 3$

b) $(a^2 + 2a)^2 - 2(a^2 + 2a) - 3$

11. (88) Factorize

$a^2 - a - 6$ and $a^3 + 8$

Hence find their L.C.M.

12. (89) Given that $(x + 1)$ is a factor of $x^4 + x^3 - 8x + k$ where k is a constant.

a) Find the value of k.

b) Factorize $x^4 + x^3 - 8x + k$

13. (90) a) Find the remainder when $x^{1000} + 6$ is divided by $x + 1$.

b) i) Using (a) or otherwise, find the remainder when $8^{1000} + 6$ is divided by 9.

ii) What is the remainder when 8^{1000} is divided by 9?

Function - Polynomials

1. a) $a(3b-c)+c-3b$.

$=a(3b-c)-(3b-c)$

$=(a-1)(3b-c)$.

b. x^4-1

$=(x^2-1)(x^2+1)$

$=(x-1)(x+1)(x^2+1)$.

2. i) $f(x)=2x^2+ax+b$.

$f(1)=-5=2(1)^2+a(1)+b$.

$-5=2+a+b$.

$\therefore a+b=-7$ —— ①.

$f(-2)=4=2(-2)^2+a(-2)+b$.

$4=8-2a+b$.

$2a-b=4$ —— ②.

① + ② $3a=-3$.

$\therefore a=-1$.

$b=-6$.

$\therefore f(x)=2x^2-x-6$.

ii. 4. $f(x)=0$.

$\therefore 2x^2-x-6=0$.

$(2x-3)(x+2)=0$.

$\therefore x=3/2$ or -2.

3. $f(x)=(x+2)(x-3)+3$.

$f(k)=k=(k+2)(k-3)+3$

$k=k^2-k-6+3$.

$\therefore k^2-2k-3=0$

$(k-3)(k+1)=0$

$\therefore k=3$ or -1.

4. $(1+x)^4-(1-x^2)^2$ P.1

$=[(1+x)^2-(1-x^2)][(1+x)^2+(1-x^2)]$

$=[1+2x+x^2-1+x^2][1+2x+x^2+1-x^2]$

$=(2x^2+2x)(2+2x)$

$=2x(x+1)\cdot 2(x+1)$

$=4x(x+1)^2$.

5. $(x^2+4x+4)-(y-1)^2$

$=(x+2)^2-(y-1)^2$

$=[(x+2)-(y-1)][(x+2)+(y-1)]$

$=[x+2-y+1][x+2+y-1]$

$=(x-y+3)(x+y+1)$.

6. Let. $f(x)=3x^2-kx-2$.

$f(k)=0=3(k)^2-k(k)-2$.

$3k^2-k^2-2=0$

$2k^2-2=0$

$k^2=1$

$k=\pm 1$.

7. a) $x^2y+2xy+y$

$=y(x^2+2x+1)$

$=y(x+1)^2$.

b, $x^2y+2xy+y-y^3$

$=y(x+1)^2-y^3$

$=y[(x+1)^2-y^2]$

$=y[(x+1)-y][(x+1)+y]$

$=y[(x-y+1)(x+y+1)$.

8. a) $a^4 - 16$.

$= (a^2 - 4)(a^2 + 4)$

$= (a+2)(a-2)(a^2+4)$.

b) $a^3 - 8$

$= (a-2)(a^2 + 2a + 4)$.

$\therefore$ L.C.M. $= (a-2)(a+2)(a^2+4)(a^2+2a+4)$.

9. $f(x) = ax^2 + bx - 1$.

$f(1) = 0 = a(1)^2 + b(1) - 1$

$\therefore a + b - 1 = 0$ —— ①.

$f(-1) = 4 = a(-1)^2 + b(-1) - 1$

$a - b = 5$ —— ②.

① + ② $2a = 6$

$a = 3$.

$\therefore b = -2$.

10. a) $x^2 - 2x - 3$.

$= (x-3)(x+1)$.

b) $(a^2+2a)^2 - 2(a^2+2a) - 3$

Let $x = (a^2+2a)$

$= x^2 - 2x - 3$.

$= (x-3)(x+1)$

$= (a^2+2a-3)(a^2+2a+1)$

$= (a+3)(a-1)(a+1)^2$.

11. a) $a^2 - a - 6$ P.2.

$= (a-3)(a+2)$

$a^3 + 8$

$= (a+2)(a^2 - 2a + 4)$.

$\therefore$ L.C.M. $= (a+2)(a-3)(a^2-2a+4)$.

12. Let. $f(x) = x^4 + x^3 - 8x + k$.

$f(-1) = 0 = (-1)^4 + (-1)^3 - 8(-1) + k$

$0 = 1 - 1 + 8 + k$.

$\therefore k = -8$.

$\therefore f(x) = x^4 + x^3 - 8x - 8$.

$x=-1$	1	1	0	-8	-8
		-1	0	0	8
	1	0	0	-8	0

$\therefore f(x) = (x+1)(x^3 - 8)$

$= (x+1)(x-2)(x^2+2x+4)$.

13. a) Let. $f(x) = x^{1000} + 6$.

$\therefore$ the remainder $= f(-1)$.

$= (-1)^{1000} + 6$

$= 1 + 6 = 7$.

b i) when put. $x = 8$.

$\therefore 8^{1000} + 6$. divided by $(8+1) = 9$.

$\therefore$ the remainder $= 7$.

ii) Let. $8^{1000} + 6 = 9 \cdot Q + 7$ where Q is quotient

$8^{1000} = 9Q + 1$.

$\therefore$ when 8^{1000} divided by 9

the remainder. $= 1$.

HKCEE M.C.(83-88)
POLYNOMIALS AND ALGEBRAIC FRACTIONS

1. (83) $\dfrac{6}{x^2-9} - \dfrac{5}{x^2+x-6} =$

A. $\dfrac{1}{(x-2)(x-3)}$ B. $\dfrac{1}{(x-2)(x+3)}$ C. $\dfrac{1}{(x+2)(x-3)}$

D. $\dfrac{1}{(x+2)(x+3)}$ E. $\dfrac{x-27}{(x-2)(x+3)(x-3)}$

2. (83) $\dfrac{1/a^2 + 1/b^2}{1/a + 1/b} =$

A. $\dfrac{1}{a^2} + \dfrac{1}{b^2}$ B. $\dfrac{1}{a^2} + \dfrac{1}{ab} + \dfrac{1}{b^2}$ C. $\dfrac{1}{a^2} - \dfrac{1}{ab} + \dfrac{1}{b^2}$

D. $a^2 - ab + b^2$ E. $a^2 + ab + b^2$

3. (83) The H.C.F. of $a^3 - 1$ and $a^4 - 1$ is
A. 1 B. a+1 C. a−1 D. a^2+1 E. a^2-1

4. (83) When f(x) is divided by (2x+1), the remainder is
A. f(2) B. f(1) C. f(−1) D. f(½) E. f(−1/2)

5. (84) $\dfrac{4}{(x-2)(x+1)} - \dfrac{3}{x^2-1} =$

A. $\dfrac{1}{(x-1)^2(x+1)}$ B. $\dfrac{x+2}{(x-2)(x+1)(x-1)}$ C. $\dfrac{x+10}{(x-2)(x+1)(x-1)}$

D. $\dfrac{x-10}{(x-2)(x+1)(x-1)}$ E. $\dfrac{x^2-3x-10}{(x-2)(x+1)(x-1)^2}$

6. (84) If x+2 is a factor of x^2+ax+b, then $2a-b+3 =$
A. −7 B. −1 C. 0 D. 1 E. 7

7. (85) $\dfrac{2}{1+x} - \dfrac{1}{1-x} - \dfrac{4x}{x^2-1} =$

A. $\dfrac{1}{1-x}$ B. $\dfrac{1}{x-1}$ C. $\dfrac{1-7x}{x^2-1}$ D. $\dfrac{1-7x}{1-x^2}$ E. $\dfrac{3x+1}{1-x^2}$

8. (85) $\dfrac{b/a - a/b}{1/a - 1/b} =$
A. a+b B. a−b C. −a+b D. −a−b E. 1/a + 1/b

9. (85) $(x-y)^{-1}(x^{-2} - y^{-2}) =$

A. $\dfrac{1}{x^3} - \dfrac{1}{y^3}$ B. $\dfrac{-1}{x^2y} - \dfrac{1}{xy^2}$ C. $\dfrac{1}{xy^2} - \dfrac{1}{x^2y}$ D. $\dfrac{1}{x^2} - \dfrac{1}{y^2}$ E. $\dfrac{1}{x^2y} + \dfrac{1}{xy^2}$

10. The L.C.M. of $2a^2 - 2b^2$ and $a^3 - 2a^2b + ab^2$ is
(85) A. $a-b$ B. $a(a-b)(a+b)$ C. $2a(a-b)(a+b)$ D. $2a(a-b)^2(a+b)$
E. $2a(a-b)^3(a+b)$

11. Let a and b be constants. If $3x^3 - ax^2 + 5x - 3b$ is
(85) divisible by $x + 3$, then $3a + b = ?$
A. -32 B. -22 C. 22 D. 32 E. It cannot be determined

12. $$\frac{\frac{x^2}{3} - 3y^2}{\frac{3}{2}(x - 3y)} =$$
(86)
A. $\frac{1}{2}(x-3y)$ B. $\frac{2}{9}(x-3y)$ C. $2(x+3y)$ D. $\frac{1}{2}(x+3y)$ E. $\frac{2}{9}(x+3y)$

13. The L.C.M. of $12a^2b$ and $18ab^3c$ is
(86) A. $6ab$ B. $6a^2b^3c$ C. $36ab$ D. $36a^2b^3c$ E. $216a^3b^4c$

14. Let $F(x) = 2x^3 + 3x^2 - 11x - 6$. Given that $F(2) = 0$ and
(86) $F(-3) = 0$, then $F(x)$ can be factorized as
A. $(x+2)(x-3)(2x+1)$ B. $(x+2)(x-3)(2x-1)$ C. $(x-2)(x+3)(2x+1)$
D. $(x-2)(x-3)(2x+1)$ E. $(x-2)(x+3)(2x-1)$

15. $$\left(\frac{x+1}{x}\right)^2 - \left(\frac{x-1}{x}\right)^2 =$$
(87)
A. $2/x$ B. $4/x$ C. $2/x^2$ D. $4/x^2$ E. 0

16. If $1/x - 1/y = 1/z$, and $x = 1/2$, $z = 1/3$, then $y =$
(87) A. -1 B. 1 C. 5 D. 6 E. 1/6

17. When the expression x^2+px+q is divided by $x+1$, then
(87) remainder is 4. Find the value of $2p-2q+1$
A. -3 B. -5 C. -7 D. -9 E. It cannot be determined.

18. Find the H.C.F. of $(2x-1)(x^2-6x+9)$ and $(x^2-3x)(4x^2-1)$
(87) A. $(x-2)$ B. $(2x-1)$ C. $(x-3)(2x-1)$
D. $x(x-3)^2(2x-1)(2x+1)$ E. There is no H.C.F.

19. $$\frac{x^2 - 2x}{x^3 - 25x} \times \frac{x^2 - 2x - 15}{x^2 + x - 6} =$$
(88)
A. $\frac{1}{x-5}$ B. $\frac{x-2}{(x+2)(x-5)}$ C. $\frac{1}{x+5}$ D. $\frac{1}{x}$ E. $\frac{x-3}{(x+3)(x-5)}$

20. Let $f(x)=ax^2+bx+c$. When $f(x)$ is divided by $(x-1)$, the
(88) remainder is 10. When $f(x)$ is divided by $(x+1)$, the
remainder is 6. Find the value of b.
A. -4 B. -2 C. 2 D. 4 E. It cannot be found.

21. (88) $\dfrac{1}{2x - x^2} + \dfrac{1}{x^2 + x - 6} =$

A. $\dfrac{3}{x(2-x)(x+3)}$ B. $\dfrac{-3}{x(x+2)(x-3)}$ C. $\dfrac{6-x}{x(2-x)(x+2)(x-3)}$

D. $\dfrac{x-6}{x(2-x)(x+2)(x-3)}$ E. $\dfrac{2x+3}{x(2-x)(x+3)}$

22. (88) Which of the following expressions CANNOT be factorized ?
A. x^3-125 B. $4x^2-9y^2$ C. x^3+125 D. $4x^2+9y^2$
E. $3x^2+6xy+3y^2$

23. (88) $8abc^3$ is the H.C.F. of $24ab^2c^3$ and
A. $12a^2bc^4$ B. $30a^2bc^3$ C. $32a^2bc^3$ D. $40ab^2c^3$ E. $48a^3bc^2$

ANSWERS :

1.A 2.C 3.C 4.E 5.B 6.E 7.A 8.A 9.B 10.D
11.A 12.E 13.D 14.C 15.B 16.A 17.B 18.C 19.C 20.C
21.A 22.D 23.C

Polynomials and Algebraic fractions

1. $\frac{6}{x^2-9} - \frac{5}{x^2+x-6}$

$= \frac{6}{(x-3)(x+3)} - \frac{5}{(x+3)(x-2)}$

$= \frac{6(x-2) - 5(x-3)}{(x-3)(x-2)(x+3)}$

$= \frac{6x-12-5x+15}{(x-3)(x-2)(x+3)}$

$= \frac{x+3}{(x-3)(x-2)(x+3)}$.

$\frac{1}{(x-2)(x-3)}$. (A.)

2. $\frac{1/a^3 + 1/b^3}{1/a + 1/b}$.

$= \frac{b^3+a^3/a^3 \cdot b^3}{b+a/ab}$.

$= \frac{b^3+a^3}{(ab)^3} \cdot \frac{ab}{b+a}$.

$= \frac{(b+a)(b^2-ab+a^2) \cdot ab}{(ab)^3 (b+a)}$

$= \frac{b^2-ab+a^2}{(ab)^2}$

$= \frac{1}{a^2} - \frac{1}{ab} + \frac{1}{b^2}$ (C.)

3. $(a^3-1) = (a-1)(a^2+a+1)$

$(a^4-1) = (a^2-1)(a^2+1)$

$= (a-1)(a+1)(a^2+1)$.

$\therefore$ H.C.F. $= (a-1)$ (C.)

4. $f(x)$ is divided by $(2x+1)$.

the remainder $= f(-1/2)$ (E.)

5. $\frac{1}{(x-2)(x+1)} - \frac{\quad}{(x^2-1)}$

$= \frac{4}{(x-2)(x+1)} - \frac{3}{(x-1)(x+1)}$

$= \frac{4(x-1) - 3(x-2)}{(x-2)(x-1)(x+1)}$

$= \frac{4x-4-3x+6}{(x-2)(x-1)(x+1)}$

$= \frac{x+2}{(x-2)(x-1)(x+1)}$ (B.)

6. $(x+2)$ is a factor of x^2+ax+b.

$\therefore (-2)^2 + a(-2) + b = 0$.

$4 - 2a + b = 0$.

$\therefore 2a - b = 4$.

$2a - b + 3$

$= 4 + 3 = 7$ (E.)

7. $\frac{2}{1+x} - \frac{1}{1-x} - \frac{4x}{x^2-1}$.

$= \frac{2}{1+x} - \frac{1}{1-x} - \frac{4x}{(x-1)(x+1)}$

$= \frac{2(x-1) - (-1)(x+1) - 4x}{(x-1)(x+1)}$

$= \frac{2x-2+x+1-4x}{(x-1)(x+1)}$

$= \frac{-x-1}{(x-1)(x+1)}$

$= \frac{-(x+1)}{(x-1)(x+1)}$

$= \frac{-1}{x-1}$

$= \frac{1}{1-x}$ (A.)

8. $\frac{1/a - 1/b}{1/a - 1/b}$ P.1

$= \frac{b^2-a^2/ab}{b-a/ab}$.

$= \frac{b^2-a^2}{ab} \cdot \frac{ab}{b-a}$.

$= \frac{b^2-a^2}{b-a}$

$= \frac{(b-a)(b+a)}{(b-a)}$

$= a+b$ (A.)

9. $(x-y)^{-1}(x^{-2}-y^{-2})$.

$= \left(\frac{1}{x-y}\right)\left(\frac{1}{x^2} - \frac{1}{y^2}\right)$

$= \left(\frac{1}{x-y}\right)\left[\frac{y^2-x^2}{(x \cdot y)^2}\right]$

$= \frac{(y-x)(x+y)}{(x-y)(x \cdot y)^2}$

$= \frac{-(x+y)}{(x \cdot y)^2}$

$= -\frac{x}{(x \cdot y)^2} - \frac{y}{(x \cdot y)^2}$

$= \frac{-1}{x \cdot y^2} - \frac{1}{x^2 y}$ (B.)

10. $2a^2 - 2b^2$

$= 2(a-b)(a+b)$

$a^3 - 2a^2b + ab^2$

$= a(a^2 - 2ab + b^2)$

$= a(a-b)^2$

$\therefore$ L.C.M. $= 2a(a+b)(a-b)^2$

(D.)

11. Let $f(x)=3x^3-ax^2+5x-3b$.

$f(-3)=0=3(-3)^3-a(-3)^2+5(-3)-3b$.

$-81-a(9)-15-3b=0$.

$-96-9a-3b=0$

$9a+3b=-96$

$3a+b=-32$. (A.)

2. $\dfrac{\frac{x^2}{3}-3y^2}{\frac{3}{2}(x-3y)}$

$=\dfrac{x^2-9y^2}{3}\cdot\dfrac{2}{3(x-3y)}$

$=\dfrac{2}{9}\left[\dfrac{(x-3y)(x+3y)}{(x-3y)}\right]$

$=\dfrac{2(x+3y)}{9}$ (E)

13. $\begin{cases}12a^2b\\18ab^3c\end{cases}$.

$\therefore$ L.C.M. $=36a^2b^3c$ (D.)

14. $F(x)=2x^3+3x^2-11x-6$.

$F(2)=0$.

	2	3	-11	-6
		4	14	6
	2	7	3	0

$F(x)=(x-2)(2x^2+7x+3)$

$=(x-2)(x+3)(2x+1)$ (C.)

15. $\left(\dfrac{x+1}{x}\right)^2-\left(\dfrac{x-1}{x}\right)^2$

$=\dfrac{(x+1)^2-(x-1)^2}{x^2}$

$=\dfrac{x^2+2x+1-(x^2-2x+1)}{x^2}$

$=\dfrac{4x}{x^2}=\dfrac{4}{x}$ (B.)

16. $\dfrac{1}{x}-\dfrac{1}{y}=z$.

If $x=\frac{1}{2}$, $z=\frac{1}{3}$.

$\Rightarrow \frac{1}{x}=2$, $\frac{1}{z}=3$.

$2-\frac{1}{y}=3$.

$\therefore \frac{1}{y}=2-3$

$\frac{1}{y}=-1$

$y=-1$. (A.)

17. Let $f(x)=x^2+px+q$.

$f(-1)=4=(-1)^2+p(-1)+q$.

$4=1-p+q$.

$p-q=-3$.

$2p-2q+1$

$=2(p-q)+1$

$=2(-3)+1=-5$. (B.)

18. $(2x-1)(x^2-6x+9)$

$=(2x-1)(x-3)^2$

$(x^2-3x)(4x^2-1)$

$=x(x-3)(2x-1)(2x+1)$

$\therefore$ H.C.F. $=(x-3)(2x-1)$

(C.)

19. $\dfrac{x^2-2x}{x^3-25x}\cdot\dfrac{x^2-2x-15}{x^2+x-6}$.

$=\dfrac{x(x-2)\cdot(x-5)(x+3)}{x(x-5)(x+5)\cdot(x+3)(x-2)}$

$=\dfrac{1}{x+5}$ (C)

20. $f(x)=ax^2+bx+c$.

$f(1)=10=a(1)^2+b(1)+c$.

$a+b+c=10$ — ①

$f(-1)=6=a(-1)^2+b(-1)+c$.

$a-b+c=6$ — ②.

①-②.

$2b=10-6$.

$2b=4$

$b=2$. (C.)

21. $\dfrac{1}{2x-x^2}+\dfrac{1}{x^2+x-6}$

$=\dfrac{1}{x(2-x)}+\dfrac{1}{(x-2)(x+3)}$

$=\dfrac{(x+3)+(-1)x}{x(2-x)(x+3)}$

$=\dfrac{x+3-x}{x(2-x)(x+3)}$

$=\dfrac{3}{x(2-x)(x+3)}$ (A.)

22. A, x^3-125

$=(x-5)(x^2+5x+25)$.

B, $4x^2-9y^2$

$=(2x-3y)(2x+3y)$.

C, x^3+125.

$=(x+5)(x^2-5x+25)$.

D, $4x^2+9y^2$ cannot be factorized.

E, $3x^2+6xy+3y^2$

$=3(x^2+2xy+y^2)$

$=3(x+y)^2$. (D.)

23. H.C.F. $=8abc^3$.

$\therefore$ 8, a, b and c^3 are highest common factor of $24ab^2c^3$ & $32a^2bc^3$ (C.)