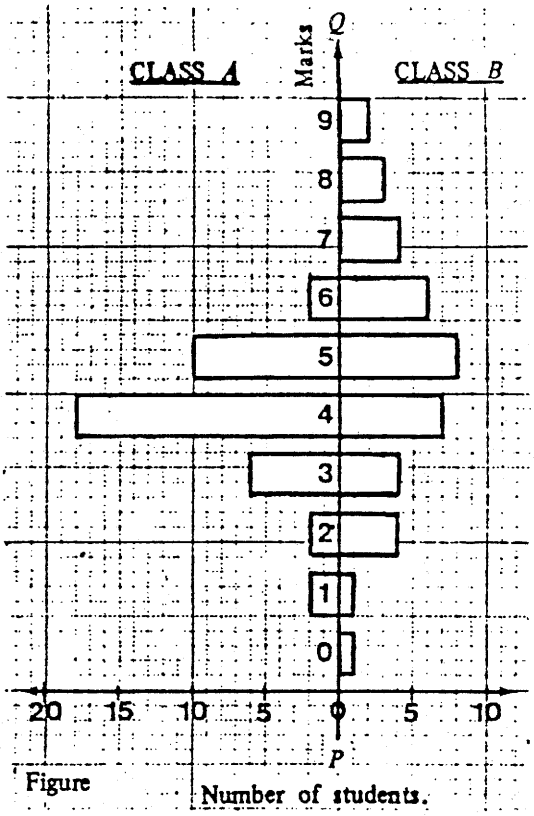
1(80) Two classes, A and B, each of 40 students, took a test. In the test, students may score 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9 marks. In Figure , the distribution of marks of class A is shown in the bar chart on the left of PQ and that of class B is shown on the right.

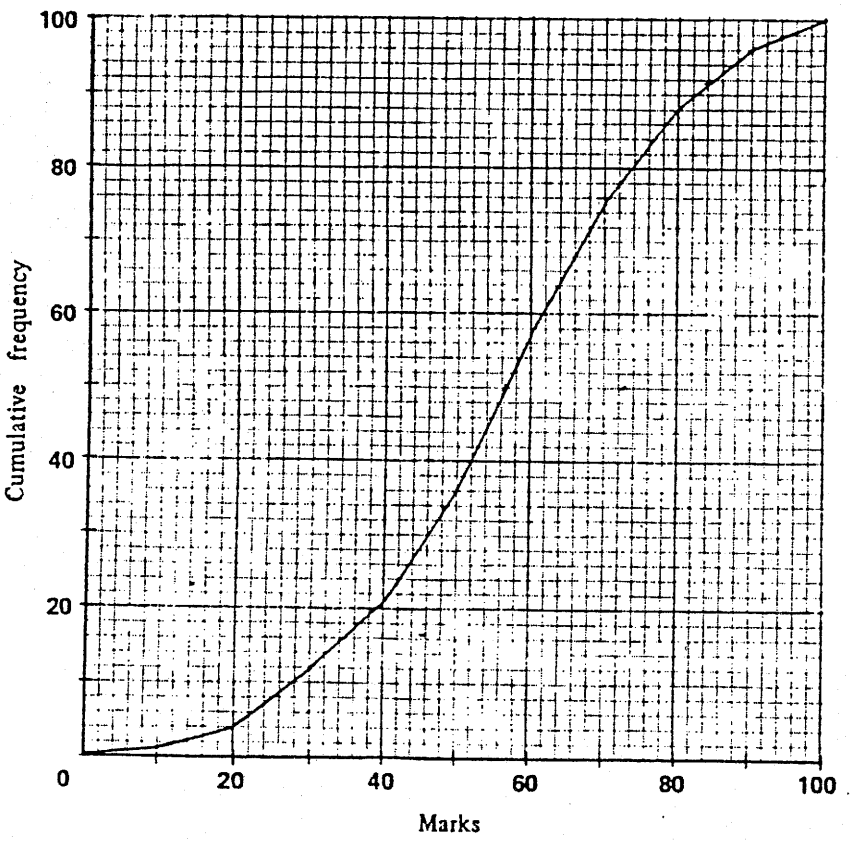
(a) Find, by inspection, which class has a greater standard deviation of marks.

(b) If 70 students from the two classes pass the test, what is the minimum mark that a student should get in order to obtain a pass?



Figure

2(81) Figure- shows the cumulative frequency polygon of the marks obtained by 100 students taking a mathematics test.



Figure

(a) If 75% of the students pass the test, what is the pass mark, correct to the nearest integer ?

(b) If the pass mark were 40 , how many students would pass the test ?

(c) Find the inter-quartile range.

3(81) The heights of 1000 students form a symmetrical distribution with a mean of 1.70 m and a standard deviation of 0.02 m. If 67% of the students lie within one standard deviation of the mean and 97% lie within two standard deviations of the mean, find

(a) the number of students who are shorter than 1.74 m,

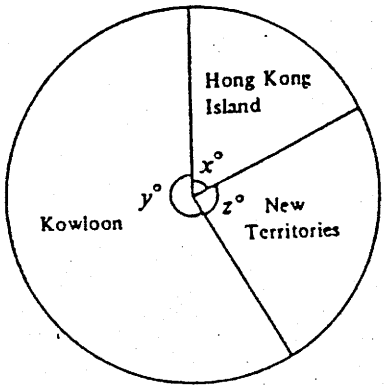
(b) the number of students whose heights lie between 1.68 m and 1.74 m.

4(82) In a certain school, the numbers of students living on Hong Kong Island, in Kowloon and the New Territories are in the ratios 2 : 7 : 3. The pie-chart in Figure shows the distribution.

(a) Find x, y and z.

(b) If the number of students living on Hong Kong Island is 240, find the total number of students in the school.

(6 marks)



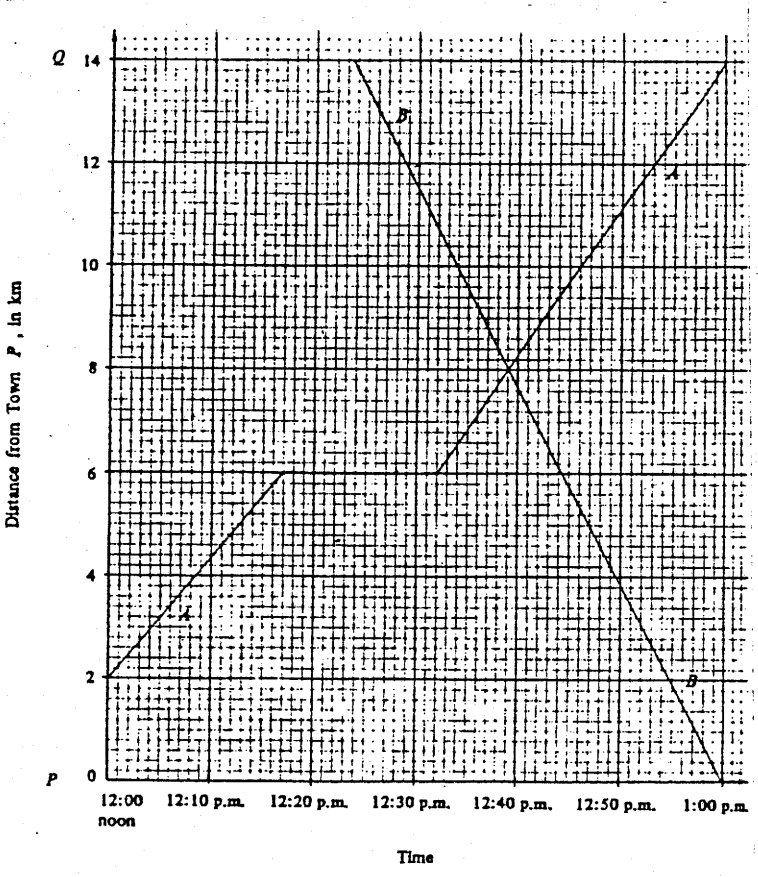
Figure

5(83) Given five real numbers $a - 6$, a, $a + 2$, $a + 3$, $a + 6$,

find (a) the mean,

(b) the standard deviation.

6(84)



Figure

Figure shows the travel graphs of two cyclists A and B travelling on the same road between towns P and Q, 14 km apart.

(a) For how many minutes does A rest during the journey ?

(b) How many km away from P do A and B meet ?

7(82)(a) The pie-chart in Figure (a) shows how Mr Wong's income was distributed between his expenses and savings for March.

If his rent is $2000, find Mr Wong's income for that month.

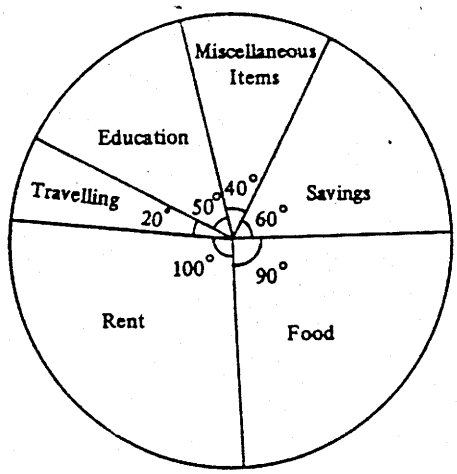


Figure (a)

(b) The table below shows the percentage changes when each item of Mr Wong's expenses in April is compared with that in March.

Item	Food	Rent	Travelling	Education	Miscellaneous Items	Savings
Percentage Change	Increased by 10%	Increased by 30%	Increased by 30%	No change	No change	?

The pie-chart in Figure (b) shows how Mr Wong's income was distributed between his expenses and savings for April.

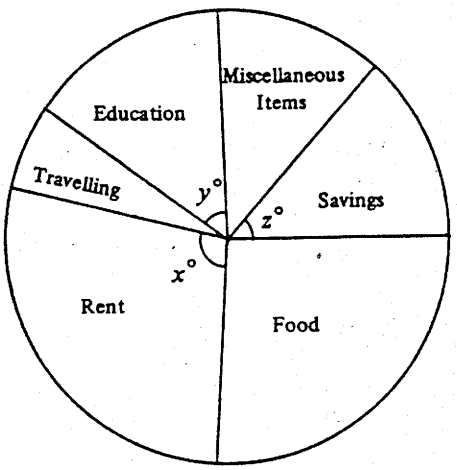


Figure (b)

(i) Suppose that Mr Wong's income in March and April were the same.

(1) Find x, y and z in Figure (b).

(2) Calculate the percentage change in Mr Wong's savings for April when compared with those for March.

(ii) If Mr Wong's income in April actually increased by 37.5%, what percentage of his income in April was spent on food ?

8(83). The following table shows the distribution of the marks of 1000 students in a mathematics test:

Class of Marks	Number of Students
40 – 49	100
50 – 59	300
60 – 69	400
70 – 79	200

(a) Find the mid-value of the class 50 – 59.
(b) Estimate the mean of the above distribution of marks.

(84) The table below shows the distribution of the marks of a group of students in a short test :

Marks	1	2	3	4	5
Number of Students	10	10	5	20	x

If the mean of the distribution is 3 , find the value of x .

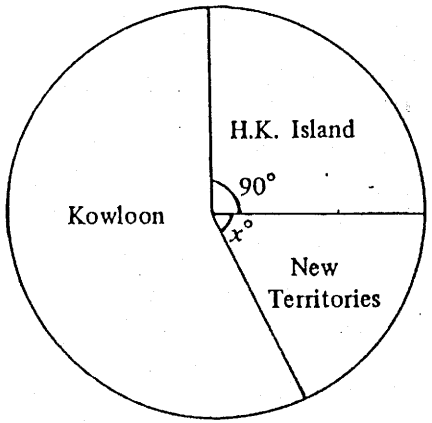


Figure 2

(85) The pie-chart in Figure 2 shows the distribution of traffic accidents in Hong Kong in 1983. There were 4200 traffic accidents on H.K. Island, 9240 accidents in Kowloon and n accidents in the New Territories. Find n and x.

(86) **The table below shows the number of students in three classes of a school and their average marks in a test :**

Class	No. of Students	Average Mark
F.5A	40	61
F.5B	x	70
F.5C	35	50

If the overall average mark of the three classes is 60, find x.

12(90) (a) The distribution of the monthly salaries of 100 employees in a firm is shown in the histogram in Figure

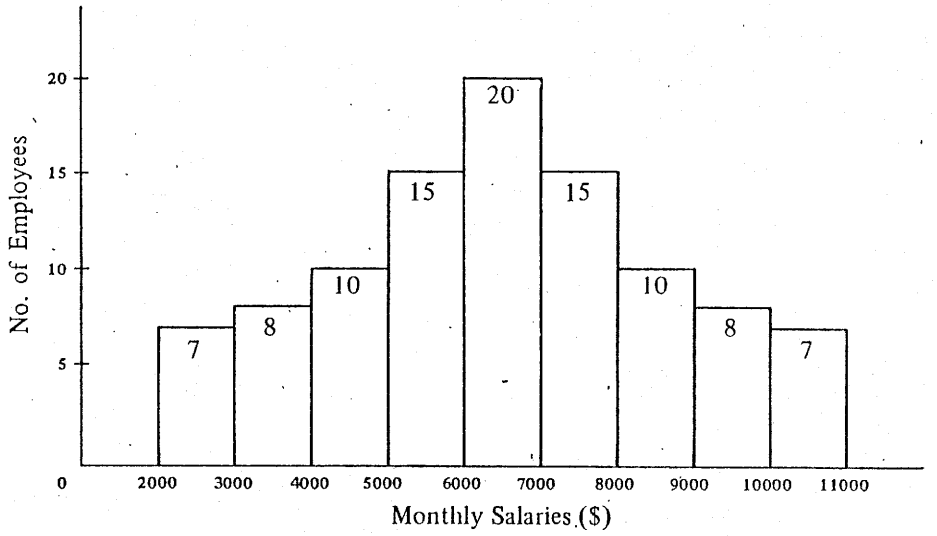


Figure : Distribution of monthly salaries of 100 employees

(i) Find the modal class, median, mean, interquartile range and mean deviation of the monthly salaries of the 100 employees.

(ii) Now the firm employs 10 more employees whose monthly salaries are all $6500. Will the standard deviation of the monthly salaries of all the employees in the firm become greater, smaller or remain unchanged? Explain briefly.

(b) The mean of 7 numbers x_1 , x_2 , . . . , x_7 is $\bar{x}$ and the *squares* of their deviations from $\bar{x}$ are 9, 4, 1, 0, 1, 4, 9 respectively. Find the standard deviation of the 7 numbers.

Figure 4 shows the cumulative frequency curve of the marks of 600 students in a mathematics contest.

(a) From the curve, find

(i) the median, and

(ii) the interquartile range of the distribution of marks.

(4 marks)

(b) A student with marks greater than or equal to 100 will be awarded a prize.

(i) Find the number of students who will be awarded prizes.

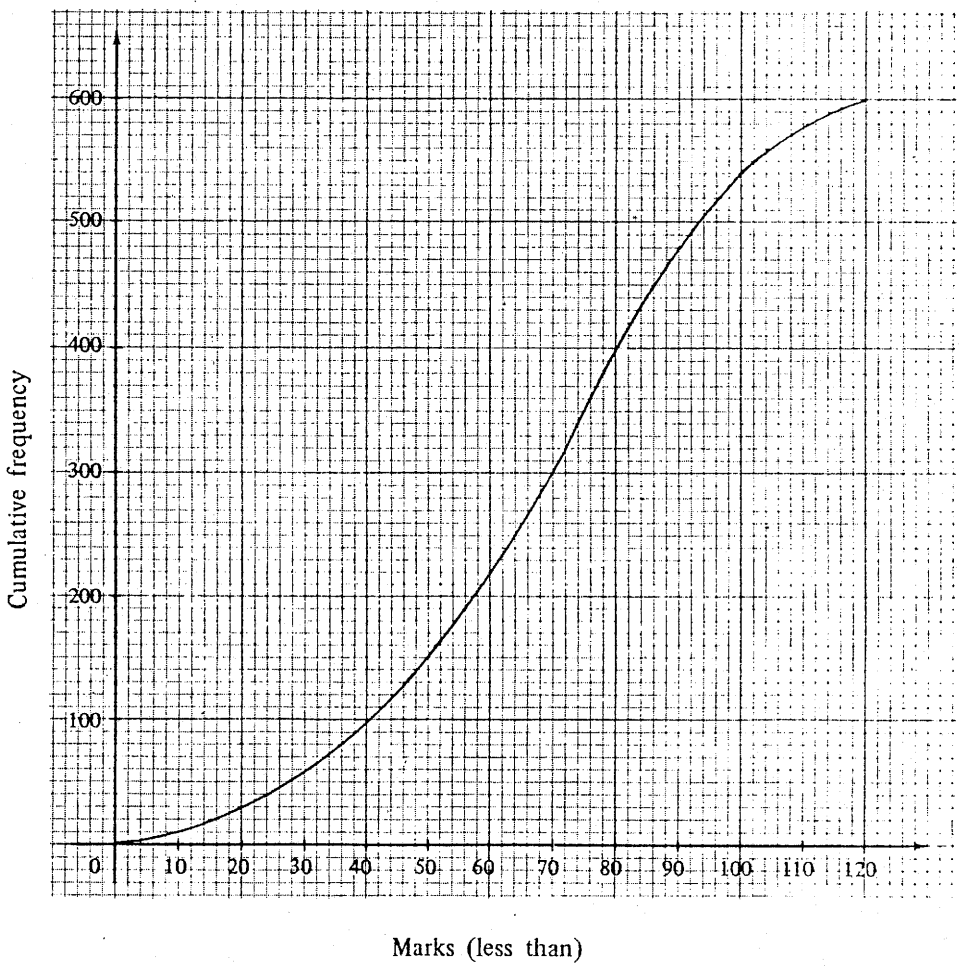
(ii) If one student is chosen at random from the 600 students, find the probability that the student is a prize-winner.

(iii) If two students are chosen at random, find the probability that

(1) both of them are prize-winners,

(2) at least one of them is a prize-winner.

(8 marks)



4(91) In Figure 1, the cumulative frequency polygon shows the distribution of the marks of 80 students in a Mathematics test.

(a) From the figure, write down the median of the distribution.

(b) Copy the table below onto your answer book and complete it.

Hence find the mean mark of the students in the test.

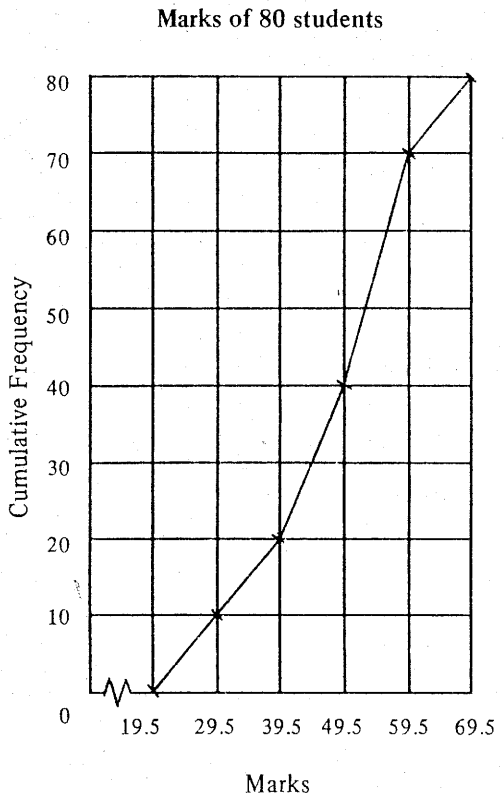


Figure 1

Marks	No. of Students
20 – 29	
30 – 39	
40 – 49	
50 – 59	
60 – 69	

(5 marks)

HKCEE Problems Statistics

1. a) Class A has a greater standard deviation of marks since its variation is greatest.

b) If 70 students pass the test.

∴ 10 students who score lowest marks are failed.

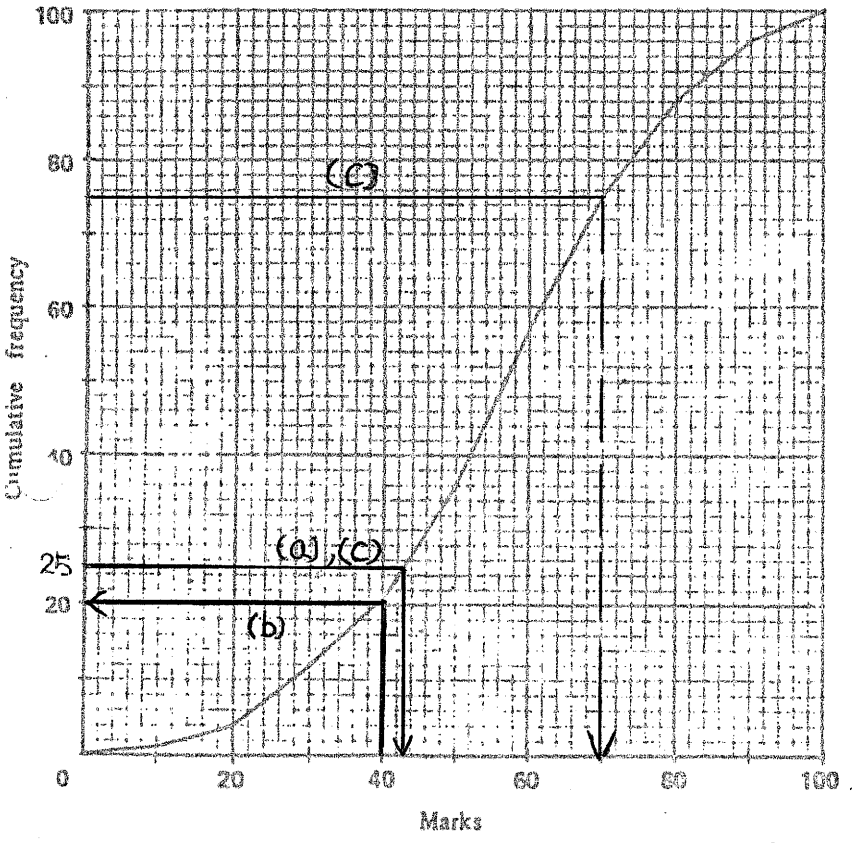
1 students score 0 marks.

3 students score 1 marks.

6 students score 2 marks.

10 students

∴ the pass mark is 3 marks.



a) If 75% pass the test, the lowest 25% students failed the test.

No. of students failed = 100 × 25% = 25.

∴ the pass mark is 43 (nearest integer)

b) If the pass mark were 40,

no. of students pass the test.

= 100 − 20

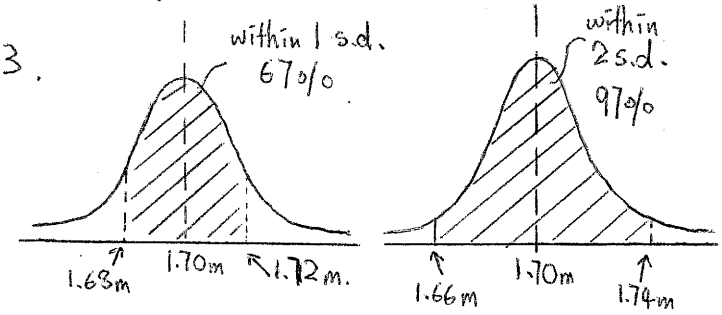
= 80.

c) the inter-quartile range.

= (70 − 43)

= 27.

3.



a) no. of students ≤ 1.74m

$= 1000 \times \left(\frac{1-97\%}{2}\right)$

$= 1000 \times \frac{3}{2}\%$

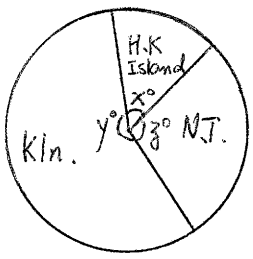
= 15.

b) 1.68 < no. of students < 1.74

$= 1000 \times \left(\frac{67\%}{2} + \frac{97\%}{2}\right)$

= 820.

4a) $x^\circ : y^\circ : z^\circ = 2 : 7 : 3$.



$x^\circ = 360^\circ \times \frac{2}{12}$

$x^\circ = 60^\circ$

$x = 60$

$y^\circ = 360^\circ \times \frac{7}{12}$

$y = 210$

$z^\circ = 360^\circ \times \frac{3}{12}$

$z = 90$.

b) Let n be total no. of student.

$\frac{n}{240} = \frac{12}{2}$

$n = 1440$.

5. $a-6,\ a,\ a+2,\ a+3,\ a+6$.

a) the mean.

$$\bar{x} = \frac{(a-6)+a+(a+2)+(a+3)+(a+6)}{5}$$

$$= \frac{5a+5}{5} = a+1.$$

b) the standard deviation.

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{[(a-6)-(a+1)]^2 + [a-(a+1)]^2 + [(a+2)-(a+1)]^2 + [(a+3)-(a+1)]^2 + [(a+6)-(a+1)]^2}{5}}$$

$$= \sqrt{\frac{49+1+1+4+25}{5}}$$

$$= \sqrt{16} = 4.$$

6 a) the time taken for A to rest.

$= (12:32 - 12:17)$

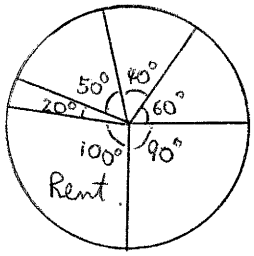
$= 15$ mins.

b) when A and B meet, they are 8km away from P.

7a) Let x be Mr Wong's income.

$$\frac{x}{\$2000} = \frac{360^\circ}{100^\circ}$$

$$x = \$7200$$



b) i) For rent,

(1) $x^\circ = 100^\circ \times (1+30\%)$

$\therefore x = 130$

For Education, (no change).

$y^\circ = 50^\circ$

$\therefore y = 50$

For food,

$= 90^\circ \times (1+10\%) = 99^\circ$

$= 20^\circ \times (1+30\%)$

$= 26^\circ$

For Mrs collaneous items (no. change)

$= 40^\circ$

For savings,

$z^\circ = 360^\circ - 130^\circ - 50^\circ - 99^\circ - 26^\circ - 40^\circ$

$= 15^\circ$.

(2). the % change. for April.

$$= \frac{15^\circ - 90^\circ}{90^\circ} \times 100\%$$

$= -83.33\%$

decrease 83.33%

ii) Mr. Wong's income

$= \$7200 \times (1+37.5\%)$

$= \$9900$.

spend on food.

$= \$9900 \times \left(\frac{99^\circ}{360^\circ}\right)$

$= \$2722.5$.

% of his income spend on food

$$= \frac{2722.5}{9900} \times 100\%$$

$= 27.5\%$

8 a) mid-value of 50 ~ 59.

$$= \frac{59.5 - 49.5}{2}$$

$= 54.5$.

b) the mean.

$$\bar{x} = \frac{44.5 \times 100 + 54.5 \times 300 + 64.5 \times 400 + 74.5 \times 200}{1000}$$

$= 61.5$.

1.

Marks	1	2	3	4	5
no. of students	10	10	5	20	x

mean = 3.

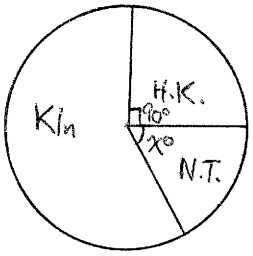
$$\therefore 3 = \frac{1\times10+2\times10+3\times5+4\times20+5\times x}{10+10+5+20+x}$$

$$3(45+x) = 5x + 125$$

$$10 = 2x$$

$$x = 5.$$

10. Let y be total no. of accidents in 1983.



$$\therefore \frac{y}{4200} = \frac{360^\circ}{90^\circ}$$

$$y = 16800.$$

no. of accidents in N.T.

$$= 16800 - 4200 - 9240$$

$$= 3360.$$

$$\frac{x^\circ}{360^\circ} = \frac{3360}{16800}$$

$$x = 72$$

11. average mark = 60

$$60 = \frac{(61)(40)+(70)(x)+(50)(35)}{40+x+35}$$

$$(75+x)60 = 70x + 4190$$

$$10x = 310$$

$$x = 31$$

12(a)

i) the greatest no. of employees is the class \$6000 ~ \$7000.

∴ the modal class is \$6000 ~ \$7000

median is $\left(\frac{100}{2}\right)^{th}$ term = 50^{th} term.

∴ median = \$6500

since the distribution is symmetric,

∴ the mean = \$6500.

interquartile range = $Q_3 - Q_1$

$Q_3 = (100\times\frac{3}{4})^{th}$ term = 75^{th} term.

$Q_1 = (100\times\frac{1}{4})^{th}$ term = 25^{th} term.

Q_3 = \$5000

Q_1 = \$8000.

∴ interquartile range = \$8000 − \$5000 = \$3000.

mean deviation

$$= \frac{\sum f_i|x_i-\bar{x}|}{n}$$ ~ absolute value.

$$= \frac{\$1}{100}[7\times|2500-6500| + 8|3500-6500| + 10|4500-6500| + 15|5500-6500| + 20|6500-6500| + 15|7500-6500| + 10|8500-6500| + 8|9500-6500| + 7|10500-6500|]$$

$$= \frac{\$1}{100}[7\times4000 + 8\times3000 + 10\times2000 + 15\times1000 + 20\times0 + 15\times1000 + 10\times2000 + 8\times3000 + 7\times4000].$$

= \$1740

aii) if 10 employees with \$6500 added.

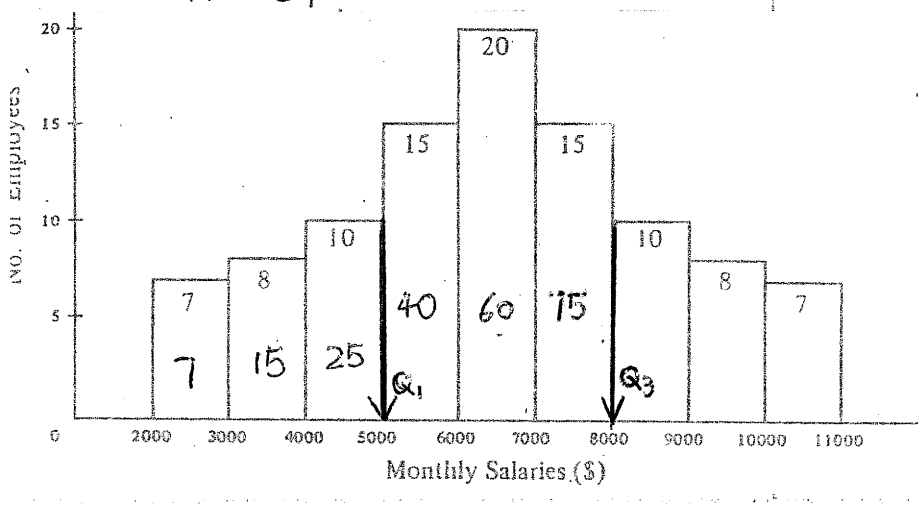
$$\text{S.D. } \sigma = \sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{n}}$$

since $\bar{x}$ = \$6500

∴ numerator is not change.

but denominator is increased 10.

∴ σ is smaller.



12 b) The nos, $x_1, x_2, \ldots, x_7$

their mean $= \bar{x}$.

$(x_1-\bar{x})^2 = 9$ $\quad (x_5-\bar{x})^2 = 1$

$(x_2-\bar{x})^2 = 4$ $\quad (x_6-\bar{x})^2 = 4$

$(x_3-\bar{x})^2 = 1$ $\quad (x_7-\bar{x})^2 = 9$.

$(x_4-\bar{x})^2 = 0$

the standard deviation.

$$\sigma = \sqrt{\frac{\sum f_i (x_i-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{9+4+1+0+1+4+9}{7}}$$

$$\sigma = \sqrt{4} = 2.$$

b) i) the number of students awarded prizes.

$= (600-540)$

$= 60$.

ii) P(the student is a prize-winner.)

$= \frac{60}{600}$

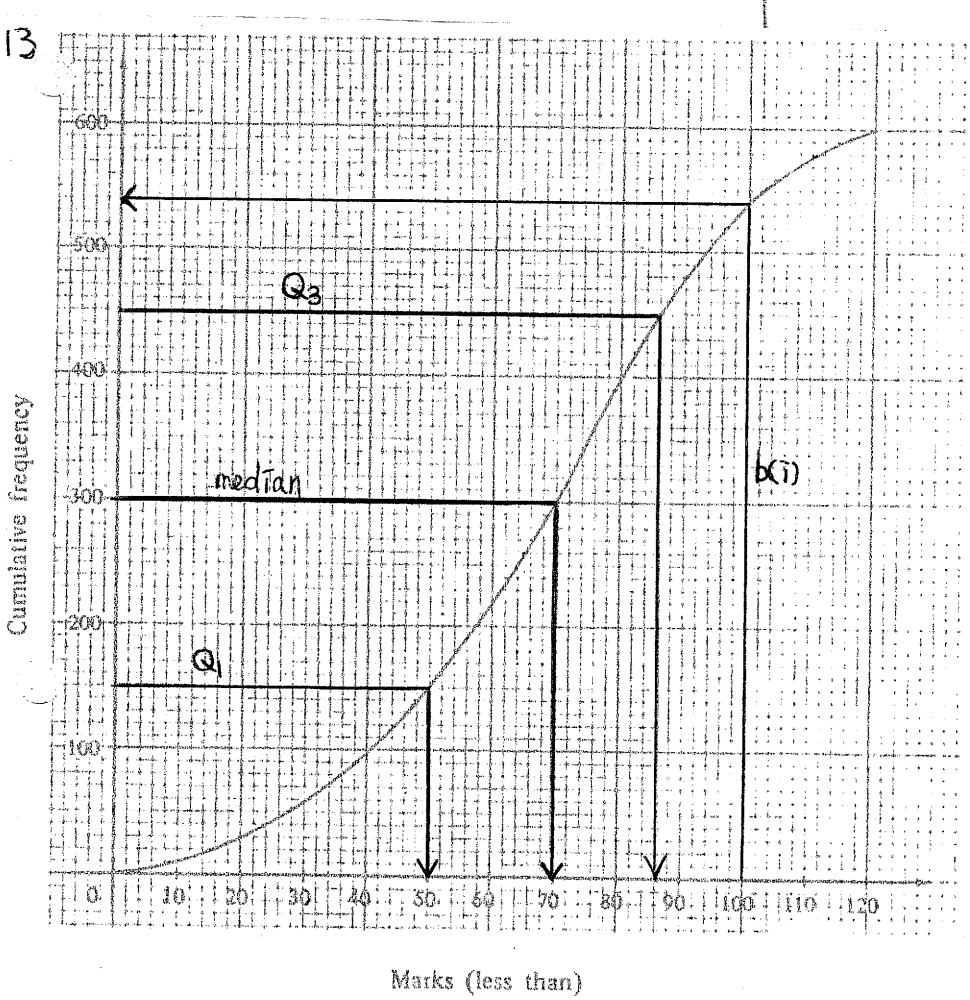
$= \frac{1}{10}$

iii)(1) P(both of them are prize-winners.)

$= (\frac{1}{10})(\frac{1}{10})$

$= \frac{1}{100}$

13



a) From the curve,

i) the median $= (\frac{600}{2})^{th}$ term.

$= 70$ marks.

ii) the interquartile range

$= Q_3 - Q_1$

$= (86-50)$ marks

$= 36$ marks.

(2). P(at least one of them is a prize-winner.)

$= 1 - P$(they are not prize-winner)

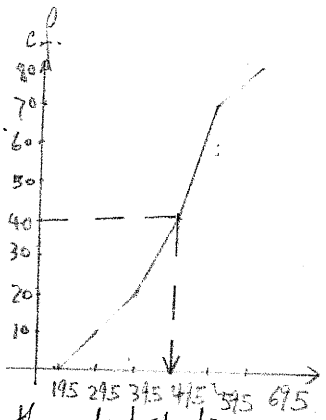
$= 1-(1-\frac{1}{10})(1-\frac{1}{10})$

$= 1-(\frac{81}{100})$

$= \frac{19}{100}$.

14.

Marks.	No. of student.
20-29	10
30-39	10
40-49	20
50-59	30
60-69	10.



the median of the distribution.

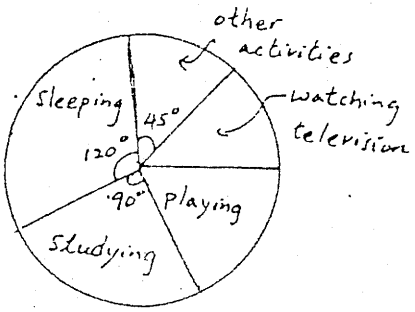
$= 49.5$.

the mean mark of the students in the test.

$$= \frac{[24.5\times10 + 34.5\times10 + 44.5\times20 + 54.5\times30 + 64.5\times10]}{80}$$

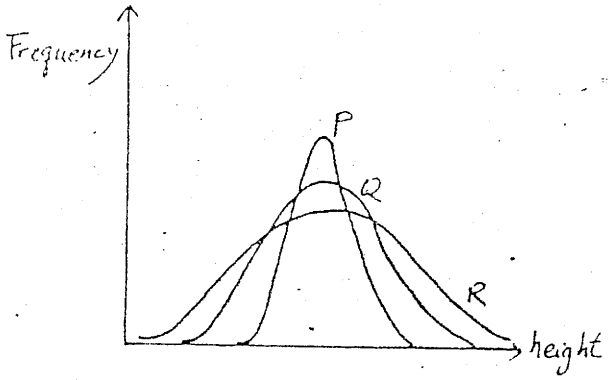
$= 47$.

1. (83) The pie chart shows how a boy spends the 24 hours of a day. If the boy spends 4 hours playing, how much time does he spend watching television ?



A. 1 hours
B. 2 hours
C. 3 hours
D. 4 hours
E. 5 hours

2. (83) In the figure, P, Q, R are curves showing the frequency distributions of the heights of students in three schools, each having the same number of students. Which distribution has the greatest standard deviation and which the smallest ?



	Greatest	Smallest
A.	P	Q
B.	P	R
C.	Q	R
D.	R	P
E.	R	Q

3. (84) The standard deviation of the five numbers a−2d, a−d, a, a+d, a+2d is

A. 0 B. d C. $\sqrt{2}$ d D. $\sqrt{5}$ d E. $\sqrt{10}$ d

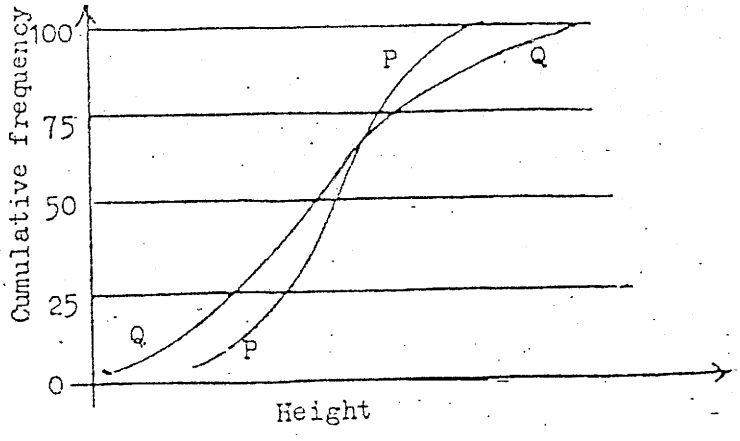
4. (85)

Class mid-value	Frequency
m − 8	3
m − 4	1
m	2
m + 4	6

The mean of the distribution is

A. m − 1/3 B. m − 1/2 C. m − 2 D. m − 4 E. m

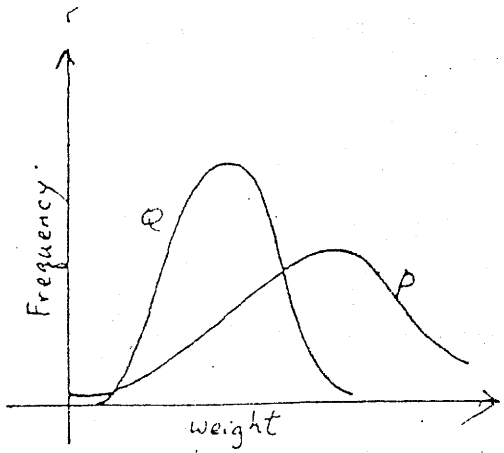
5. (85) In the figure, P and Q are the cumulative frequency curves for the heights of two groups of students, each having 100 students. Which of the following must be true ?



I. range of P < range of Q
II. median of P < median of Q
III. the 3rd quartile of P < the 3rd quartile of Q

A. I only B. II only
C. I and II only
D. I and III only
E. I, II and III

6. (85) In the figure, P and Q are curves showing the distribution of weights of students in two schools, each having the same number of students. Which of the following must be true ?



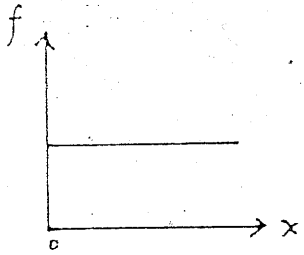
I. standard deviation of P > standard deviation of Q
II. mode of P > mode of Q
III. median of P > median of Q

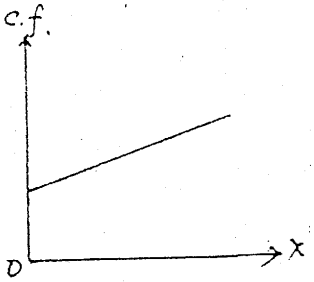
A. I only B. I and II only
C. I and III only
D. II and III only
E. I, II and III

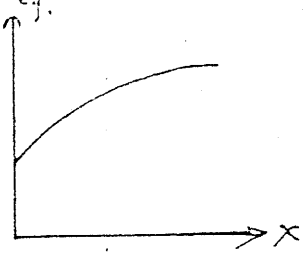
7. (86) Given two groups of numbers $a+1, a+2, a+3$ and $b+1, b+2, b+3$ where $a > b$. m_1 and m_2 are respectively the means of the two groups, and s_1 and s_2 are respectively their standard deviations. Which of the following is true ?

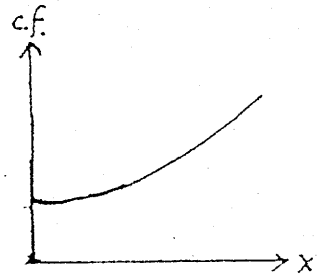
A. $m_1 > m_2$ and $s_1 > s_2$ B. $m_1 > m_2$ and $s_1 = s_2$
C. $m_1 = m_2$ and $s_1 > s_2$ D. $m_1 = m_2$ and $s_1 = s_2$
E. $m_1 > m_2$ and $s_1 < s_2$

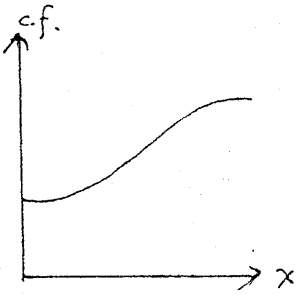
8. (86) The figure shows the frequency curve of a certain distribution. Which of the following can be the distribution's cumulative frequency curve ?



A.


B.


C.


D.


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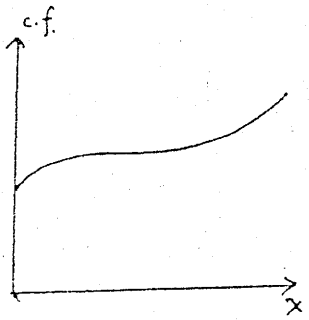
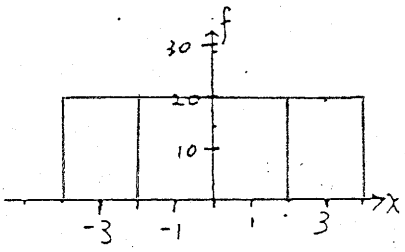


9. (87) If the median of the 5 different integers $2, 7, 10, x, 2x-3$ is 7, then $x =$

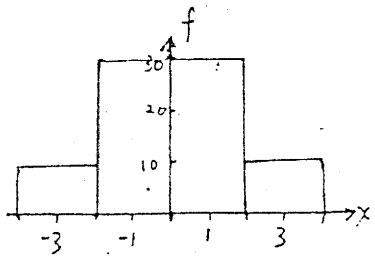
A. 3 B. 4 D. 5 C. 6 E. 8

10. (87) The figures shows the histograms of three frequency distributions. Arrange their standard deviations in ascending order of magnitude.

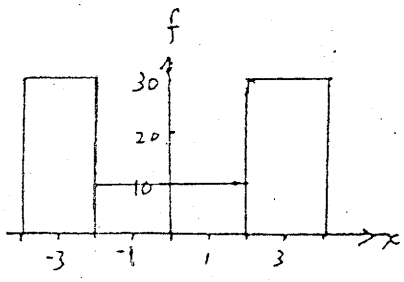
(1)



(2)



(3)



A. (1),(2),(3) B. (1),(3),(2) C. (2),(1),(3)
D. (2),(3),(1) E. (3),(2),(1)

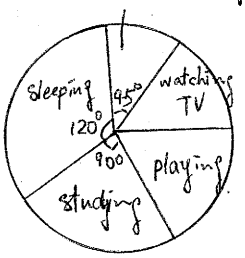
11. (88) The maximum load a lift can carry is 600 kg. 11 men with a mean weights of 49 kg are already in the lift. If one more man is to enter the lift, his weight must not exceed
A. 49 kg B. 50 kg C. 51 kg D. 59 kg E. 61 kg

12. (88) The mean length of 30 rods is 80 cm. If one of these rods of length 68 cm is taken out and replaced by another rod of length 89 cm, then the new mean length is
A. 79.3 cm B. 79.7 cm C. 80 cm D. 80.3 cm E. 80.7 cm

ANSWERS
1.C 2.D 3.C 4.A 5.D 6.C 7.B 8.A 9.D 10.C
11.E 12.E

Statistic

1. Let $x°$ be angle of playing.



$$\frac{x°}{360°} = \frac{4}{24}$$

$x° = 60°$.

∴ angle of watching TV

$= 360° - 60° - 90° - 120° - 45°$

$= 45°$.

∴ time he spends.

$= 24 \cdot \left(\frac{45°}{360°}\right)$ hrs

$= 3$ hrs. (C.)

2. The greater the standard deviation, the more dispersion of the distribution.

Sine R. is wide,

s.d. is greatest

P is narrow,

s.d. is smallest. (D.)

3. the mean.

$$= \frac{(a-2d)+(a-d)+a+(a+d)+(a+2d)}{5}$$

$$= \frac{5a}{5} = a.$$

$$\text{s.d.} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{1}{5}\left[(a-2d-a)^2+(a-d-a)^2+(a-a)^2 + (a+d-a)^2+(a+2d-a)^2\right]}$$

$$= \sqrt{\frac{(2d)^2+d^2+0+d^2+(2d)^2}{5}}$$

$$= \sqrt{\frac{10d^2}{5}} = \sqrt{2}d. \quad (C.)$$

4. The mean

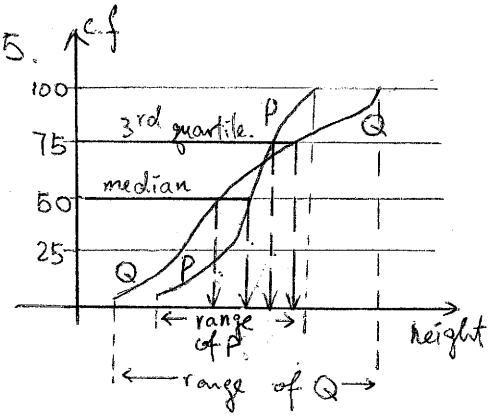
$$= \frac{3(m-8)+(m-4)+2m+6(m+4)}{12}$$

$$= \frac{3m-24+m-4+2m+6m+24}{12}$$

$$= \frac{12m-4}{12}$$

$= m - \frac{1}{3}$. (A.)

5.



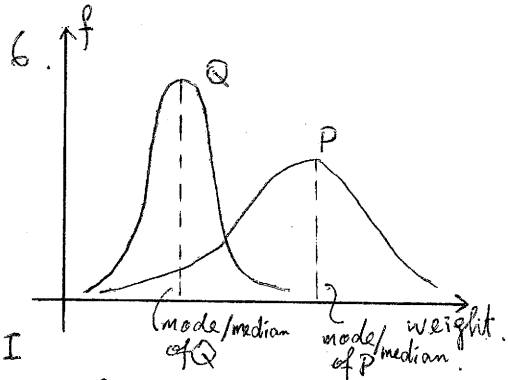
I range of P < range of Q.

II. median of P > median of Q

III 3rd quartile of Q > 3rd quartile of P.

∴ (I) & (III) only (D.)

6.



I

since P is wider than Q.

∴ s.d. of P > s.d. of Q

II since Q is higher than P.

∴ mode of P < mode of Q

III median of P > median of Q.

∴ (I) & (III) are true.

(C)

7. $a+1, a+2, a+3$; P.1

$b+1, b+2, b+3$.

$m_1 = \frac{1}{3}(a+1+a+2+a+3)$

$= a+2$.

$m_2 = \frac{1}{3}(b+1+b+2+b+3)$

$= b+2$.

since $a > b$.

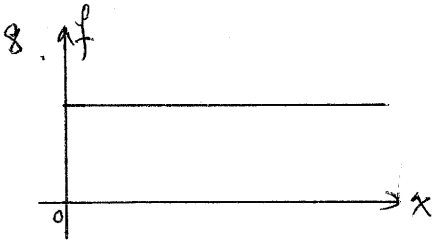
∴ $a+2 > b+2$.

$m_1 > m_2$.

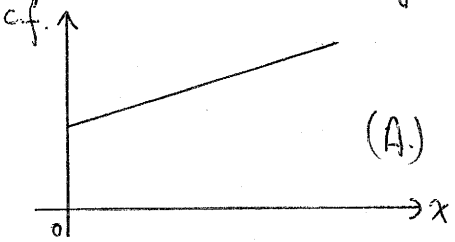
since the difference between the numbers of two groups are equal.

∴ $s_1 = s_2$. (B.)

8.



since the frequecy is constant, therefore c.f. is increasing steady.



(A.)

9. 2, 7, 10, x, 2x-3.

median = 7.

∴ 7 is the 3rd number.

$x < 7 < 2x-3$

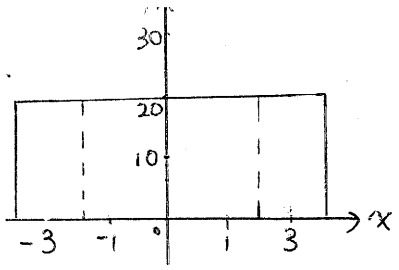
$\begin{cases} x < 7 \\ 7 < 2x-3 \end{cases}$

$\begin{cases} x < 7 \\ 5 < x \end{cases}$

∴ $x = 6$. (D.)

10.

(1).



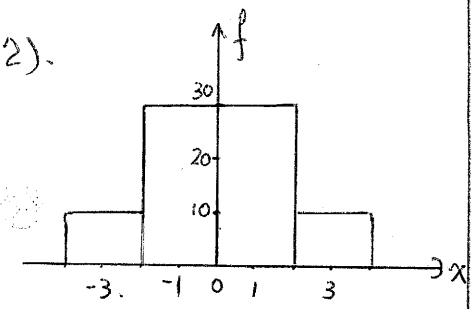
since it is symmetric,

$\bar{x} = 0$.

$$S_1 = \sqrt{\frac{(-1)^2 \times 20 + (-3)^2 \times 20 + (1)^2 20 + (3)^2 \cdot 20}{80}}$$

$= \sqrt{5}$.

(2).

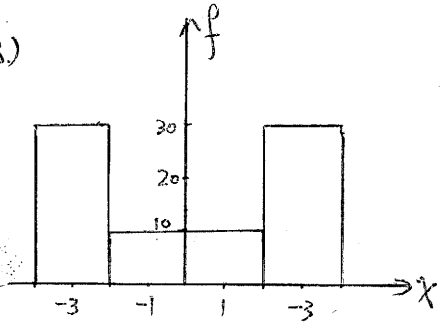


symmetrize, $\bar{x} = 0$,

$$S_2 = \sqrt{\frac{(-1)^2 \cdot 30 + (-3)^2 \cdot 10 + 1^2 \cdot 30 + 3^2 \cdot 10}{80}}$$

$= \sqrt{3}$.

(3)



symmetrize, $\bar{x} = 0$,

$$\therefore S_3 = \sqrt{\frac{(-1)^2 \cdot 10 + (-3)^2 \cdot 30 + (1)^2 \cdot 10 + (3)^2 \cdot 30}{80}}$$

$= \sqrt{7}$.

$\therefore \sqrt{3} < \sqrt{5} < \sqrt{7}$

$S_2 < S_1 < S_3$

ascending order of magnitude (2), (1), (3)

(C.)

11. Let his weight be x kg

$x + 11(49) \leq 600$

$x \leq 61$.

$\therefore$ his weight must not exceed 61 kg. (E.)

12. the new mean length.

$$\bar{x} = \frac{30 \times 80 - 68 + 89}{30} \text{ cm}.$$

$= 80.7$ cm (E.)