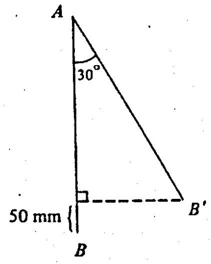
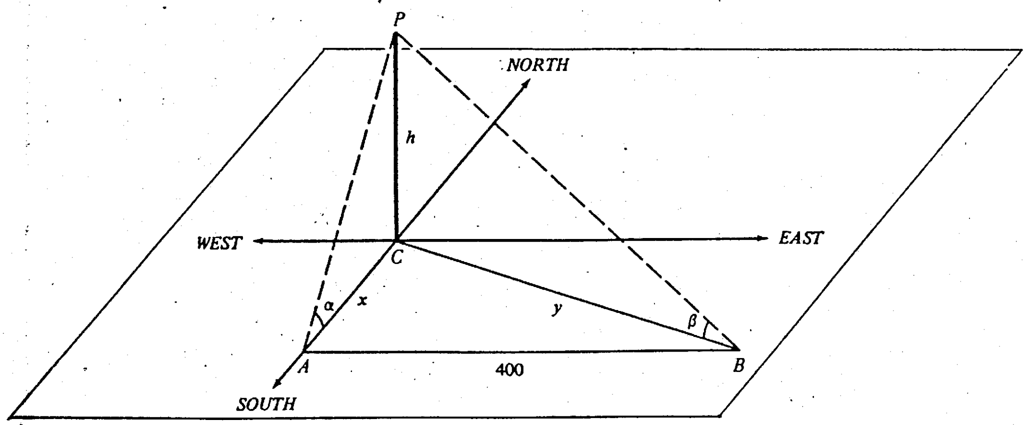
1.(80) If $0^\circ < \theta < 360^\circ$ and $\sin\theta = \cos 120^\circ$, find θ.



2.(80) In figure, AB is a vertical thin rod, It is rotated about A to position AB′ such that BAB′ = 30°. If B′ is 50mm higher than B, find the length of the rod, correct to 3 significant figures.



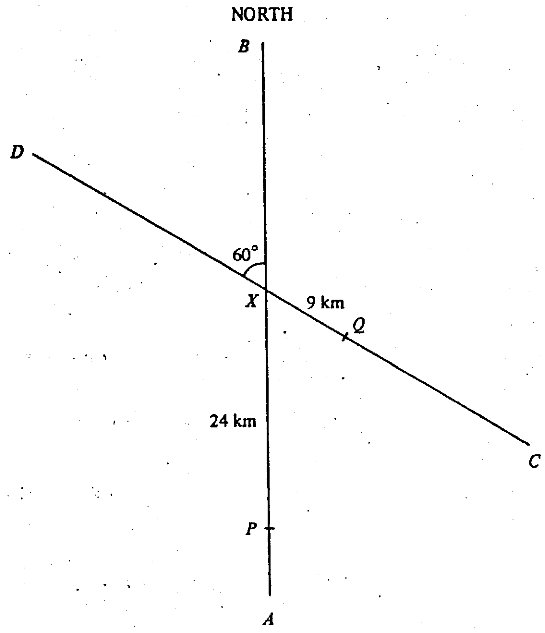
3.(80) In figure , PC represents a vertical object of height h meters. From a point A, aouth of C, the angle of elevation of P is α. From a point B, 400 meters east of A , the angle of elevation of P is β . AC and BC are x meters and y meters respectively.

(a) i) Express x in terms of h and α .
   ii) Express y in terms of h amd β .

(b) If $\alpha = 60^\circ$ and $\beta = 30^\circ$, find the value of h correct to 3 significant figures.

4.(81) Solve $\cos(200^\circ + \theta) = \sin 120^\circ$ where $0^\circ < \theta < 180^\circ$

5.(81)



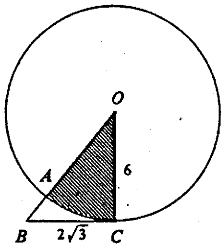
AB and CD are two straight roads intersting at X. AB runsNorth and makes an angle of 60° with CD. At noon, two people P and Q are respectively 24 km and 9 km from X as shown in figure. P walks at a speed of 4.5km/h towards B and Q walks at speed of 6 km/h towards D.

(a) Calculate the distance between P and Q at noon.

(b) What are the distance of P and Q from X at 4 p.m. ?

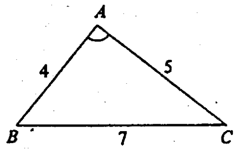
(c) Calculate the bearing of Q from P at 4p.m. to the nearest degree.

6.(82) In figure , the circle, centre and radius 6 , touches the straight line BC at C. BC = $2\sqrt{3}$. OAB is a straight line. Find the area of the shaded sector in term of π .

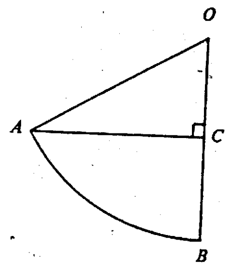


7.(82) Solve $2\sin^2\theta + 5\sin\theta - 3 = 0$ for θ , where $0^\circ \leqslant \theta \leqslant 360^\circ$

8.(82) In figure AB = 4, AC = 5 and BC = 7. Calculate $\angle A$ to the nearest degree.

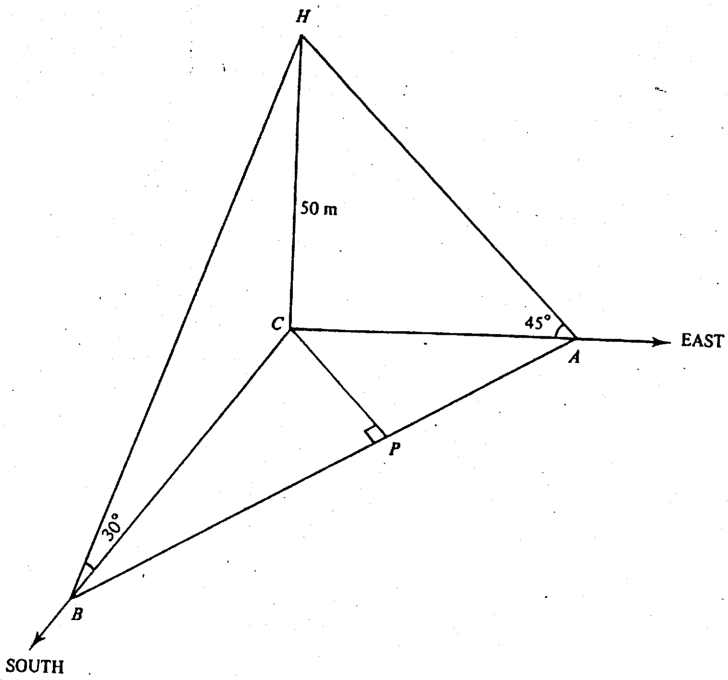


9.(83) In figure , O is the centre of the sector OAB, OA = 30, CB = 15 and AC $\perp$ OB.
Find a) $\angle$AOC.
b) the length of the arc AB in terms of π .



10.(83)Find all the values of θ , where $0^\circ \leqslant \theta \leqslant 360^\circ$, such that $2\cos^2\theta + 5\sin\theta + 1 = 0$.

11.(83)



In figure , A,B and C are three points on the same horizontal groud. HC is a vertical tower 50m high.A and B repectively due east and due south of the tower. The angles of elevation of H observed from A and B are respectively 45° and 30° .

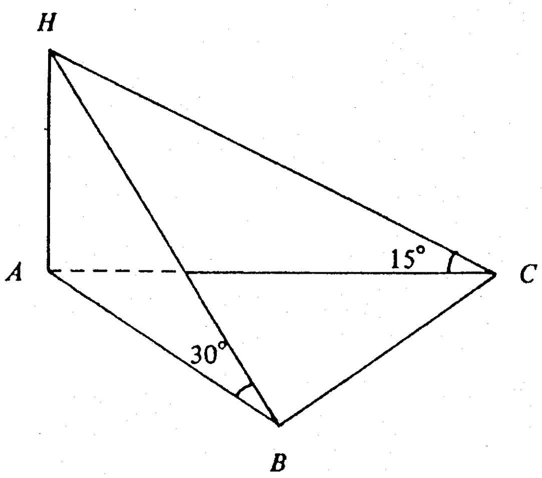
(a) Find the distance between A and B.
(b) Find the angle of elevation of H observed from P to the nearest degree.

12(84). Given $\tan\theta = \dfrac{1+\cos\theta}{\sin\theta}$ $(0° < \theta < 90°)$,

(a) rewrite the above equation in the form $a\cos^2\theta + b\cos\theta + c = 0$ where a, b and c are integers;

(b) hence, solve the given equation, giving your answer in degrees.

(6 marks)



Figure

3(84). In Figure , A, B and C lie in a horizontal plane. $AC = 20$ m. HA is a vertical pole. The angles of elevation of H from B and C are 30° and 15° respectively.

(In this question, give your answers correct to 2 decimal places.)

(a) (i) Find, in m, the length of the pole HA.

(ii) Find, in m, the length of AB.

(6 marks)

(b) If A, B and C lie on a circle with AC as diameter,

(i) find, in m, the distance between B and C;

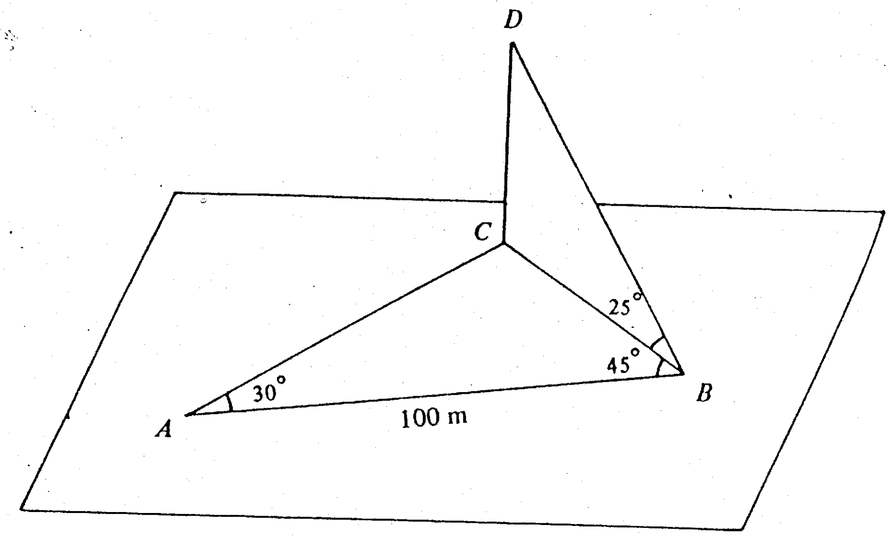
(ii) find, in m², the area of $\triangle ABC$.

(6 marks)

14(85) Solve $2\tan^2\theta = 1 - \tan\theta$, where $0° \leqslant \theta < 360°$. (Give your answers correct to the nearest degree.)

(6 marks)

15(85)



Figure

In Figure , A, B and C are three points in a horizontal plane. $AB = 100$ m. $\angle CAB = 30°$, $\angle ABC = 45°$.

(a) Find BC and AC, in metres, correct to 1 decimal place.

(5 marks)

(b) D is a point vertically above C. From B, the angle of elevation of D is 25°.

(i) Find CD, in metres, correct to 1 decimal place.

(ii) X is a point on AB such that $CX \perp AB$.

(1) Find CX, in metres, correct to 1 decimal place.

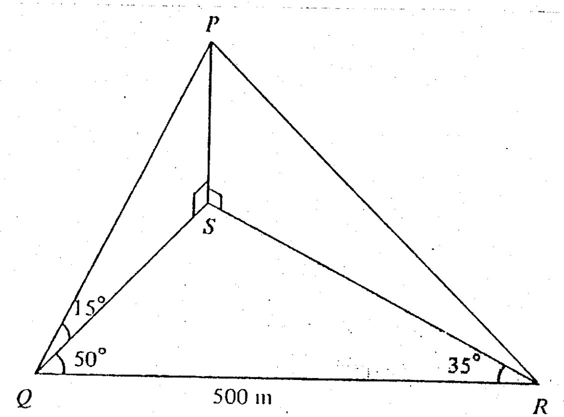
(2) Find the angle of elevation of D from X, correct to the nearest degree.

(7 marks)

6(86) Solve $\sin^2\theta + 7\sin\theta = 5\cos^2\theta$ for $0° \leqslant \theta < 360°$.

(6 marks)

Figure



17(86) In Figure , Q, R and S are three points on the same horizontal plane. $QR = 500$ m, $\angle SQR = 50°$ and $\angle QRS = 35°$. P is a point vertically above S. The angle of elevation of P from Q is $15°$.

(a) Find the distance, in metres, from P to the plane, correct to 3 significant figures.

(6 marks)

(b) Find the angle of elevation of P from R, correct to the nearest degree.

(6 marks)

18(87.)

Figure

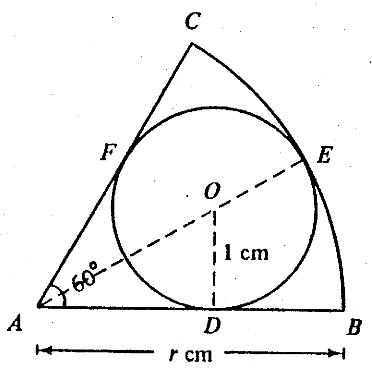


Figure shows a circle, centre O, inscribed in a sector ABC. D, E and F are points of contact. $OD = 1$ cm, $AB = r$ cm and $\angle BAC = 60°$. Find r.

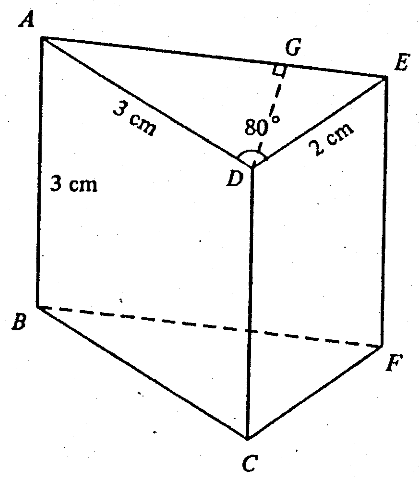
(6 ma

19(87) Solve the equation $\sin^2\theta = \frac{3}{2}\cos\theta$, where $0° \leqslant \theta < 360°$.

(6 marks)

20(87)

Figure



In this question, you should give your answers in cm or degrees, correct to 3 decimal places.

Figure shows a solid in which $ABCD$, $DCFE$ and $ABFE$ are rectangles. DG is the perpendicular from D to AE. $AB = 3$ cm, $AD = 3$ cm and $DE = 2$ cm. $\angle ADE = 80°$.

(a) Find AE.

(3 marks)

(b) Find $\angle DAE$.

(3 marks)

(c) Find DG.

(2 marks)

(d) Find BD.

(2 marks)

(e) Find the angle between the line BD and the face $ABFE$.

(2 marks)

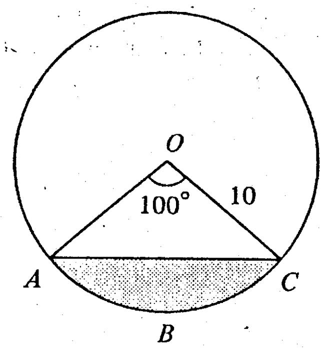
21.(88) Simplify

(a) $\dfrac{\sin(180^\circ - \theta)}{\sin(90^\circ + \theta)}$,

(b) $\sin^2(\pi - \phi) + \sin^2\left(\dfrac{3\pi}{2} + \phi\right)$.

(5 marks)

Figure



22(88) In Figure , ABC is a circle with centre O and radius 10. $\angle AOC = 100^\circ$. Calculate, correct to 2 decimal places,

(a) the area of sector $OABC$,

(b) the area of ΔOAC,

(c) the area of segment ABC.

(6 marks)

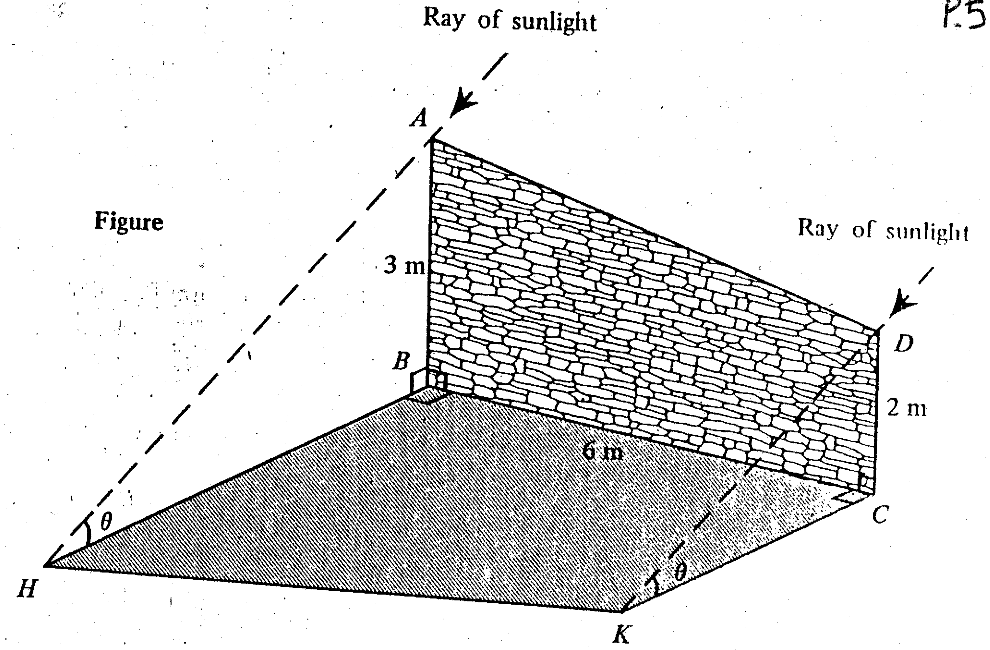
23(89). **Rewrite the equation $3\tan\theta = 2\cos\theta$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are integers.**

Hence solve the equation for $0^\circ < \theta < 360^\circ$.

(7 marks)

24(88).

Figure



In Figure , $ABCD$ is a wall in the shape of a trapezium with AB and DC vertical. Rays of sunlight coming from the back of the wall cast a shadow $HBCK$ on the horizontal ground such that the edges HB and KC of the shadow are perpendicular to BC. Suppose the angle of elevation of the sun is θ, $AB = 3$ m, $CD = 2$ m and $BC = 6$ m.

(a) Express HB and KC in terms of θ.

(3 marks)

(b) (i) Find the area S_1 of the wall.

(ii) Find, in terms of θ, the area S_2 of the shadow.

Hence show that $\dfrac{S_1}{S_2} = \tan\theta$.

(3 marks)

(c) If $\theta = 30^\circ$, find the length of the edge HK, leaving your answer in surd form.

(6 marks)

25(89) **Answers in this question should be given correct to at least 3 significant figures or in surd form.**

In Figure , a triangular board ABC, right-angled at A with $AB = AC = 10$ m, is placed with the vertex A on the horizontal ground. AB and AC make angles of 45° and 30° with the horizontal respectively. The sun casts a shadow $AB'C'$ of the board on the ground such that B' and C' are vertically below B and C respectively.

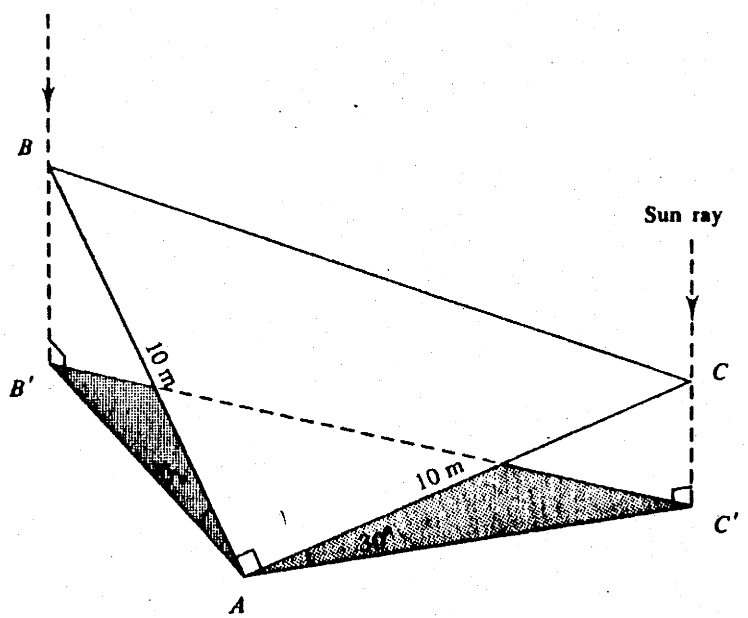
(a) Find the lengths of AB' and AC'. (2 marks)

(b) Find the lengths of BC, BB' and CC'. (3 marks)

(c) Using the results of (b), or otherwise, find the length of $B'C'$. (3 marks)

(d) Find $\angle B'AC'$.

Hence find the area of the shadow. (4 marks)

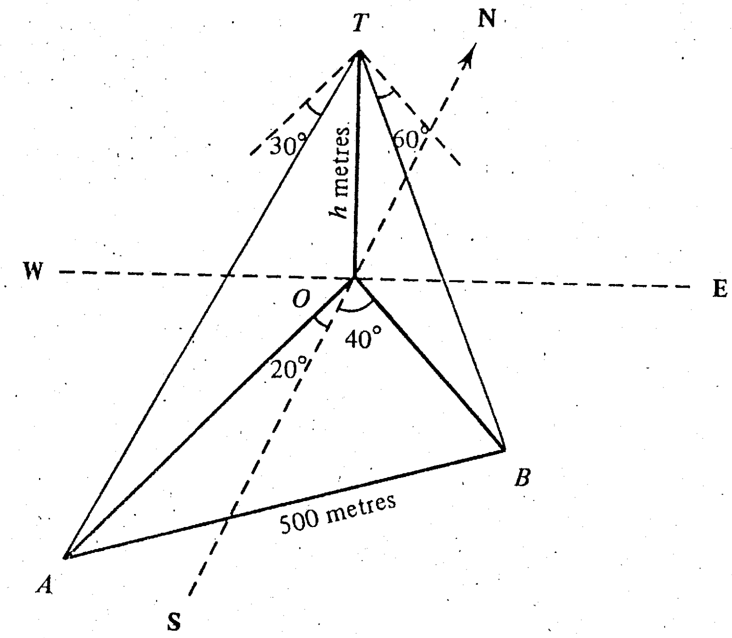


26(90) Rewrite $\sin^2\theta : \cos\theta = -3 : 2$ in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a, b and c are integers.

Hence solve for θ, where $0^\circ \leqslant \theta < 360^\circ$.

(6 marks)

27(90)



Figure

In Figure , OT represents a vertical tower of height h metres. From the top T of the tower, two landmarks A and B, 500 metres apart on the same horizontal ground, are observed to have angles of depression 30° and 60° respectively. The bearings of A and B from the tower OT are S20°W and S40°E respectively.

(a) Find the lengths of OA and OB in terms of h. (3 marks)

(b) Express the length of AB in terms of h. Hence, or otherwise, find the value of h. (5 marks)

(c) Find $\angle OAB$, correct to the nearest degree.

Hence write down

(i) the bearing of B from A,

(ii) the b ... ıg of A from B.

28(91) Solve $\sin^2\theta - 3\cos\theta - 1 = 0$ for $0° \leq \theta < 360°$.

(6 marks)

29(91)

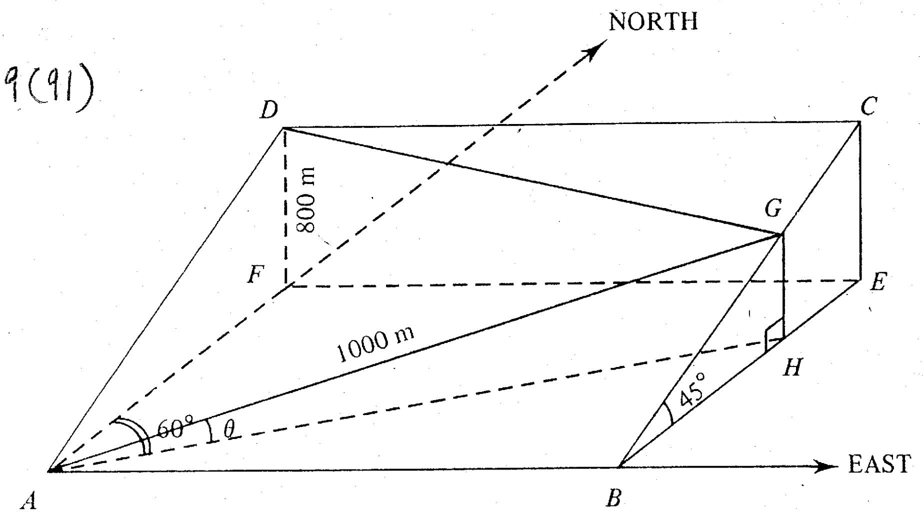


Figure 8

Figure 8 shows a rectangular plane $ABCD$ which inclines at 45° to the horizontal plane $ABEF$, where E and F are vertically below C and D respectively. B is due east of A . D is due north of A and 800 m vertically above F . G is a point on BC vertically above a point H on BE . Let $\angle GAH = \theta$, $\angle FAH = 60°$ and $AG = 1000$ m.

(a) Express GH and AH in terms of θ .

(2 marks)

(b) By considering $\triangle ABH$, express BH in terms of θ .
Hence find θ .

(5 marks)

(c) Find EF and EH .

Hence find the bearing of G from D .

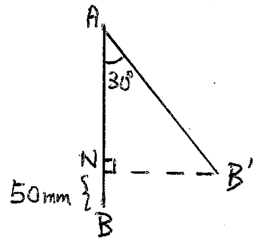
(5 marks)

1. $\sin\theta = \cos 120^\circ$. $\quad 0 < \theta < 360^\circ$

$\sin\theta = -\frac{1}{2}$

$\therefore \theta = 210^\circ$ or 330°.

2. Let x be the length of the rod.



$\frac{AN}{AB'} = \cos 30^\circ$.

$\frac{AN}{x} = \cos 30^\circ$.

$AN = x\cos 30^\circ$.

$AB = AN + NB$

$x = x\cos 30^\circ + 50\,mm$

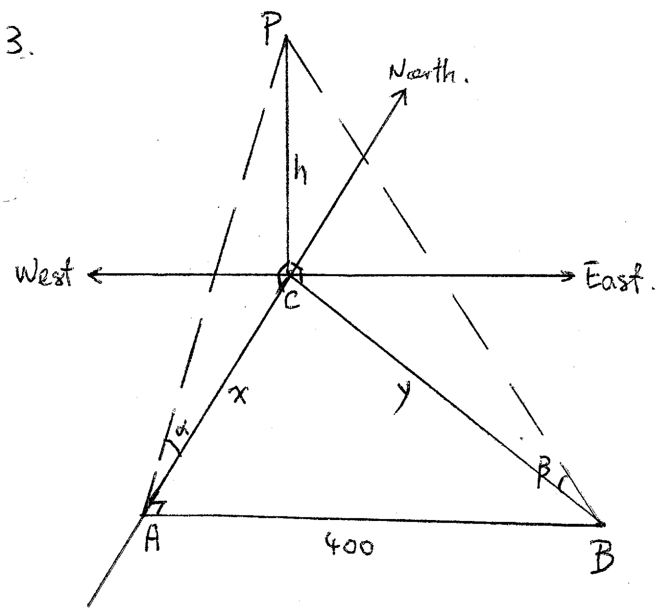
$x(1-\cos 30^\circ) = 50\,mm$

$x = \frac{50}{1-\sqrt{3}/2}\,mm$

$= \frac{100}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}\,mm.$

$= \frac{100(2+\sqrt{3})}{4-3} = 100(2+\sqrt{3})\,mm.$

3.



a i) In ΔACP.

$\frac{h}{x} = \tan\alpha$.

$x = \frac{h}{\tan\alpha}$.

a ii) In ΔBCP.

$\frac{h}{y} = \tan\beta$.

$\therefore y = \frac{h}{\tan\beta}$.

b) if. $\alpha = 60^\circ$, $\beta = 30^\circ$.

$\therefore x = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}} =$

$y = \frac{h}{\tan 30^\circ} = \frac{h}{1/\sqrt{3}} = \sqrt{3}h$.

In ΔABC.
$\angle CAB = 90^\circ$.

$\therefore x^2 + 400^2 = y^2$.

$\left(\frac{h}{\sqrt{3}}\right)^2 + 400^2 = (\sqrt{3}h)^2$.

$\frac{h^2}{3} + 400^2 = 3h^2$.

$\frac{8}{3}h^2 = 400^2$

$h^2 = 60{,}000$

$h = 245.\,m$ (3 sig. fig.).

4. $\cos(200^\circ + \theta) = \sin 120^\circ \quad 0 \le \theta \le 180^\circ$

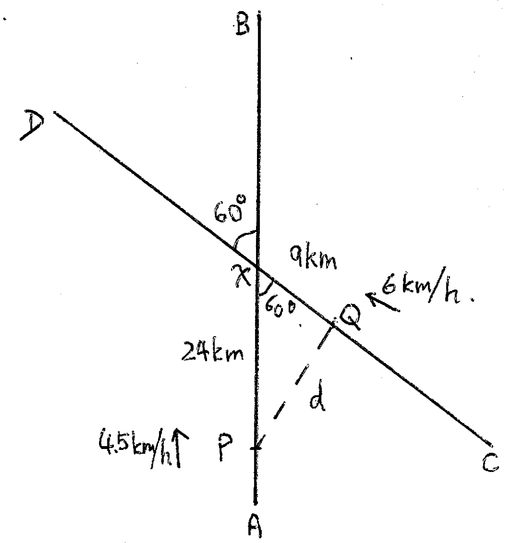
$\cos(200^\circ + \theta) = \frac{\sqrt{3}}{2}$.

$200^\circ + \theta = 30^\circ, 330^\circ$,

$\therefore \theta = -170^\circ, 130^\circ$.

$\theta = 130^\circ$.

5.



5a) Let d be the distance between P & Q at noon.

By cosine rule,

$d^2 = 24^2 + 9^2 - 2(24)(9)\cos 60^\circ$

$d^2 = 441$

$d = 21$ km.

b) At. 4 p.m.

distance of P travelled.

= speed × time.

= (4.5×4) km

= 18 km.

distance of P from X.

= $(24 - 18)$ km.

= 6 km.

distance of Q travelled.

= (6×4) km

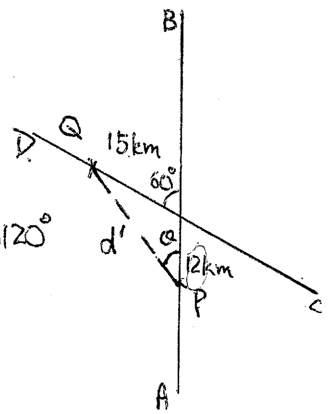
= 24 km.

distance of Q from X

= $(24 - 9)$ km.

= 15 km.

at 4 p.m.



By cosine rule,

$d'^2 = 15^2 + 6^2 - 2(6)(15)\cos 120^\circ$

$d'^2 = 351$

$d' = 18.73$ km.

c) By sine rule,

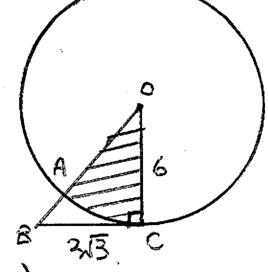
$$\frac{\sin 120^\circ}{d'} = \frac{\sin\theta}{15}$$

$$\sin\theta = \frac{15}{18.73}\sin 120^\circ$$

$\theta = 43.90^\circ$.

the bearing of Q from P is. N43.90°W.

6. In $\triangle OBC$.



$$\tan\angle BOC = \frac{2\sqrt{3}}{6}$$

$$\tan\angle BOC = \frac{\sqrt{3}}{3}$$

$\angle BOC = \pi/6$ (in radian.)

the area of the shaded sector.

$= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(6)^2(\pi/6)$

$= 3\pi$.

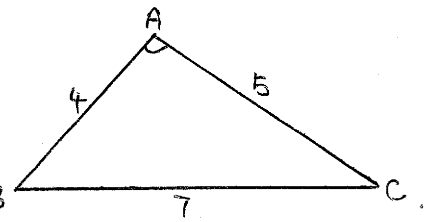
7. $2\sin^2\theta + 5\sin\theta - 3 = 0$. $0^\circ \le \theta \le 360^\circ$

$(2\sin\theta - 1)(\sin\theta + 3) = 0$

$\therefore \sin\theta = \frac{1}{2}$ or -3 (rejected)

$\theta = 30^\circ, 150^\circ$.

8.



By cosine rule,

$a^2 = b^2 + c^2 - 2bc\cos A$.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

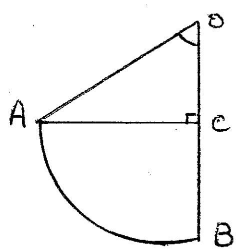
$$\cos A = \frac{5^2 + 4^2 - 7^2}{2(5)(4)}$$

$$\cos A = -\frac{1}{5}$$

$A = 101.5^\circ$

$= 102^\circ$ (nearest degree)

1. a) $OA = OB = 30$.



$OC = OB - BC$
$= 30 - 15$
$= 15$.

In ΔOAC.

$\frac{OC}{OA} = \cos\angle AOC$

$\cos\angle AOC = \frac{15}{30} = \frac{1}{2}$

$\angle AOC = 60^\circ = \frac{\pi}{3}$.

b, the length of the arc AB

$= r\theta$
$= 30 \cdot (\frac{\pi}{3})$
$= 10\pi$.

10. $2\cos^2\theta + 5\sin\theta + 1 = 0 \quad 0^\circ < \theta \leq 360^\circ$

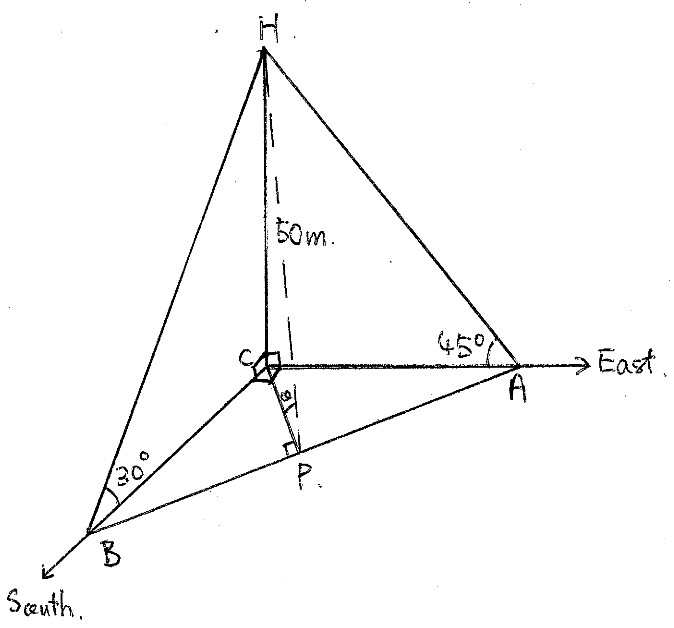
$2(1-\sin^2\theta) + 5\sin\theta + 1 = 0$

$2\sin^2\theta - 5\sin\theta - 3 = 0$.

$(2\sin\theta + 1)(\sin\theta - 3) = 0$

$\sin\theta = -\frac{1}{2}$ or 3 (rejected.)

$\theta = 210^\circ$ or 330°.



a) $\frac{HC}{BC} = \tan 30^\circ$.

$BC = \frac{50}{1/\sqrt{3}}$
$= 50\sqrt{3}$.

In ΔACH.

$\frac{CH}{AC} = \tan 45^\circ$

$AC = \frac{50}{1}$
$= 50$.

the distance between A and B.

$AB^2 = BC^2 + AC^2$
$= (50\sqrt{3})^2 + 50^2$
$= 10000$

$AB = 100\text{ m}$.

b, i) Let the distance between C and P be x.

In ΔABC.

area of ΔABC:

$\frac{1}{2}(AC)(BC) = \frac{1}{2}(CP)(AB)$

$(50)(50\sqrt{3}) = (x)(100)$

$x = 43$ m (nearest metre.)

ii) Let θ be the angle of elevation.

$\tan\theta = \frac{CH}{CP}$

$\tan\theta = \frac{50}{43}$

$\theta = 41^\circ$ (nearest degree.)

12. $\tan\theta = \frac{1+\cos\theta}{\sin\theta}$ $\quad 0^\circ < \theta < 90^\circ$

$$\frac{\sin\theta}{\cos\theta} = \frac{1+\cos\theta}{\sin\theta}.$$

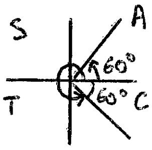
$$\therefore \sin^2\theta = \cos\theta(1+\cos\theta)$$

$$1-\cos^2\theta = \cos\theta + \cos^2\theta.$$

$$\therefore 2\cos^2\theta + \cos\theta - 1 = 0.$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0.$$

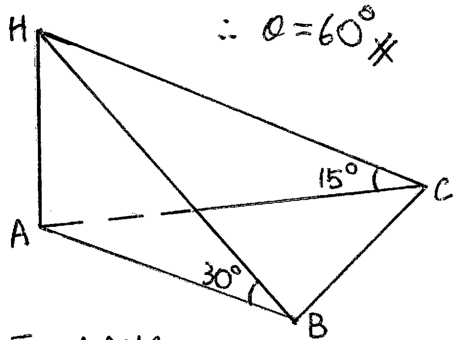
$$\therefore \cos\theta = \tfrac{1}{2} \text{ or } -1.$$



$$\theta = 60^\circ, 300^\circ \text{ or } 180^\circ.$$

$$\therefore \theta = 60^\circ \#$$

13.



a i) In ΔAHC.

$$\frac{HA}{AC} = \tan 15^\circ$$

$$HA = 20 \cdot \tan 15^\circ$$

$$= 5.36\text{m} \text{ (2 dec. places.)}$$

ii) In ΔABH.

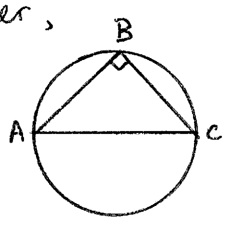
$$\frac{HA}{AB} = \tan 30^\circ.$$

$$AB = 5.36/\tan 30^\circ$$

$$= 9.28\text{m} \text{ (2 dec. places.)}$$

b i) since AC is a diameter,

$\angle ABC = 90^\circ$.



$$\therefore BC^2 = AC^2 - AB^2$$

$$BC = \sqrt{20^2 - 9.28^2}$$

$$BC = 17.72\text{m} \text{ (2 dec. places.)}$$

b ii) the area of ΔABC

$$= \tfrac{1}{2}(AB)\cdot(BC)$$

$$= \tfrac{1}{2}(17.72)(9.28)\ \text{m}^2$$

$$= 82.22\ \text{m}^2 \text{ (2 dec. places.)}$$

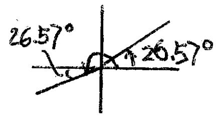
$2\tan^2\theta - 1 - \tan\theta$. $\quad 0 \leq \theta < 360$ $\quad$ P.4

$$2\tan^2\theta + \tan\theta - 1 = 0$$

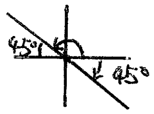
$$(2\tan\theta - 1)(\tan\theta + 1) = 0$$

$$\tan\theta = \tfrac{1}{2} \text{ or } -1$$

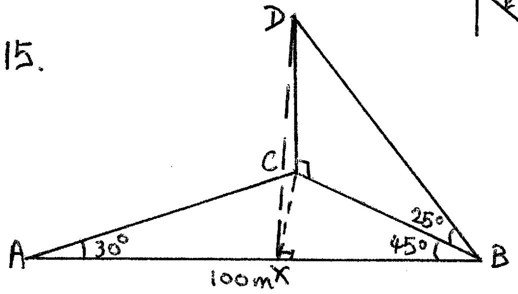
$$\theta = 26.57^\circ,\ 206.57^\circ$$

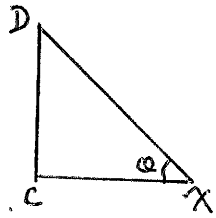


$$\text{or } 135^\circ,\ 315^\circ$$



15.





a) In ΔABC,.

$$\angle ACB = 180^\circ - 30^\circ - 45^\circ$$

$$= 105^\circ.$$

By sine rule,

$$\frac{AC}{\sin 45^\circ} = \frac{AB}{\sin 105^\circ}$$

$$AC = \frac{\sin 45^\circ}{\sin 105^\circ}(100)$$

$$AC = 73.2\text{m}. \text{ (1 dec. place.)}$$

$$\frac{BC}{\sin 30^\circ} = \frac{AB}{\sin 105^\circ}.$$

$$BC = \frac{\sin 30^\circ}{\sin 105^\circ}(100).$$

$$BC = 51.8\text{m} \text{ (1 dec. place.)}$$

b i) In ΔBCD.

$$\angle DCB = 90^\circ.$$

$$\frac{CD}{BC} = \tan 25^\circ.$$

$$CD = 51.8 \cdot \tan 25^\circ$$

$$= 24.1\text{m} \text{ (1 dec. place.)}$$

ii) In ΔBCX,

(1) $\angle CXB = 90^\circ$

$$\frac{CX}{BC} = \sin 45^\circ.$$

$$CX = 51.8 \cdot \sin 45^\circ$$

$$= 36.6\text{m}.$$

b ii) (2). Let θ be the angle of elevation of D from X.

$$\therefore \frac{CD}{CX} = \tan\theta.$$

$$\tan\theta = \frac{24.1}{36.6}$$

$$\theta = 33^\circ \text{ (nearest degree.)}$$

16. $\sin^2\theta + 7\sin\theta = 5\cos^2\theta \quad 0^\circ \le \theta < 360^\circ$

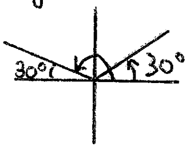
$\sin^2\theta + 7\sin\theta = 5(1-\sin^2\theta)$

$6\sin^2\theta + 7\sin\theta - 5 = 0.$

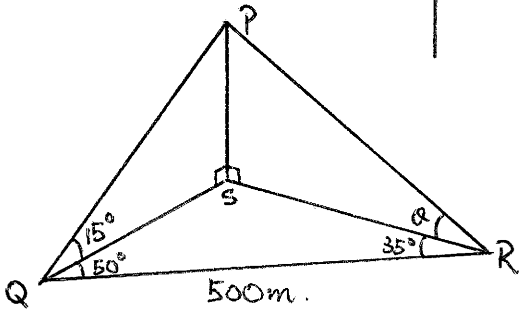
$(2\sin\theta - 1)(3\sin\theta + 5) = 0$

$\therefore \sin\theta = \frac{1}{2}$ or $-\frac{5}{3}$ (rejected).

$\theta = 30^\circ, 150^\circ.$



17.



a) In ΔQRS, $\angle QSR = 180^\circ - 50^\circ - 35^\circ$
$= 95^\circ.$

In ΔPQS,

$\frac{PS}{QS} = \tan 15^\circ.$

$PS = QS \cdot \tan 15^\circ$ —— ①.

In ΔQRS, (By sine rule)

$\frac{QS}{\sin 35^\circ} = \frac{QR}{\sin 95^\circ}.$

$QS = \frac{\sin 35^\circ}{\sin 95^\circ}(500\,m)$

$= 287.88\ m.$

From ①

$PS = 287.88(\tan 15^\circ)\ m$

$= 77.1\ m$ (3 sig. fig.).

b) Let θ be the angle of elevation,
By sine rule, (In ΔQRS)

$\frac{RS}{\sin 50^\circ} = \frac{500}{\sin 95^\circ}$

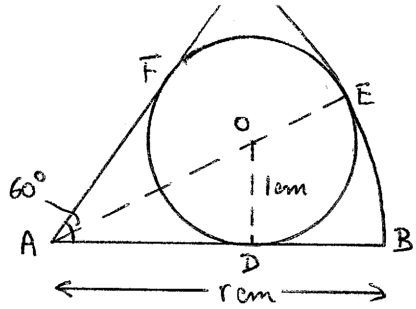
$RS = 384.48\,m.$

$\tan\theta = \frac{RS}{RS} = \frac{77.1}{384.48}.$

$\theta = 11^\circ.$ (nearest degree.)

18.

P.5.



$AE = AB = r\,cm.$ (radius of sector.)

$OE = OD = 1\,cm$ (radius of circle.)

$AE = AO + OE.$ —— ①.

In ΔAOD.

$\frac{OD}{OA} = \sin 30^\circ.$

$OA = 1/\sin 30^\circ$

From ① $OA = 2\,m.$

$AE = AO + OE$

$r = 2 + 1.$

$r = 3.$ ✱.

19. $\sin^2\theta = \frac{3}{2}\cos\theta. \quad 0^\circ \le \theta < 360^\circ$

$(1-\cos^2\theta) = \frac{3}{2}\cos\theta.$

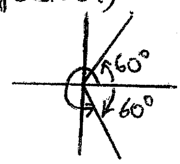
$2 - 2\cos^2\theta = 3\cos\theta$

$2\cos^2\theta + 3\cos\theta - 2 = 0.$

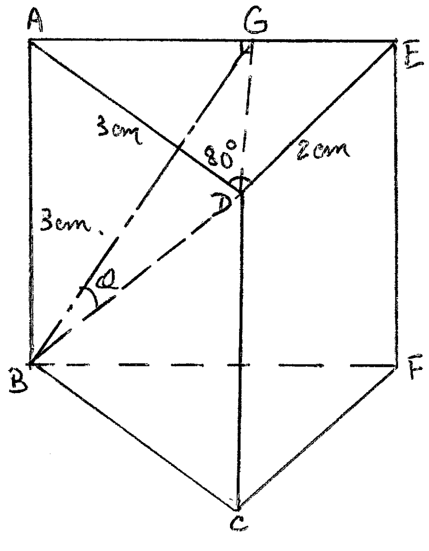
$(2\cos\theta - 1)(\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2}$ or -2 (rejected)

$\theta = 60^\circ, 300^\circ.$



20.



20 a) In $\triangle ADE$,

By cosine rule,

$AE^2 = AD^2 + DE^2 - 2AD \cdot DE \cos\angle ADE$

$AE^2 = 3^2 + 2^2 - 2(3)(2)\cos 80^\circ$

$AE = 3.304$ cm (3 dec. places.)

b) By sine rule,

$$\frac{\sin\angle DAE}{DE} = \frac{\sin\angle ADE}{AE}$$

$$\sin\angle DAE = \frac{2}{3.304}\cdot\sin 80^\circ$$

$\angle DAE = 36.594^\circ$ (3 dec. places.)

c) In $\triangle ADG$,

$$\sin\angle DAE = \frac{DG}{AD}$$

$DG = 3(\sin 36.594^\circ)$

$= 1.788$ cm (3 dec. places.)

d) In ABCD.

$BD^2 = AD^2 + AB^2$

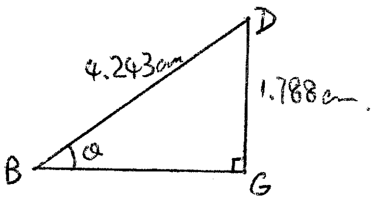
$BD = \sqrt{3^2+3^2}$

$= 4.243$ cm (3 dec. places.)

e) Let α be the angle between the line BD and the face ABFE.

In $\triangle BDG$, $\angle BGD = 90^\circ$.

$$\sin\alpha = \frac{DG}{BD}$$



$$\sin\alpha = \frac{1.788}{4.243}$$

$\alpha = 24.931^\circ$. (3 dec. places.)

21. a) $\dfrac{\sin(180^\circ-\alpha)}{\sin(90^\circ+\alpha)}$

$$= \frac{\sin\alpha}{\cos\alpha} = \tan\alpha.$$

21 b) $\sin^2(\pi-\phi) + \sin^2\left(\frac{3\pi}{2}+\phi\right)$

P.6.

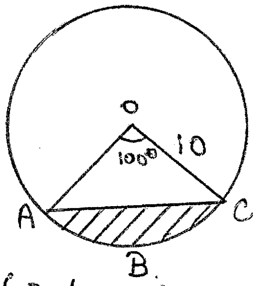
$= (\sin\phi)^2 + (-\cos\phi)^2$

$= \sin^2\phi + \cos^2\phi$

$= 1$.

22.

a) the area of sector OABC



$= \pi(10)^2 \cdot \left(\frac{100^\circ}{360^\circ}\right)$

$= 87.27$ sq unit. (2 dec. pl.)

b) the area of $\triangle OAC$.

$= \frac{1}{2}(OC)(OA)\sin\angle AOC$

$= \frac{1}{2}(10)(10)\sin 100^\circ$.

$= 49.24$ sq. unit. (2 dec. pl.)

c) the area of segment ABC

= the area of sector OABC − the area of $\triangle OAC$

$= (87.27 - 49.24)$ sq. unit

$= 38.03$ sq. unit. (2 dec. pl.)

23. $3\tan\alpha = 2\cos\alpha$ $\quad 0^\circ \le \alpha < 360^\circ$

$$\frac{3\sin\alpha}{\cos\alpha} = 2\cos\alpha$$

$3\sin\alpha = 2\cos^2\alpha$

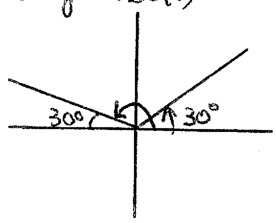
$3\sin\alpha = 2(1-\sin^2\alpha)$

$2\sin^2\alpha + 3\sin\alpha - 2 = 0$

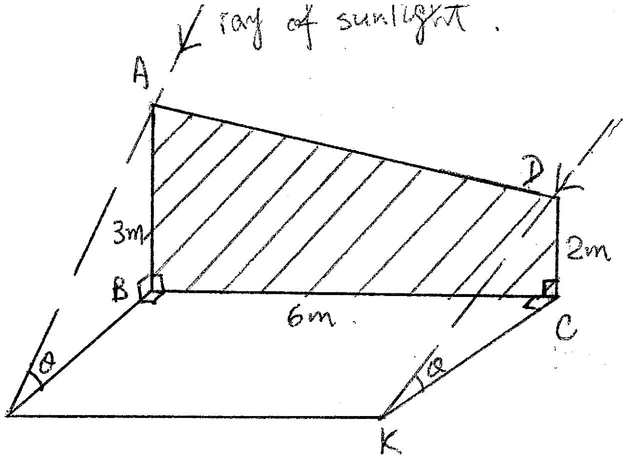
$(2\sin\alpha - 1)(\sin\alpha + 2) = 0$

$\sin\alpha = \frac{1}{2}$ or -2 (rejected.)

$\alpha = 30^\circ, 150^\circ$.



24



a) In ΔABH

$$\frac{AB}{HB} = \tan\theta$$

$$\frac{3}{HB} = \tan\theta$$

$$HB = 3/\tan\theta$$

In ΔCDK.

$$\frac{CD}{KC} = \tan\theta$$

$$KC = 2/\tan\theta$$

bi) the area S_1 of the wall

$$= \frac{1}{2}(AB+CD)\cdot BC$$

$$= \frac{1}{2}(3+2)\cdot 6 \text{ m}^2$$

$$= 15\text{m}^2$$

ii) the area S_2 of the shadow.

$$= \frac{1}{2}(HB+KC)\cdot BC$$

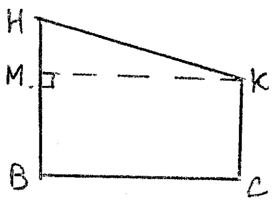
$$= \frac{1}{2}\left(\frac{3}{\tan\theta}+\frac{2}{\tan\theta}\right)\cdot 6 \text{ m}^2$$

$$= \frac{1}{2}\left(\frac{5}{\tan\theta}\right)\cdot 6 \text{ m}^2$$

$$= 15/\tan\theta \text{ m}^2.$$

$$\therefore \frac{S_1}{S_2} = \frac{15}{15/\tan\theta} = \tan\theta.$$

c) if $\theta = 30^\circ$,



$HB = 3/\tan 30^\circ = 3\sqrt{3}$ m

$KC = 2/\tan 30^\circ = 2\sqrt{3}$ m

$$\therefore HM = HB - KC$$

$$= \sqrt{3} \text{ m}$$

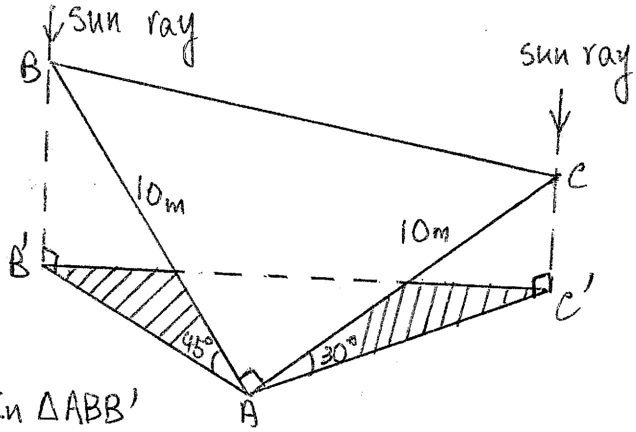
In ΔHKM.

$$HK^2 = HM^2 + MK^2$$

$$HK^2 = (\sqrt{3})^2 + 6^2$$

$$HK = \sqrt{39} \text{ m}.$$

25.



a) In $\Delta ABB'$

$$\frac{AB'}{AB} = \cos 45^\circ$$

$$AB' = 10\cdot\cos 45^\circ$$

$$= 5\sqrt{2}\text{m} \quad \text{or } 7.07\text{m (3 sig. fig.)}$$

In $\Delta ACC'$

$$\frac{AC'}{AC} = \cos 30^\circ$$

$$AC' = 10\cdot\sqrt{3}/2$$

$$= 5\sqrt{3} \text{ m} \quad \text{or } 8.66\text{m (3 sig. fig.)}$$

b) In ΔABC,

$$BC^2 = AB^2 + AC^2$$

$$BC = \sqrt{10^2+10^2}$$

$$= 10\sqrt{2} \text{ m} \quad \text{or } 14.1 \text{ (3 sig. fig.)}$$

In $\Delta ABB'$

$$\frac{BB'}{AB} = \sin 45^\circ.$$

$$BB' = 10\cdot\sqrt{2}/2$$

$$= 5\sqrt{2} \text{ m} \quad \text{or } 7.07\text{m (3 sig. fig.)}$$

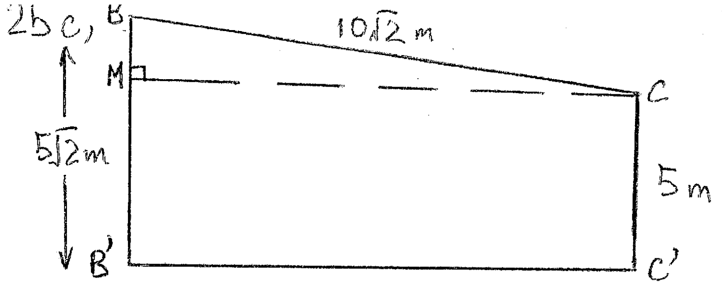
In $\Delta ACC'$

$$\frac{CC'}{AC} = \sin 30^\circ$$

$$CC' = 10\cdot\tfrac{1}{2}\text{m}$$

$$= 5\text{m}.$$

2b c)



$B'C'^2 = BC^2 - BM^2$

$B'C'^2 = BC^2 - (BB' - CC')^2$

$B'C'^2 = (10\sqrt{2})^2 - (5\sqrt{2} - 5)^2$

$B'C' = \sqrt{195.7}$

$= 13.99\text{ m}$ (4 sig. fig.)

d) In $\Delta AB'C'$.

By cosine rule,

$$\cos\angle B'AC' = \frac{AB'^2 + AC'^2 - B'C'^2}{2(AB')(AC')}$$

$$\cos\angle B'AC' = \frac{(5\sqrt{2})^2 + (5\sqrt{3})^2 - (13.99)^2}{2(5\sqrt{2})(5\sqrt{3})}$$

$\cos\angle B'AC' = -0.577$

$\angle B'AC' = 125.3°$ (4 sig. fig.)

the area of the shadow,

$=$ the area of $\Delta AB'C'$

$= \frac{1}{2}(AB')(AC')\sin\angle B'AC'$

$= \frac{1}{2}(5\sqrt{2})(5\sqrt{3})\sin 125.3°$

$= 25\text{ m}^2$.

26. $\sin^2\theta : \cos\theta = -3 : 2$

$\therefore \frac{\sin^2\theta}{-3} = \frac{\cos\theta}{2}$ $\quad 0° \leq \theta < 360°$

$2\sin^2\theta = -3\cos\theta$.

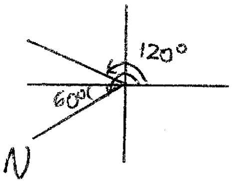
$2(1 - \cos^2\theta) = -3\cos\theta$.

$2\cos^2\theta - 3\cos\theta - 2 = 0$.

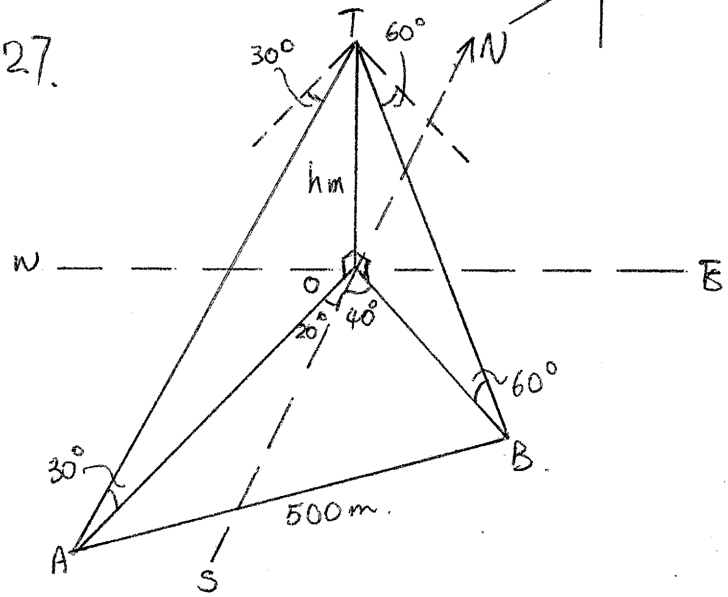
$(2\cos\theta + 1)(\cos\theta - 2) = 0$

$\cos\theta = -\frac{1}{2}$ or 2 (rejected.)

$\theta = 120°, 240°$.



27.



a) In ΔOAT.

$\frac{OT}{OA} = \tan 30°$

$\therefore OA = h / \frac{1}{\sqrt{3}} = \sqrt{3}h$.

In ΔOBT.

$\frac{OT}{OB} = \tan 60°$

$OB = \frac{h}{\sqrt{3}} = \frac{\sqrt{3}}{3}h$.

b) In ΔOAB.

By cosine rule,

$AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos 60°$.

$AB^2 = 3h^2 + \frac{1}{3}h^2 - 2(\sqrt{3}h)(\frac{\sqrt{3}}{3}h)\frac{1}{2}$.

$AB^2 = 3h^2 + \frac{1}{3}h^2 - h^2$

$AB^2 = \frac{7}{3}h^2$

$AB = \sqrt{\frac{7}{3}}h$

since $AB = 500\text{ m}$.

$\therefore \sqrt{\frac{7}{3}}h = 500\text{ m}$.

$h = 500\sqrt{3/7}$

$= \frac{500}{7}\sqrt{21}\text{ m}$.

27c) $h = \frac{500}{7}\sqrt{21}$

$OB = \frac{\sqrt{3}}{3} \cdot \left(\frac{500}{7}\sqrt{21}\right)$

$= \frac{500\sqrt{7}}{7}$ m.

By sine rule,

$\frac{\sin\angle OAB}{OB} = \frac{\sin\angle AOB}{AB}$.

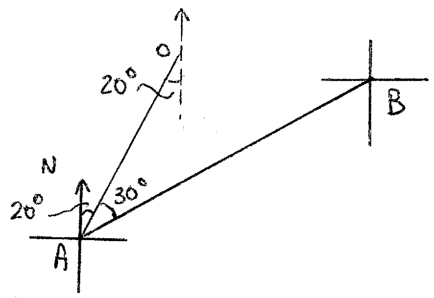
$\sin\angle OAB = \frac{\frac{500\sqrt{7}}{7}}{\frac{500}{7}\sqrt{21}} \cdot \sin 60°$

$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{2}$.

$\therefore \angle OAB = 30°$.

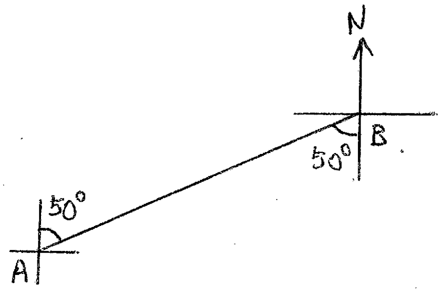
i)



$\therefore$ the bearing of B from A

$= N50°E$.

ii)



$\therefore$ the bearing of A from B

$= S50°W$.

28. $\sin^2\theta - 3\cos\theta - 1 = 0 \quad 0 \le \theta < 360$

$(1-\cos^2\theta) - 3\cos\theta - 1 = 0$.

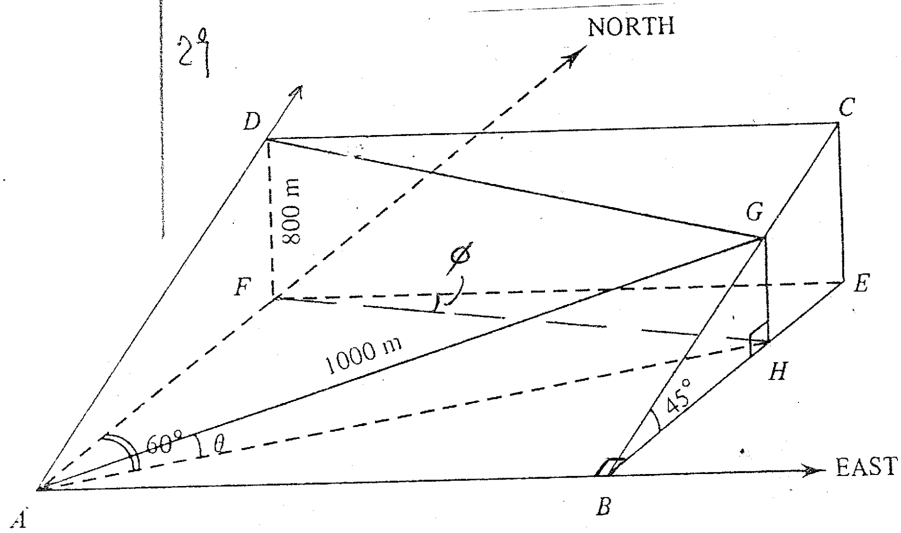
$\cos^2\theta + 3\cos\theta = 0$.

$\cos\theta(\cos\theta + 3) = 0$

$\cos\theta = 0$ or -3 (rejected).

$\therefore \theta = 90°, 270°$.

29



In ΔAGH,

a) $\sin\theta = \frac{GH}{AG}$.

$\therefore GH = 1000\sin\theta$ m

$\cos\theta = \frac{AH}{AG}$.

$AH = 1000\cos\theta$ m.

b) Consider ΔABH

$\angle HAB = 90° - \angle FAH$

$= 90° - 60° = 30°$.

$\therefore \sin 30° = \frac{BH}{AH}$

$\therefore BH = \frac{1}{2}(1000)\cos\theta$

$= 500\cos\theta$ m.

In ΔBHG,

$\frac{GH}{BH} = \tan 45°$.

$\therefore \frac{1000\sin\theta}{500\cos\theta} = 1$.

$\tan\theta = \frac{1}{2}$.

$\theta = 26.57°$ (4 sig. fig.)

c) since $EF = AB$.

In ΔABH,

$EF = AB = BH/\tan 30°$.

$= \frac{500\cos 26.57°}{\tan 30°} = 774.6$ m (4 sig. fig.)

c) cont'd

In ΔBCE.

$\tan 45° = \frac{CE}{BE}$

$\therefore BE = 800/1$.

$= 800$.

$\therefore HE = BE - BH$

$= 800 - 500\cos 26.57°$

$= 352.8$ m (4 sig. fig.)

the bearing of G from D

= the bearing of H from F.

(on the horizontal plane.)

$\tan\phi = \frac{HE}{EF}$

$\therefore \phi = 24.49°$.

$\therefore$ the bearing of G from D

$= 90° + 24.49°$.

$= 114.5°$ (4 sig. fig.)