1(80) The examination for a professional qualification consists of a theory paper and a practical paper. To obtain the qualification, a candidate has to pass both papers. If a candidate fails in either paper, he need only sit that paper again.

The probabilities of passing the theory paper for two candidates A and B are both $\frac{9}{10}$ and their probabilities of passing the practical paper are both $\frac{2}{3}$. Find the probabilities of the following events :

(a) Candidate A obtaining the qualification by sitting each paper only once. (3 marks)

(b) Candidate A failing in one of the two papers but obtaining the qualification with one re-examination. (4 marks)

(c) At least one of the candidates A and B obtaining the qualification without any re-examination. (3 marks)

2(81) In a class of 42 students, 28 have been to Ocean Park and 34 have been to the Space Museum.

(a) Find the least number of students who have been to both Ocean Park and the Space Museum.

(b) If 7 of the 42 students have never been to Ocean Park or the Space Museum, find the number of students who have been to both places.

(6 marks)

3(81) There are 40 students in a class, including students A and B. If two students are to be chosen at random as class representatives, find the probability that both A and B are chosen.

(5 marks)

4(82) If two dice are thrown once, find the probability that the sum of the numbers on the dice is

(a) equal to 4 ,

(b) less than 4 ,

(c) greater than 4 .

(6 marks)

5(83) In a short test, there are 3 questions. For each question, 1 mark will be awarded for a correct answer and no marks for a wrong answer. The probability that John correctly answers a question in the test is 0.6. Find the probability that

(a) John gets 3 marks in the test, (3 marks)

(b) John gets no marks in the test, (3 marks)

(c) John gets 1 mark in the test, (4 marks)

(d) John gets 2 marks in the test. (2 marks)

6(84)(a) There are two bags. Each bag contains 1 red, 1 black and 1 white ball. One ball is drawn randomly from each bag. Find the probability that

(i) the two balls drawn are both red;

(ii) the two balls drawn are of the same colour;

(iii) the two balls drawn are of different colours.

(6 marks)

(b) A box contains 2 red, 2 black and 3 white balls. One ball is drawn randomly from the box. After putting the ball back into the box, one ball is again drawn randomly. Find the probability that

(i) the two balls drawn are both red;

(ii) the two balls drawn are of the same colour;

(iii) the two balls drawn are of different colours.

(6 marks)

7(85) (a) If two dice are thrown,

(i) find the probability that the sum of the numbers on the two dice is greater than 9 ;

(ii) find the probability that the sum of the numbers on the two dice is greater than 9 or the numbers on the two dice are equal.

(6 marks)

(b) In a game, two dice are thrown. In each throw, 2 points are gained if the sum of the numbers on the two dice is greater than 9 or the numbers on the two dice are equal; otherwise 1 point is lost. Using the result in (a)(ii), find the probability of

(i) losing a total of 2 points in two throws,

(ii) gaining a total of 1 point in two throws. (6 marks)

8(86) A box contains wooden blocks of 5 different shapes A, B, C, D and E. For each shape, there are 5 different colours red, orange, yellow, green and blue. For each colour of each shape, there is one block of each of the sizes L, M and S. (Hint: There are altogether 75 blocks in the box.)

(a) When a block is picked out randomly from the box, what is the probability that it is of

(i) red colour ?

(ii) blue colour and shape C ?

(iii) size S, shape A or E but not yellow ?

(6 marks)

(b) Two blocks are drawn at random from the box, one after the other. The first block drawn is put back into the box before the second is drawn. Find the probability that

(i) the first block drawn is of size L and the second block is of size S,

(ii) one of the blocks drawn is of size L and the other of size S,

(iii) the two blocks drawn are of different sizes.

(6 marks)

9(87) P, Q and R are three bags. P contains 1 black ball, 2 green balls and 3 white balls; Q contains 4 green balls; R contains 5 white balls. A ball is drawn at random from P and is put into Q ; then a ball is drawn at random from Q and is put into R . Find the probability that

(a) the black ball still remains in P ,

(2 marks)

(b) the black ball is in Q ,

(4 marks)

(c) the black ball is in R ,

(3 marks)

(d) all the balls in R are white.

(3 marks)

10(89) (a) Bag A contains a number of balls. Some are black and the rest are white. A ball is drawn at random from bag A. Let p be the probability that the ball drawn is black and q be the probability that the ball drawn is white. If $p = 3q$, find q.

(2 marks)

(b) Bag C contains 10 balls of which n $(2 \leq n \leq 10)$ balls are black.

(i) If two balls are drawn at random from bag C, find the probability, in terms of n, that both balls are black.

(ii) If the probability obtained in (i) is greater than $\frac{1}{3}$, find the possible values of n

(7 marks)

(c) Bag M contains 1 red and 1 green ball. Bag N contains 3 red and 2 green balls. A ball is drawn at random from bag M and put into bag N; then a ball is drawn at random from bag N. Find the probability that the ball drawn from bag N is red.

(3 marks)

12(91) The practical test for a driving licence consists of two independent parts A and B. To pass the practical test, a candidate must pass in both parts. If a candidate fails in any one of these parts, the candidate may take that part again. Statistics show that the passing percentages for Part A and Part B are 70% and 60% respectively.

(a) A candidate takes the practical test. Find the probabilities that the candidate

(i) fails Part A on the first attempt and passes it on the second attempt,

(ii) passes Part A in no more than two attempts,

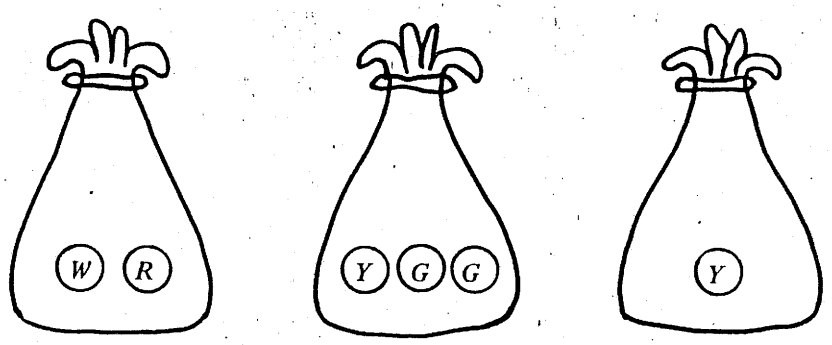
(iii) passes the practical test in no more than two attempts in each part.

(10 marks)

(b) In a sample of 10 000 candidates taking the practical test, how many of them would you expect to pass the practical test in no more than two attempts in each part?

(2 marks)

11(90)



Bag A Bag B Bag C

Figure

Figure shows 3 bags A, B and C.

Bag A contains 1 white ball (W) and 1 red ball (R).
Bag B contains 1 yellow ball (Y) and 2 green balls (G).
Bag C contains only 1 yellow ball (Y).

(a) Peter chooses one bag at random and then randomly draws one ball from the bag. Find the probability that

(i) the ball drawn is green;

(ii) the ball drawn is yellow.

(6 marks)

(b) After Peter has drawn a ball in the way described in (a), he puts it back into the original bag. Next, Alice chooses one bag at random and then randomly draws one ball from the bag. Find the probability that

(i) the balls drawn by Peter and Alice are both green;

(ii) the balls drawn by Peter and Alice are both yellow and from the same bag.

(6 marks)

HKCEE Problem: Probability

1.

	Theory paper	Practical Paper
	9/10	3/3

a) P(candidate A obtaining the qualification by sitting each paper only once)

$= \frac{9}{10} \times \frac{2}{3}$

$= \frac{3}{5}$.

b) P(candidate A failing in one of the two papers but qualified with one-exam.)

= P(A fails theory or practical & pass.)

$= \frac{1}{10} \cdot \frac{2}{3} \cdot \frac{9}{10} + \frac{9}{10} \cdot \frac{1}{3} \cdot \frac{2}{3}$

$\frac{13}{50}$

c) P(at least one of candidates obtaining qualification without re-examination)

= 1 − P(A and B not qualification in first exam.)

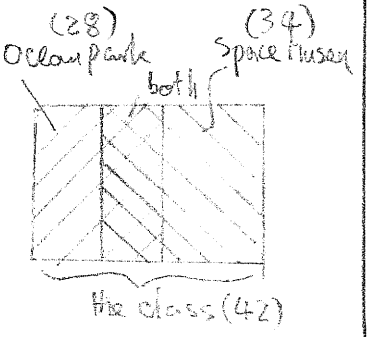
$= 1 - (1-\frac{3}{5})(1-\frac{3}{5})$. from the result a)

$= \frac{21}{25}$.

2a) the least no. of students have been to both places.

= (34 + 28) − 42

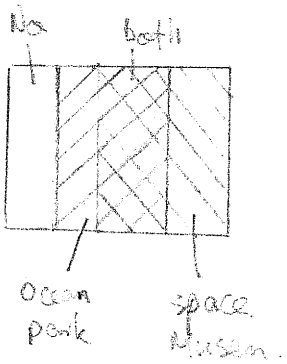
= 20.



b) the no. of students have been to both places.

= (34 + 28 + 7) − 42

= 27.



alternative method in 5d)

P(John gets 2 marks)

= 1 − P(no marks) − P(1 marks) − P(3 marks)

= 1 − 0.064 − 0.288 − 0.216

= 0.432.

3. P(both A and B are chosen.)

$= \frac{2}{40} \cdot \frac{1}{39}$.

$= \frac{1}{780}$

4. a) P(sum of no. = 4)

1, 3
2, 2
3, 1.

$= \frac{3}{6 \times 6}$.

$= \frac{1}{12}$.

b) P(sum of no. < 4)

1, 1
1, 2
2, 1

$= \frac{3}{36}$

$= \frac{1}{12}$

c) P(sum of no. > 4)

= 1 − P(sum of no = 4) − P(sum of no. < 4).

$= 1 - \frac{1}{12} - \frac{1}{12}$

$= \frac{5}{6}$.

5a) P(John gets 3 marks in the test)

= P(3 questions are correct.)

= (0.6)(0.6)(0.6)

= 0.216.

b) P(John gets no marks)

= P(3 questions are wrong).

= (0.4)(0.4)(0.4)

= 0.064.

c) P(John gets 1 marks)

= P(1 answer correct, 2 wrong.)

= (0.6)(0.4)(0.4) + (0.4)(0.6)(0.4) + (0.4)(0.4)(0.6)

= 0.288.

d) P(John gets 2 marks)

= P(2 correct answer, 1 wrong.)

= (0.6)(0.6)(0.4) + (0.6)(0.4)(0.6) + (0.4)(0.6)(0.6)

= 0.432.

6.a)

i) P(two balls draw. are both red)

1R
1B
1W

1R
1B
1W

$= \frac{1}{3} \cdot \frac{1}{3}$

$= 1/9.$

ii) P(two ball draw. are of the same colour.)

$= P(RR \text{ or } BB \text{ or } WW)$

$= \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3}$

$= \frac{1}{3}$

iii) P(two ball draw. are of different colour.)

$= 1 - P(\text{two ball are same colour.})$

$= 1 - \frac{1}{3}$

$= 2/3.$

b)

i) P(two balls. draw are both red)

2 R
2 B
3 W.

$= \frac{2}{7} \cdot \frac{2}{7}$

$= 4/49.$

ii) P(two balls draw are the same colour.)

$P(RR \text{ or } BB \text{ or } WW)$

$= \frac{2}{7} \cdot \frac{2}{7} + \frac{2}{7} \cdot \frac{2}{7} + \frac{3}{7} \cdot \frac{3}{7}.$

$= \frac{17}{49}.$

iii) P(two balls draw are different colour.)

$= 1 - P(\text{two ball are same colour.})$

$= 1 - \frac{17}{49}$

$= 32/49.$

7.a)

i) P(sum of no. > 9.)

$= P(\text{sum of no} = 10, 11, 12)$

10 { 4,6 ; 5,5 ; 4,6.

11 { 5,6 ; 6,5

12 { 6,6

$= \frac{6}{6 \times 6}$

$= 1/6.$

ii) P(sum of no. > 9 or two dice equal)

$= P(\text{sum of no.} = 10, 11, 12 \text{ or } 1,1; 2,2; \cdots 6,6)$

$= \frac{1}{6} + \frac{4}{36}$ since (5,5 & 6,6 are appear in sum of no. > 9)

$= \frac{5}{18}.$

b) i) P(lost a 2 pts in two throws.)

$= P(\text{sum. of no} \neq 9 \text{ and two dice not equal})$

$= \left(1 - \frac{5}{18}\right)\left(1 - \frac{5}{18}\right)$ by using aii)

$= \frac{169}{324}.$

ii) P(gain 1 pt in two throws.)

$= P(1 \text{ throws is sum of no.} > 9 \text{ two dice equal and 1 throws is not.})$

$= \left(\frac{5}{18}\right)\left(\frac{13}{18}\right) + \left(\frac{13}{18}\right)\left(\frac{5}{18}\right)$

$= \frac{130}{324}.$

8.

5 shapes { A, C, E, T, O }
5 colour { Y, R, B }
3 size { L, M, S }
(a total 75 blocks.)

a) i) P(red colour.)
$= \frac{5 \times 3}{75}$
$= \frac{1}{5}$

[since for red colour blocks, there are 5 shapes and 3 size.].

ii) P(blue colour and shape C)
$= \frac{3}{75}$
$= \frac{1}{25}$

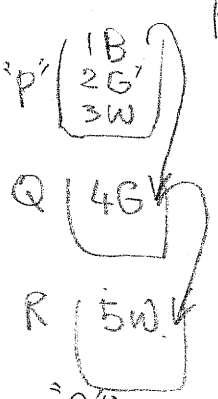
iii) P(size S, shape A or E but not yellow)
$= \frac{2 \times 4}{75}$
$= 8/75$

b) i) P(1st is L & 2nd is S)
$= (\frac{5 \times 5}{75})(\frac{5 \times 5}{75})$
$= \frac{1}{9}$

ii) P(one is L & another is S)
$= (\frac{5 \times 5}{75})(\frac{5 \times 5}{75}) \times 2$
$= \frac{2}{9}$

iii) P(two blocks are different sizes)
$= P(LS, LM, MS)$
$= \frac{2}{9} \times 3$
$= \frac{2}{3}$

9 a)



P("the black ball still remains in "P")
= P(the ball drawn from "P" is not black)
$= \frac{5}{6}$

b) P("the black ball is in "Q")
= P(the ball drawn from "P" is black & the ball drawn from "Q" is not)
$= \frac{1}{6} \cdot \frac{4}{4+1}$
$= \frac{2}{15}$

c) P("the black ball is in "R")
= P(the ball drawn from "P" & "Q" are black).
$= \frac{1}{6} \cdot \frac{1}{4+1}$
$= \frac{1}{30}$

d) P(all the balls in R are white.)
= P(the ball drawn from "P" & "Q" are white.)
$= \frac{3}{6} \cdot \frac{1}{4+1}$
$= \frac{1}{10}$

10. a). Since the balls in bag A are black and white.

$\therefore P(\text{black balls}) + P(\text{white ball}) = 1.$

$\begin{cases} p + q = 1. & \text{——①} \\ p = 3q & \text{——②.} \end{cases}$

sub ② into ①

$3q + q = 1$

$\therefore q = \frac{1}{4}.$

$p = 3/4.$

b) i) P(both balls are black)

$= \left(\frac{n}{10}\right)\left(\frac{n-1}{9}\right)$

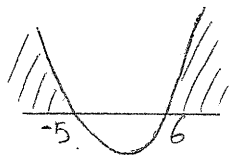
ii) P(both balls are black) $> \frac{1}{3}$.

$\therefore \frac{n(n-1)}{90} > \frac{1}{3}.$

$\therefore n^2 - n > 30.$

$n^2 - n - 30 > 0$

$(n-6)(n+5) > 0$



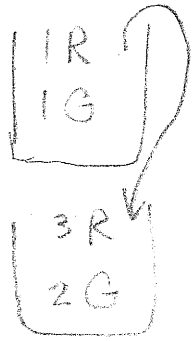
$\therefore n > 6$ or $n < -5$ (rejected $\because n > 0$)

$\therefore$ the possible values of n are

$6 < n \leq 10$

or 7, 8, 9, 10.

c)



P(the ball drawn from bag N is red)

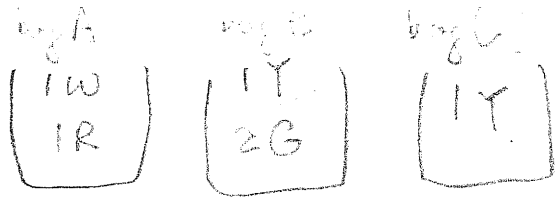
$= P(\text{M is R \& N is red or M is G \& N is red})$

$= \left(\frac{1}{2}\right)\left(\frac{3+1}{5+1}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{5+1}\right)$

$= \frac{1}{3} + \frac{1}{4}$

$= 7/12.$

11.



a i) P(the ball drawn is green)

$= P(\text{green in A or green in B or green in C})$

$= \left(\frac{1}{3}\right)(0) + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)(0)$

$= 2/9.$

ii) P(the ball drawn is yellow.)

$= P(\text{yellow in A or yellow in B or yellow in C})$

$= \left(\frac{1}{3}\right)(0) + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{1}\right)$

$= \frac{4}{9}.$

b) i) P(the balls are both green.)

$= \left(\frac{2}{9}\right)\left(\frac{2}{9}\right)$ by using a i)

$= 4/81.$

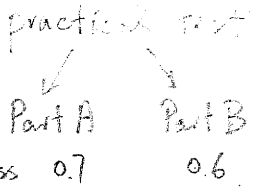
ii) P(the balls are both yellow & same bag.)

$= P(\text{the balls are both yellow in B or. the balls are both yellow in C.})$

$= \left[\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\right]\left[\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\right] + \left[\left(\frac{1}{3}\right)(1)\right]\left[\left(\frac{1}{3}\right)(1)\right]$

$= \frac{10}{81}.$

12.



a) i)

P(fails Part A on the first attempt and passes it on the 2nd attempt).

$= (1-0.7)(0.6)(0.7)$

$= 0.126.$

ii) P(pass Part A in no more than two attempts)

$= (0.7)(0.6) + (1-0.7)(0.6)(0.7)$

$= 0.546.$

iii). P(passes the practical test in no more than two attempts in each part).

$= (0.7)(0.6) + (1-0.7)(0.6)(0.7) + [(1-0.6)(0.7)(0.6)] + (1-0.7)(1-0.6)(0.6)(0.7).$

$= 0.7644.$

b) the expected candidates to pass the practical test in no more than two attempts in each part.

$= 10,000 \times 0.7644$

$= 7,644.$

1. (83) There are 12 boys and 8 girls in a class. 1/4 of the boys and 1/4 of the girls wear glasses. What if the probability that a student chosen at random from the class is a boy not wearing glasses or a girl wearing glasses ?
A. 5/20 B. 9/20 C. 11/20 D. 15/20 E. 9/100

2. (84) The probability that John will win a game is 1/3 and the probability that he will lose is 2/3. What is the probability that, in three games, he will win any two games and lose one game ?
A. 4/27 B. 2/27 C. 1/27 D. 2/9 E. 1/9

3. (84) Two dices are thrown. What is the probabililty of getting a a sum of 8 ?
A. 1/12 B. 1/11 C. 5/36 D. 1/6 E. 2/9

4. (85) There are four balls, numbered 1, 2, 5 and 10 in a bag. If 2 balls are taken out at random, the probability that the sum on the two balls drawn is greater than or equal to 7 is
A. 1/2 B. 5/8 C. 2/3 D. 3/4 E. 5/6

5. (85) Two dice are thrown. The probability of getting at least one '6' is
A. 1/6 B. 1/3 C. 11/36 D. 25/36 E. 35/36

6. (86) In a shooting game, the probability for John and Mary to hit a target are 4/5 and 3/5 respectively. When both shoot at the target, what is the probability that they both miss ?
A. 2/25 B. 3/25 C. 8/25 D. 12/25 E. 13/25

7. (87) One letter is taken from each of the words "MAN" and "ART" at random. Find the probability that the two letters are not the same.
A. 1/9 B. 1/3 C. 4/9 D. 2/3 E. 8/9

8. (87) Four persons A, B, C and D sit randomly around a round table. The probability that A sits next to B is
A. 1/4 B. 1/3 C. 1/2 D. 2/3 E. 5/6

9. (87) A die is thrown twice. Find the probability that the number obtained at the first throw is greater than that at the second throw.

10. (88) The figure shows 3 paths joining A and B. A man walks from A to B and another man walks from B to A at the same time. If they choose their paths at random, what is the probability that they will meet ?

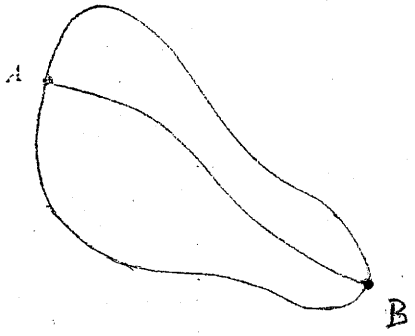
A. $1 - \frac{1}{9}$

B. 1/3

C. $1 - \frac{1}{3}$

D. $\frac{1}{2} \times \frac{1}{3}$

E. $\frac{1}{3} \times \frac{1}{3}$

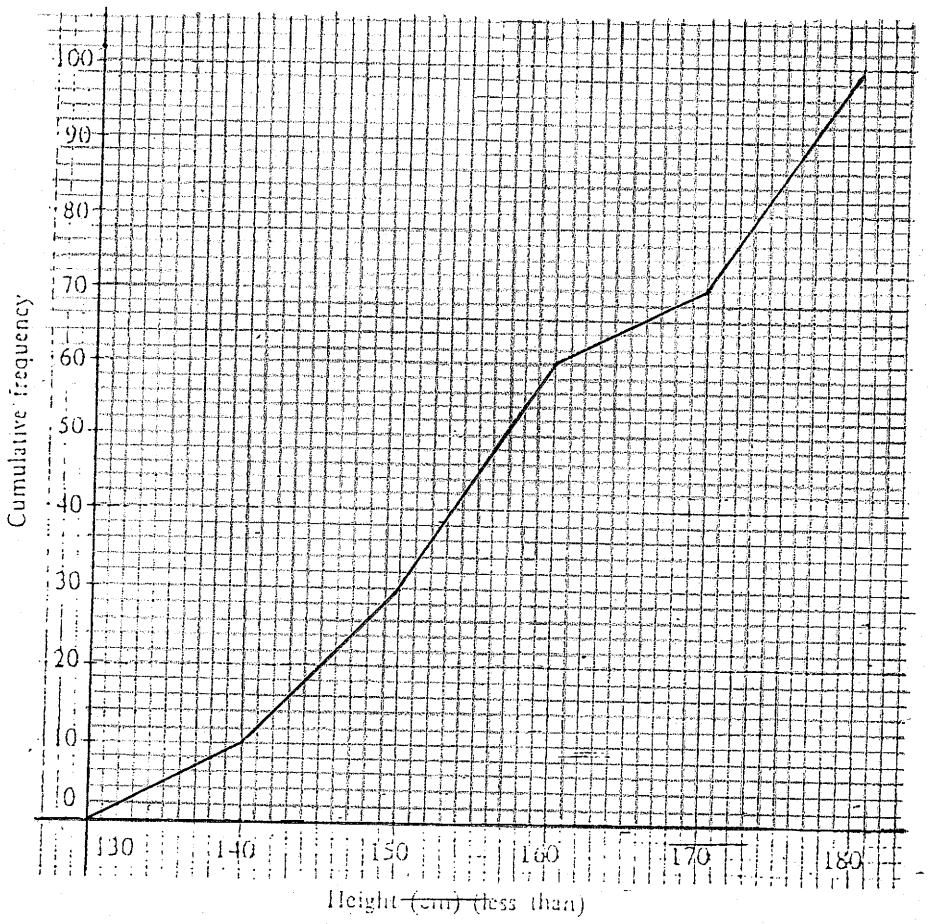


11. (88) The figure shows the cumulative frequency polygon of the heights of 100 persons. If one person is selected at random from the group, find the probability that his height is less than 170 cm but not less than 150 cm.

A. 1/5
B. 2/5
C. 3/10
D. 1/2
E. 7/10

ANSWERS

1.C	2.D	3.C	4.C	5.C
6.A	7.E	8.D	9.B	10.B
11.B				



Probability

1. 12 boys. 8 girls.

P(a boy not wearing glasses or a girl wearing glasses.)

$= (\frac{12}{20})(1-\frac{1}{4}) + (\frac{8}{20})(\frac{1}{4})$

$= \frac{9}{20} + \frac{2}{20}$

$= \frac{11}{20}$ (C.)

2. P(Win) = 1/3.

P(lose) = 2/3.

∴ P(win any two games and lose one game.)

$= \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$.

$= \frac{2}{9}$ (D.)

3. sum of 8

= 2, 6
6, 2
3, 5
5, 3
4, 4.

} 5 outcomes.

∴ P(getting a sum of 8)

$= \frac{5}{6\times 6} = \frac{5}{36}$. (C.)

4. sum of two ball. (1, 2, 5, 10.)

6 ample space:
1+2 = 3
1+5 = 6
1+10 = 11
2+5 = 7
2+10 = 12
5+10 = 15

} 4 outcomes.

P(sum. of two balls > 7)

$= \frac{4}{6} = \frac{2}{3}$. (C.)

5. P(getting at least one '6')

= P(get one '6' and two '6')

$= \frac{1}{6}\cdot\frac{5}{6} + \frac{5}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}$.

$= \frac{11}{36}$

or P(getting at least one '6')

= 1 − P(getting no '6')

$= 1 - (\frac{5}{6})(\frac{5}{6})$

$= \frac{11}{36}$ (C.)

6. P(John hit) = 4/5.

P(Mary hit) = 3/5.

P(they both miss)

= P(John and Mary miss.)

$= (1-\frac{4}{5})\cdot(1-\frac{3}{5})$

$= \frac{2}{25}$ (A.)

7. In "MAN" and "ART" only letter "A" is the same.

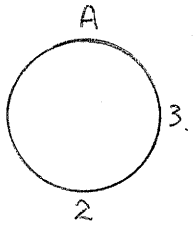
P(two letters are not the same.)

= 1 − P(two letters are the same)

$= 1 - \frac{1}{3}\cdot\frac{1}{3}$.

$= \frac{8}{9}$ (E.)

8. P(A sits next to B)



= P(B sits at 1 or 3)

$= \frac{2}{3}$ (D.)

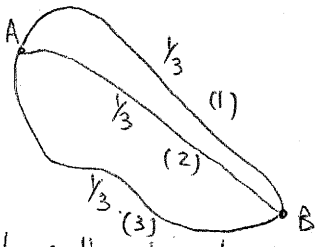
9. 1, 2, 3, 4, 5, 6.

P(1st throw > 2nd throw)

$= (\frac{1}{6})\cdot(\frac{5}{6}) + (\frac{1}{6})(\frac{4}{6}) + (\frac{1}{6})(\frac{3}{6}) + (\frac{1}{6})(\frac{2}{6}) + (\frac{1}{6})(\frac{1}{6})$

$= \frac{15}{36} = \frac{5}{12}$. (B.)

10.



Each paths has 1/3 chances

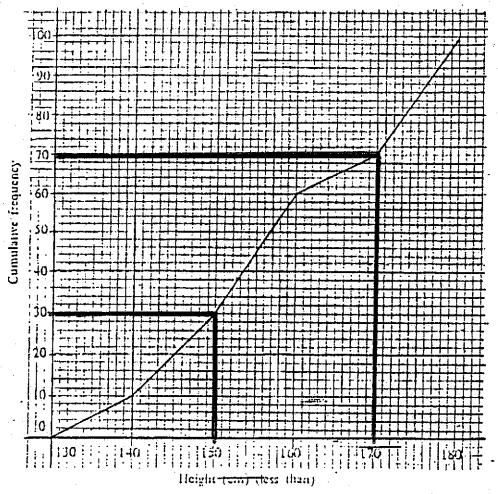
P(they will meet)

= P(they meet at (1), (2) or (3))

$= \frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3}$.

$= \frac{1}{3}$ (B.)

11.



P(his height is less than 170cm but not less than 150 cm.)

$= \frac{70-30}{100}$

$= \frac{2}{5}$. (B.)