1. If $2x=3y=5z$, then $x : y : z =$
(83) A. 2:3:5 B. 5:3:2 C. 6:10:15 D. 15:10:6 E. 25:9:4

2. Two men cycle round a circular track which is 3 km long. If
(83) they start at the same time and at the same spot but go in opposite direction with speeds 6 km/h and 9 km/h respectively, for how long must they cycle before they meet for the first time ?
A. 12 minutes B. 15 minutes C. 18 minutes
D. 24 minutes E. 60 minutes

3. The scale of a map is 1 : 20000. On the map, the area of a
(83) farm is 2 cm^2. The actual area of the farm is
A. 400 m^2 B. 800 m^2 C. 40000 m^2 D. 80000 m^2
E. 8000000 m^2

4. Three numbers are in the ratio 2:3:5. The ratio of their
(83) average to the largest of the three numbers is
A. 1:3 B. 1:2 C. 3:5 D. 2:3 E. 2:1

5.
(84) If $\dfrac{3x + 2y}{x + 5y} = 1$, then $x + y : x - y =$
A. 1 : 5 B. 3 : 2 C. 5 : 6 D. 5 : 1 E. 7 : 2

6. A man drives a car at 30 km/h for 3 hours and then at 40
(84) km/h for 2 hours. His average speed for the whole journey is
A. 14 km/h B. 30 km/h C. 34 km/h D. 35 km/h E. 70 km/h

7. A alone can complete a job in 8 hours. B alone takes 12
(84) hours and C alone takes 6 hours. After A and B have worked together on the job for 3 hours, C joins them. How much longer will they take to complete the job ?
A. 1 hour B. 1½ hours C. 2 hours D. 2½ hours E. 3 hours

8. a, b and c are positive numbers such that $a/b = b/c = k$ (k
(84) being a constant), which of the following must be true ?
(1) $b^2 = k^2$ (2) $(a+b)/(b+c) = k$ (3) $a/c = k^2$
A. (2) only B. (3) only C. (1) and (2) only
D. (2) and (3) only E. (1), (2) and (3)

9. If $a : b = 1 : 2$ and $b : c = 1 : 3$, then $a+b : b+c =$
(85) A. 1 : 5 B. 2 : 3 C. 3 : 4 D. 3 : 5 E. 3 : 8

10. It takes John 40 minutes to walk from his home to school. If
(85) he increases his walking speed by 2 km/h, then it takes only 30 minutes. What is the distance between John's home and his school ?
A. 1 km B. 4 km C. 6 km D. 8 km E. 12 km

11. A man drives a car at 45 km/h for 3 hours and then at 50
(86) km/h for 2 hours. His average speed for the whole journey is
A. 47 km/h B. 47.5 km/h C. 48 km/h D. 48.5 km/h
E. 49 km/h

12. If A, B and C can finish running the same distance in 3, 4
(86) and 5 minutes respectively, then A's speed : B's speed : C's speed =

A. 3 : 4 : 5 B. 20 : 15 : 12 C. 5 : 4 : 3
D. 25 : 16 : 9 E. 9 : 8 : 7

13. The number π is
(87) A. 22/7
B. 3.1416
C. the ratio of the area of a circle to the square of its diameter.
D. the ratio of the circumference of a circle to its radius.
E. the ratio of the circumference of a circle to its diameter.

14. If a : b = 3 : 2, b : c = 4 : 3, then a+b : b+c =
(87) A. 7 : 10 B. 5 : 7 C. 1 : 1 D. 7 : 5 E. 10 : 7

15. If 3a = 2b = 5c, then 1/a : 1/b : 1/c =
(87) A. 3 : 2 : 5 B. 5 : 2 : 3 C. 1/3 : 1/2 : 1/5
D. 1/5 : 1/2 : 1/3 E. 1/2 : 1/3 : 1/5

16. A man walks from place A to place B at a speed of 3 km/h and
(87) and cycles immediately back to place A along the same road at a speed of 15 km/h. The average speed for the whole trip is

A. 5 km/h B. 6 km/h C. 9 km/h D. 10 km/h E. 12 km/h

17. y varies inversely as x^2. If x is increased by 100%, then y
(88) is

A. increased by 100% B. increased by 300%
C. decreased by 25% D. decreased by 75%
E. decreased by 100%

18. A car travels from P to Q. If its speed is increased by k%,
(88) then the time it takes to travel the same distance is reduced by

A. k% B. 100/k % C. 100k/(100+k) %
D. k/(100+k) % E. k/(100-k) %

19. The weight of a gold coin of a given thickness varies as
(88) the square of its diameter. If the weights of two such coins are in the ratio 1 : 4, then their diameter are in the ratio

A. 1 : 2 B. 2 : 1 C. 1 : 4 D. 4 : 1 E. 1 : 16

ANSWERS

1.D 2.A 3.D 4.D 5.D 6.C 7.A 8.D 9.E 10.B
11.A 12.B 13.E 14.E 15.A 16.A 17.D 18.C 19.A

HKCEE. M.C. Ratio, Proportion, Variation

1. $2x = 3y = 5z$.

$\begin{cases} 2x = 3y \\ 3y = 5z \end{cases}$

$x : y = 3 : 2$

$\underline{y : z = 5 : 3}$

$x : y : z = 15 : 10 : 6$ (D)

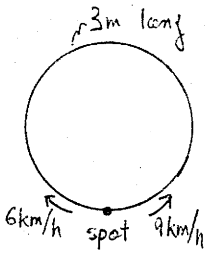
2. the time taken

$= \frac{3}{6+9}$ hr

$= \frac{1}{5}$ hr

$= \frac{1}{5} \times 60$ mins

$= 12$ mins (A)



3. Let x be the actual area.

$1 : 20000$ — scale.

$2 : x = 1^2 : (20{,}000)^2$

$\frac{x}{2} = \left(\frac{20{,}000}{1}\right)^2$

$x = 800{,}000{,}000$ cm^2

$= 80{,}000$ m^2 (D)

4. Let the three nos. be x, y, z

$\begin{cases} x = 2k \\ y = 3k \\ z = 5k \end{cases}$ where $k \neq 0$,

the average $= \frac{x+y+z}{3}$

$= \frac{10}{3} k$.

the ratio $= \frac{10}{3} : 5$

$= 2 : 3$. (D)

5. if. $\frac{3x+2y}{x+5y} = 1$

$3x + 2y = x + 5y$

$2x = 3y$

Let $\begin{cases} x = 3k \\ y = 2k \end{cases}$, $k \neq 0$

$\frac{x+y}{x-y} = \frac{3k+2k}{3k-2k} = \frac{5}{1}$

$x+y : x-y = 5 : 1$ (D)

6. Total distanced travelled

$= (30 \times 3 + 40 \times 2)$ km

$= 170$ km.

Total time taken

$= (3+2)$ hrs $= 5$ hrs.

the average speed

$= \frac{\text{total distance}}{\text{total time}}$

$= \frac{170}{5}$ km/h

$= 34$ km/h. (C)

7. Let x be the job.

speed of A : $\frac{x}{8}$

speed of B : $x/12$

speed of C : $x/6$.

A & B worked

$= \left(\frac{x}{8} + \frac{x}{12}\right) \times 3$

$= \frac{5}{8} x$

the work left

$= x - \frac{5}{8}x = \frac{3}{8}x$.

the time taken.

$= \frac{3/8\, x}{\left(\frac{x}{8} + \frac{x}{12} + \frac{x}{6}\right)}$

$= \frac{\cancel{3/8\, x}}{\cancel{3/8\, x}}$

$= 1$ hr (A.)

8. $\frac{a}{b} = \frac{b}{c} = k$.

$\therefore \begin{cases} a = bk \\ b = ck \end{cases}$ & $ac = b^2$.

(1). It cannot prove.

(2). $\frac{a+b}{b+c} = \frac{bk+ck}{b+c} = \frac{k(b+c)}{b+c}$

$= k$.

(3). $\frac{a}{c} = \frac{bk}{c} = \frac{ck^2}{c} = k^2$. (D).

9. $a : b = 1 : 2$

$\underline{b : c = 1 : 3}$

$\therefore a : b : c = 1 : 2 : 6$.

Let $\begin{cases} a = k \\ b = 2k \\ c = 6k \end{cases}$, $k \neq 0$

$\frac{a+b}{b+c} = \frac{k+2k}{2k+6k} = \frac{3k}{8k} = \frac{3}{8}$

$a+b : b+c = 3 : 8$. (E.)

10. Let x be the distance between John's home & his school.

the original speed $= \frac{x}{40/60} = \frac{3}{2}x$

the final speed $= \frac{x}{30/60} = 2x$.

$\therefore \frac{3}{2}x + 2 = 2x$

$\frac{1}{2}x = 2$

$x = 4$ km. (B)

11. total distance.

$= (45 \times 3 + 50 \times 2)$ km

$= 235$ km.

time taken

$= (3+2)$ hrs $= 5$ hrs.

average speed

$= \frac{235}{5}$ km/h $= 47$ km/h (A)

12. Let the distance be x.

A's speed $= \frac{x}{3}$

B's speed $= \frac{x}{4}$

C's speed $= x/5$.

A's speed : B's speed : C's speed.

$= \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$.

$= 20 : 15 : 12$ (B)

13 $\pi \approx 22/7$ and 3.1416

but not exactly equal.

$A = \pi r^2$ r-radius not diameter.

$S = 2\pi r$

$= \pi d$ d-diameter (E.)

14. $a : b = 3 : 2$

$\underline{b : c = 4 : 3}$

$a : b : c = 12 : 8 : 6$

$= 6 : 4 : 3$

Let $a = 6k$

$b = 4k$ $k \neq 0$

$c = 3k$

$\frac{a+b}{b+c} = \frac{6k+4k}{4k+3k} = \frac{10}{7}$.

$a+b : b+c = 10 : 7$ (E.)

15. $3a = 2b = 5c$.

$\begin{cases} 3a = 2b \\ 2b = 5c \end{cases}$ $a:b = 2:3$, $b:c = 5:2$, $a:b:c = 10:15:6$

$\therefore \frac{1}{a} : \frac{1}{b} : \frac{1}{c} = \frac{1}{10} : \frac{1}{15} : \frac{1}{6}$

$= 3 : 2 : 5$ (A)

16. Let x be the distance.

total distance $= 2x$.

total time $= \frac{x}{3} + \frac{x}{15}$

$= \frac{2}{5}x$

the average speed

$= \frac{2x}{2x/5} = 5\text{ km/h}$. (A)

17. $y = \frac{k}{x^2}$, $k \neq 0$.

x increase 100%

$\therefore x' = 2x$.

$y' = \frac{k}{(2x)^2} = \frac{k}{4x^2}$

$\therefore$ % change.

$\frac{y'-y}{y} \cdot 100\%$

$= \frac{\frac{k}{4x^2} - \frac{k}{x^2}}{k/x^2} \cdot 100\%$

$= (\frac{1}{4} - 1)(100\%)$

$= -75\%$

decreased by 75% (D.)

18. Let d – distance.

s – speed.

t – time. $t = \frac{d}{s}$.

$s' = (1 + k\%)s$

$\therefore t' = \frac{d}{s'} = \frac{d}{(1+k\%)s}$

$\frac{t'-t}{t} \cdot 100\%$

$= \frac{\frac{d}{(1+k\%)s} - \frac{d}{s}}{d/s} \cdot 100\%$

$= (\frac{1}{1+k\%} - 1)\, 100\%$

$= (\frac{100}{100+k} - 1)\, 100\%$

$= (\frac{-k}{100+k}) \cdot 100\%$

$= \frac{-100k}{100+k}\%$

reduced $\frac{100k}{100+k}\%$ (C.)

19. w – weight

d – diameter.

$w = kd^2$, $k \neq 0$

$4w = kd'^2$

$\therefore \frac{1}{4} = \frac{d^2}{d'^2}$

$d'^2 = 4d^2$

$d' = 2d$

$\therefore d : d' = 1 : 2$ (A.)

HKCEE Problems. Ratio, Proportion and Variation.

1 (80) A factory employs 10 skilled, 20 semi-skilled, and 30 unskilled workers. The daily wages per worker of the three kinds are in the ratio 4 : 3 : 2. If a skilled worker is paid \$120 a day, find the mean daily wage for the 60 workers.

(5 marks)

2 (80) Normally, a factory produces 400 radios in x days. If the factory were to produce 20 more radios each day, then it would take 10 days less to produce 400 radios. Calculate x.

(12 marks)

3(81) The capacities of two spherical tanks are in the ratio 27 : 64. If 72 kg of paint is required to paint the outer surface of the smaller tank, then how many kilograms of paint would be required to paint the outer surface of the bigger tank?

(5 marks)

4(82) The price of a certain monthly magazine is x dollars per copy. The total profit on the sale of the magazine is P dollars. It is given that $P = Y + Z$, where Y varies directly as x and Z varies directly as the square of x. When x is 20, P is 80 000; when x is 35, P is 87 500.

(a) Find P when $x = 15$. (7 marks)

(b) Using the method of completing the square, express P in the form $P = a - b(x - c)^2$ where a, b and c are constants. Find the values of a, b and c. (3 marks)

(c) Hence, or otherwise, find the value of x when P is a maximum. (2 marks)

5(83) If $a : b = 3 : 4$ and $a : c = 2 : 5$,

find (a) $a : b : c$,

(b) the value of $\dfrac{ac}{a^2 + b^2}$.

6(84) A school and a youth centre agree to share the total expenditure for a camp in the ratio 3 : 1. The total expenditure \$$E$ for the camp is the sum of two parts: one part is a constant \$$C$, and the other part varies directly as the number of participants N. If there are 300 participants, the school has to pay \$7500. If there are 500 participants, the school has to pay \$12 000.

(a) Find the total expenditure for the camp, when the school has to pa \$7500.

(2 marks)

(b) Find the value of C.

(5 marks)

(c) Express E in terms of N.

(2 marks)

(d) If the youth centre has to pay \$4750, find the number of participants.

(3 marks)

7(86) It is given that z varies directly as x^2 and inversely as y. If $x = 1$ and $y = 2$, then $z = 3$. Find z when $x = 2$ and $y = 3$.

(6 marks)

8(87) Given $p = y + z$, where y varies directly as x, z varies inversely as x and x is positive. When $x = 2$, $p = 7$; when $x = 3$, $p = 8$.

(a) Find p when $x = 4$.

(8 marks)

(b) Find the range of values of x such that p is less than 13.

(4 marks)

9(88) A variable quantity y is the sum of two parts. The first part varies directly as another variable x, while the second part varies directly as x^2. When $x = 1$, $y = -5$; when $x = 2$, $y = -8$.

(a) Express y in terms of x.

Hence find the value of y when $x = 6$.

(8 marks)

(b) Express y in the form $(x - p)^2 - q$, where p and q are constants.

Hence find the least possible value of y when x varies.

(4 marks)

10(89) **(a) Solve the simultaneous equations** $\begin{cases} x + 2y = 5 \\ 5x - 4y = 4 \end{cases}$

(b) Given that $\begin{cases} \dfrac{a}{c} + \dfrac{2b}{c} = 5 \\ \dfrac{5a}{c} - \dfrac{4b}{c} = 4 \end{cases}$**, where a, b and c are non-zero numbers, using the result of (a), find $a : b : c$.**

(6 marks)

11(91) In a joint variation, x varies directly as y^2 and inversely as z. Given that $x = 18$ when $y = 3$, $z = 2$,

(a) express x in terms of y and z,

(b) find x when $y = 1$, $z = 4$.

(5 marks)

12(91) Let $2a = 3b = 5c$.

(a) Find the ratio $a : b : c$.

(b) If $a - b + c = 55$, find c.

(6 marks)

HKCEE Problems. Ratio, Proportion & Variation.

1. Let. x - skilled
y - semi-skilled.
z - unskilled.

$\therefore\ x:y:z = 4:3:2.$

$\therefore \begin{cases} x = 4k \\ y = 3k \\ z = 2k \end{cases}, k \neq 0.$

since $x = \$120.$

$\therefore\ 4k = \$120$

$k = \$30$

$\therefore\ y = 3(\$30) = \$90.$

$z = 2(\$30) = \$60.$

$\therefore$ the mean wages,

$= \dfrac{10(\$120) + 20(\$90) + 30(\$60)}{10+20+30}$

$= \$80.$

2. the speed to produce radios (per day)

$= \dfrac{400}{x}.$

the new speed to produce radios each day

$= (\dfrac{400}{x} + 20)$

ie days needed to produce 400 radios.

$= (x-10)$

$\therefore (\dfrac{400}{x} + 20)(x-10) = 400.$

$(400 + 20x)(x-10) = 400x$

$(20+x)(x-10) = 20x$

$x^2 + 10x - 200 = 20x$

$x^2 - 10x - 200 = 0$

$(x-20)(x+10) = 0$

$x = 20$ or -10 (rejected $x > 0$).

3. Let R, r be the radius of bigger & smaller tank resp.

$r^3 : R^3 = 27:64.$

$(\dfrac{r}{R})^3 = \dfrac{27}{64}$

$\dfrac{r}{R} = \dfrac{3}{4}.$

$\dfrac{\text{area of smaller tank}}{\text{area of bigger tank}} = (\dfrac{r}{R})^2$

$\dfrac{72\text{kg}}{\text{paint for bigger}} = (\dfrac{3}{4})^2$

paint for bigger tank $= 72 \cdot (\dfrac{16}{9})$ kg

$= 128$ kg.

4. a) $P = Y + Z.$

$Y = k_1 x$

$Z = k_2 x^2.$ where $k_1, k_2 \neq 0$

$\therefore\ P = k_1 x + k_2 x^2.$

when $x = 20$, $P = 80000$

$80000 = k_1(20) + k_2(20)^2$

$4000 = k_1 + 20k_2$ —— ①

when $x = 35$, $P = 87500$

$87500 = k_1(35) + k_2(35)^2$

$2500 = k_1 + k_2(35)$ —— ②

② − ① $15k_2 = -1500$

$k_2 = -100.$

From ①

$\therefore\ k_1 = 4000 - 20(-100)$

$= 6000$

$\therefore P = 6000x - 100x^2$

$x = 15$,

$P = 6000(15) - 100(15)^2$

$= 67500.$

4b) $P = 6000x - 100x^2$

$P = -100(x^2 - 60x)$

$P = -100[x^2 - 60x + (\frac{60}{2})^2 - (\frac{60}{2})^2]$

$P = -100(x^2 - 60x + 30^2) + 100(30^2)$

$P = -100(x-30)^2 + 90000$

$P = 90000 - 100(x-30)^2$.

$\therefore a = 90000$; $b = 100$; $c = 30$.

4c) when P is max, $x = 30$.

5a) $\begin{cases} a:b = 3:4 \\ a:c = 2:5 \end{cases}$

$$\begin{array}{lcl} b:a & & 4:3 \\ \quad a:c & = & \quad 2:5 \\ \hline b:a:c & & 8:6:15 \end{array}$$

$\therefore a:b:c = 6:8:15$.

b) Let $\begin{cases} a = 6k \\ b = 8k \\ c = 15k \end{cases}$ where $k \neq 0$.

$\therefore \frac{ac}{a^2+b^2} = \frac{(6k)(15k)}{(6k)^2+(8k)^2}$

$= \frac{90k^2}{100k^2}$

$= 9/10$.

6.a) $E = C + kN$. where k is constant and $\neq 0$.

school $= \$7500$, $N = 300$.

$\therefore E = \$7500 (\frac{3+1}{3})$

$= \$10000$.

b)

$\therefore 10000 = C + k(300)$ —— ①

school $= \$12000$, $N = 500$

$E = \$12000 (\frac{4}{3})$

$= \$16000$

$16000 = C + k(500)$ —— ②.

P.2

② − ①.

$200k = 6000$

$k = 30$.

From ①

$\therefore C = 10000 - 30(300)$

$C = 1000$

$\therefore$ c) $E = 1000 + 30N$.

d) if youth centre $= \$4750$

$\therefore E = \$4750 (\frac{4}{1})$

$= \$19000$

$\therefore 19000 = 1000 + 30N$.

$30N = 18000$

$N = 600$.

7. $z = \frac{kx^2}{y}$ where $k \neq 0$.

$x = 1$, $y = 2$, $z = 3$.

$\therefore 3 = \frac{k(1)^2}{2}$

$\therefore k = 6$.

$\therefore z = \frac{6x^2}{y}$

when $x = 2$, $Y = 3$.

$\therefore z = \frac{6(2)^2}{3}$

$z = 8$.

8. a) $p = y + z$.

$y = k_1 x$; $z = \frac{k_2}{x}$ where $k_1, k_2 \neq 0$.

$\therefore p = k_1 x + \frac{k_2}{x}$, $x > 0$.

$x = 2$, $p = 7$

$7 = k_1(2) + (\frac{k_2}{2})$

$14 = 4k_1 + k_2$ — ①.

$x = 3$, $p = 8$

$8 = k_1(3) + (\frac{k_2}{3})$

$24 = 9k_1 + k_2$ — ②.

② − ①

$5k_1 = 10$

$k_1 = 2$.

From ①

$\therefore k_2 = 14 - 4(2)$

$= 6$.

$\therefore p = 2x + \frac{6}{x}$.

when $x = 4$

$\therefore p = 2(4) + \frac{6}{4}$

$= 8 + \frac{3}{2} = \frac{19}{2}$.

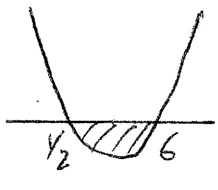
$p < 13$.

$\therefore 2x + \frac{6}{x} < 13$.

$2x^2 + 6 < 13x$.

$2x^2 - 13x + 6 < 0$.

$(2x-1)(x-6) < 0$.



$\frac{1}{2} < x < 6$

9. $y = k_1 x + k_2 x^2$ where k_1, k_2 are constant, $\neq 0$.

(a) $x = 1$, $y = -5$

$-5 = k_1(1) + k_2(1)^2$

$\therefore k_1 + k_2 = -5$ — ①.

$x = 2$, $y = -8$.

$-8 = k_1(2) + k_2(2)^2$.

$k_1 + 2k_2 = -4$ — ②.

② − ①. $k_2 = 1$.

$\therefore k_1 = -6$

$\therefore y = -6x + x^2$.

when $x = 6$,

$y = -6(6) + (6)^2$

$= 0$.

b) $y = x^2 - 6x$.

$= x^2 - 6x + (\frac{6}{2})^2 - (\frac{6}{2})^2$

$= (x-3)^2 - 9$.

$\therefore$ the least possible value of $y = -9$.

10 a) $\begin{cases} x + 2y = 5 & — ① \\ 5x - 4y = 4 & — ② \end{cases}$.

① × 2 + ②

$2x + 5x = 10 + 4$

$7x = 14$

$x = 2$.

From ①

$\therefore y = \frac{5-2}{2} = \frac{3}{2}$.

b) $\begin{cases} \frac{a}{c} + \frac{2b}{c} = 5 \\ \frac{5a}{c} - \frac{4b}{c} = 4 \end{cases}$

put $x = \frac{a}{c}$, $y = \frac{b}{c}$.

from a), $\frac{a}{c} = 2$. ; $\frac{b}{c} = \frac{3}{2}$.

$\therefore a:c = 2:1$; $b:c = 3:2$

$$\begin{array}{cc} a:c & 2:1 \\ c:b & 2:3 \\ \hline a:c:b & 4:2:3 \end{array}$$

$\therefore a:b:c = 4:3:2$