13. (a) A capacitor has two plates. Normally, the two plates hold equal but opposite charge. For instance, if one plate is earthed and the other plate stores some charges, then the earthed plate would be induced with the same quantity of unlike charge.

One plate stores +Q, the other stores -Q. The combination stores ±Q.

(b) Seemingly, a charged capacitor is neutral in charge (from (a)). Unlike an uncharged object, the +ve and -ve charges are separated by an insulator. The capacitor has stored up electric p.e. which increases with the charge stored. This energy would be released when a conducting path is established between the two plates. At the same time, the capacitor is discharged.

14. (a) When S is closed, a large initial current flows. The initial current is $I_0 = \frac{\xi}{R}$, where ξ is the e.m.f. of the battery and R is the total resistance.

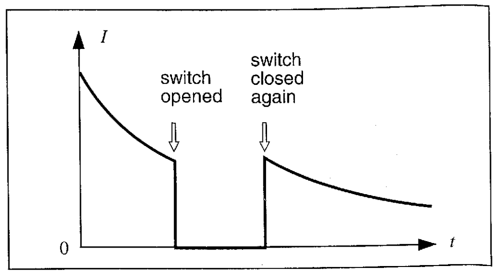
This current would decrease exponentially according to $I = I_0 e^{-\frac{t}{CR}}$. This is because the capacitor has stored charge and its p.d. opposes the current, decreasing its size. Before the current falls to zero, S is opened. Thus, the current would abruptly fall to zero.

In conclusion, the meter gives an initial large deflection. The deflection decreases and suddenly falls to zero when the switch is opened.

(b) When S is opened, the charges stored in C are unchanged. This is because there is no conducting path for charge movement.

When S is closed again, the p.d. across the resistance is less than the e.m.f. of the battery because the capacitor takes up certain amount of p.d. The current is the same as that just before S is opened. Thereafter, the current falls exponentially until it is zero.

N.B. The variation of current with time can be described as shown in the following graph:



15. (a) According to $E = \frac{V}{d}$, for the same p.d. V, the smaller is the separation, the larger is the electric field strength -- the more likely does corona discharge occur.

The minimum separation of the plate before discharge occurs is

$$d = \frac{V}{E} = \frac{5\times10^3}{3\times10^6} = 1.667\times10^{-3}\ \text{m}$$
$$\approx 1.7\ \text{mm}$$

(b) By $E = \frac{\sigma}{\varepsilon_0}$, the maximum surface charge density is

$$\sigma = \varepsilon_0 E = 8.85\times10^{-12}\times3\times10^6$$
$$= 2.655\times10^{-5} \approx 2.7\times10^{-5}\ \text{C m}^{-2}$$

N.B. When the electric field strength exceed E, then charge from one plate would jump to the other. As a result, the surface charge density decreases.

(c) By $Q = \sigma A$, the maximum amount of charge stored is

$$Q = 2.655\times10^{-5}\times(100\times10^{-4})$$
$$= 2.655\times10^{-7}\ \text{C}$$
$$\approx 0.27\ \mu\text{C}$$

16. (a) By $C = \frac{\varepsilon_r\varepsilon_0 A}{d}$, inserting the dielectrics into the plate-system would increase the capacitance from C to $C' = \varepsilon_r C = 1.5C$.

Since the charge Q remains constant, by $V = \frac{Q}{C}$, the new p.d. would decrease to

$$V' = \frac{Q}{C'} = \frac{Q}{1.5C} = 0.667\ V$$

(b) By $C = \frac{\varepsilon_0 A}{d}$, doubling the plate separation would decrease the capacitance from C to $C' = \frac{1}{2}C$.

Since the charge Q remains constant, by $V = \frac{Q}{C}$, the new p.d. would increase to

$$V' = \frac{Q}{C'} = \frac{Q}{\frac{1}{2}C} = 2\ V$$

(c) By $C = \frac{\varepsilon_0 A}{d}$, if the overlapping area A is halved, the capacitance would decrease from C to $C' = \frac{1}{2}C$.

Since the charge Q remains constant, by $V = \frac{Q}{C}$, the new p.d. would increase to

$$V' = \frac{Q}{C'} = \frac{Q}{\frac{1}{2}C} = 2\ V$$

17. (a) Inserting the dielectrics into the plate-system would increase the capacitance from C to $C' = \varepsilon_r C = 1.5C$.

Since the voltage V remains constant, by $Q = CV$, the new stored charge would increase to $Q' = C'V = 1.5\,Q$

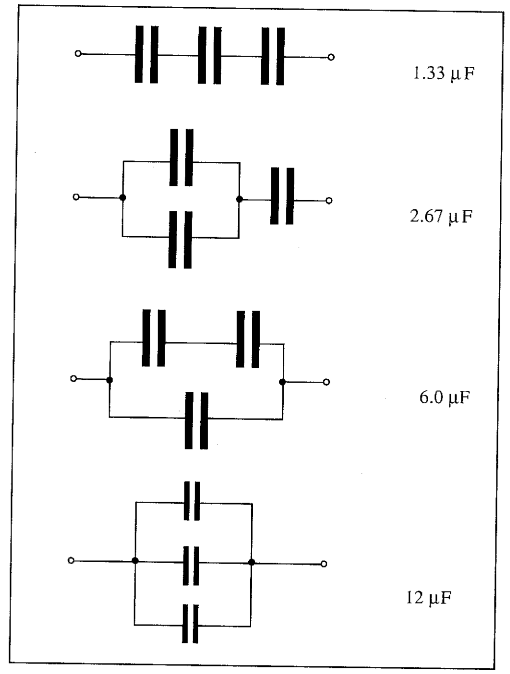
(b) Doubling the plate separation would decrease the capacitance from C to $C' = \frac{1}{2}C$.

Since the voltage V remains constant, by $Q = CV$, the new stored charge would decrease to $Q' = C'V = \frac{1}{2}CV = \frac{1}{2}Q$

(c) When the overlapping area A is halved, the capacitance would decrease from C to $C' = \frac{1}{2}C$.

Since the voltage V remains constant, by $Q = CV$, the new stored charge would decrease to $Q' = C'V = \frac{1}{2}CV = \frac{1}{2}Q$

18. There are four possible combinations:



In the first combination, the equivalent capacitance is given by

$$\frac{1}{C_1} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \Rightarrow C_1 = 1.33\ \mu\text{F}$$

In the 2nd combination, the equivalent capacitance of the two parallel capacitors is $C_{ab} = 4 + 4 = 8\ \mu\text{F}$. Thus, the overall capacitance is given by

$$\frac{1}{C_2} = \frac{1}{C_{ab}} + \frac{1}{C} = \frac{1}{8} + \frac{1}{4} \Rightarrow C_2 = 2.67\ \mu\text{F}$$

In the 3rd combination, the equivalent capacitance of the upper branch is given by

$$\frac{1}{C_{ab}} = \frac{1}{4} + \frac{1}{4} \Rightarrow C_{ab} = 2\ \mu\text{F}$$

Thus, the overall capacitance is

$$C_3 = 4 + 2 = 6\ \mu\text{F}$$

In the 4th combination, the capacitors are in parallel. Thus, their overall capacitance is

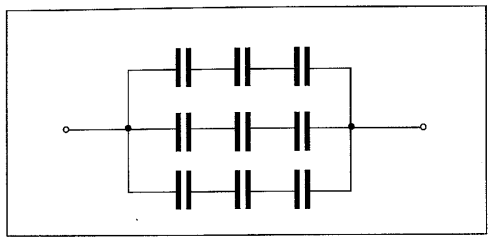
$C_4 = 4+4+4 = 12\ \mu F$

19. (a) The dielectric material breaks down. The capacitor may be damaged. Under high voltage, current flows through the insulating material, which may burn up as heat is generated.

(b) When three capacitors are joined in series to a 220 V supply, the voltage across each capacitor is only $V_c = \frac{220}{3} = 73\ V$ which is below 80 V.

However, the series combination has capacitance of $C' = \frac{5}{3} = 1.67\ \mu F$ only.

In order to multiply the capacitance, three series combinations can be joined in parallel as shown below:



The overall capacitance is exactly

$C = 3\times C' = 3\times\frac{5}{3} = 5\ \mu F$

N.B. This combination can operate safely up to a voltage of 240 V d.c.

20. To determine the total charge stored in a combination of capacitor, the best way is to join the terminals together with a connecting wire and find out how much charge will pass through a point in the wire as the combination discharges.

In network A, suppose the charge on the plate on the most right-hand side is +Q. Then, the charge on the plate on the most left-hand side is -Q. Although other plates hold excess charge, these charge would not pass out of the combination through the connecting wire. Thus, only charge Q would pass from the +ve terminal to the -ve terminal in the discharging process. In other words, the charge stored in the combination is only $\pm Q$.

In network B, suppose the charge on the right plate of each capacitor is +Q. Then, the charge on the left plate of each capacitor is -Q. There are +3Q of charge ready to leave the combination from the right side. When a connection is made, +3Q of charge would move through a point in the wire. In other words, the charge stored in the combination is $\pm 3Q$.

N.B. The capacitors in network B are identical. This is because their p.d.s must be the same in parallel connection. Since the charge stored are all +Q, they must have the same capacitance.

The capacitors in network A may not be identical, even though they have the same amount of charge.

21. (a) In series combination, the charge stored in the capacitors must be the same. This is because during charging, the same current flows through the capacitors.

Thus, $Q_1 : Q_2 = 1:1$

(b) As their stored charges are the same, by $V = \frac{Q}{C}$, we have

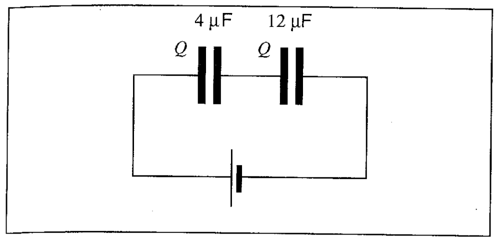
$\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{2C}{C} = \frac{2}{1}$

22. The three capacitors store the same quantity of charge. This is because the same charging current flows through all the capacitors in the same period.

By $V = \frac{Q}{C}$, $V \propto \frac{1}{C}$.

i.e. The greater is the capacitance, the smaller is the p.d. Thus, the greatest p.d. belongs to the capacitor with the smallest capacitance. i.e. C_1 has the greatest p.d.

23.



(a) In a series combination, the charges stored in the capacitors are the same. This is because the same charging current through all the capacitors in the same period.

Suppose the charge stored is Q.

The p.d. across the capacitors are

for C_1: $V_1 = \frac{Q}{C_1} = \frac{Q}{4\times10^{-6}}$

for C_2: $V_2 = \frac{Q}{C_2} = \frac{Q}{12\times10^{-6}}$

Since they are in series, we have

$150 = V_1 + V_2$

$\Rightarrow 150 = \frac{Q}{4\times10^{-6}} + \frac{Q}{12\times10^{-6}}$

$\therefore Q = 4.5\times10^{-4} = 450\ \mu C$

(b) For C_1: $V_1 = \frac{4.5\times10^{-4}}{4\times10^{-6}} = 112.5\ V$

For C_2: $V_2 = \frac{4.5\times10^{-4}}{12\times10^{-6}} = 37.5\ V$

(c) It is clear that when the charged capacitors are discharged, the quantity of charge flowing through the conducting wire is the same as those residing on the leftmost plate or on the rightmost plate. In other words, the total charge is the charge on a single capacitor. i.e. $450\ \mu C$

N.B. The equivalent capacitance is

$C = \frac{C_1C_2}{C_1+C_2} = \frac{4\times12}{4+12} = 3\ \mu F$

By $Q = CV$, the capacitance is

$C = \frac{Q}{V} = \frac{450}{150} = 3\ \mu F$

24. When the switches are closed, the two capacitors are said to join **in parallel**. This is because plates with like charge are electrically connected. Their p.d. would be the same.

Consider C_1 before connection. We have

$Q_0 = C_1V_0$ ....(1)

Note that after connection, the total charge stored in the system remains unchanged.

(a) The equivalent capacitance for the parallel combination is

$C = C_1 + C_2 = C_1 + 2C_2$
$= 3C_1$

Since the stored charge is Q_0, the final p.d. is

$V = \frac{Q_0}{C} = \frac{Q_0}{3C_1}$
$= \frac{1}{3}V_0$ from (1)

This is also the p.d.s across C_1 and C_2.

Thus, the stored charges are

for C_1: $Q_1 = C_1V = \frac{1}{3}C_1V_0 = \frac{1}{3}Q_0$

for C_2: $Q_2 = C_2V = \frac{1}{3}(2\times C_1)V_0 = \frac{2}{3}Q_0$

(b) When the plates are joined with opposite charges together, the final net charge on each plate is $Q' = \frac{2}{3}Q_0 - \frac{1}{3}Q_0 = \frac{1}{3}Q_0$.

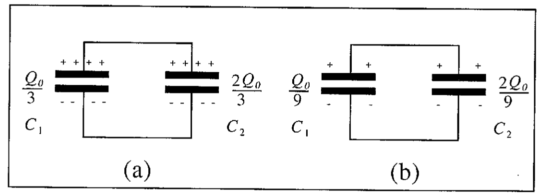
The equivalent capacitance remains unchanged at $C = 3C_1$. The final p.d. of the combination is

$V' = \frac{Q'}{C} = \frac{\frac{1}{3}Q_0}{3C_1} = \frac{1}{9}V_0$

Thus, the final stored charges are

for C_1: $Q'_1 = C_1V' = \frac{1}{9}C_1V_0 = \frac{1}{9}Q_0$

for C_2: $Q'_2 = C_2V' = \frac{1}{9}(2C_1)V_0 = \frac{2}{9}Q_0$



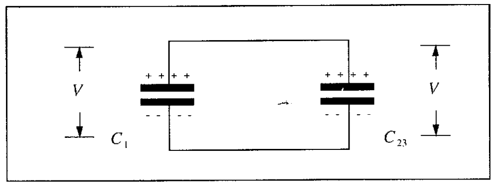
25. The initial charge stored in C_1 is

$$Q_0 = C_1 V_0 = 2\times 6 = 12\ \mu C$$

Note that this quantity of charge is unchanged by closing the switch.

As C_2 and C_3 are connected in series, their equivalent capacitance is

$$C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{2\times 2}{2+2} = 1\ \mu F$$



When the switch is closed, C_1 is to be connected in parallel with C_{23}. Thus, the overall capacitance is

$$C = C_1 + C_{23} = 2 + 1 = 3\ \mu F$$

The voltage across the combination is

$$V = \frac{Q_0}{C} = \frac{12}{3} = 4.0\ V$$

This is also the voltages across C_1 and C_{23}.

For C_1: $\underline{\underline{V_1 = 4.0\ V}}$

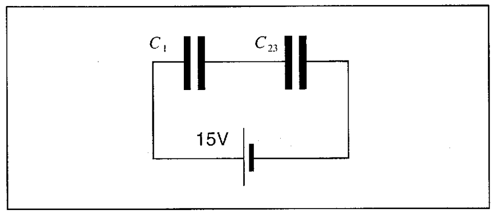
Since C_2 and C_3 are identical, they share the 4 V p.d. equally.
For C_2: $\underline{\underline{V_2 = 2.0\ V}}$

For C_3: $\underline{\underline{V_3 = 2.0\ V}}$

26. Since C_2 and C_3 are in parallel, their equivalent capacitance is

$$C_{23} = C_2 + C_3 = 2 + 3 = 5\ \mu F$$

The combination is the same as follows:



Since C_1 and C_{23} are in series, the overall capacitance is

$$C = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{10\times 5}{10+5} = 3.33\ \mu F$$

The total charge stored in the system of capacitors is

$$Q = CV = 3.33\times 15 = 50\ \mu F$$

This is also the charge stored in C_1 and C_{23}.

For C_1: $\underline{\underline{Q_1 = Q = 50\ \mu F}}$

For C_{23}: $Q_{23} = Q = 50\ \mu F$ and the p.d. is

$$V_{23} = \frac{Q_{23}}{C_{23}} = \frac{50}{5} = 10\ V$$

The p.d.s across C_2 and C_3 are both 10 V.

For C_2: $\underline{\underline{Q_2 = C_2 V_{23} = 2\times 10 = 20\mu C}}$

For C_3: $\underline{\underline{Q_3 = C_3 V_{23} = 3\times 10 = 30\mu C}}$

27. (a) The charge in C_1 is Q.

For C_2: The p.d. is the same as C_1 and its capacitance is the same as C_1. Thus, the charge stored is also $\underline{\underline{Q_2 = Q}}$.

The parallel combination of C_1 and C_2 holds charge of $Q_{12} = 2Q$

For C_3: The charge in C_3 is the same as that in C_{12} because C_3 and C_{12} are in series. Thus, we have $\underline{\underline{Q_3 = 2Q}}$.

The p.d. across C_{12} is $V_{12} = \frac{Q_1}{C_1} = \frac{Q}{C}$ and the p.d. across C_3 is $V_3 = \frac{Q_3}{C_3} = \frac{2Q}{C}$

Thus, the overall p.d. is

$$V = V_{12} + V_3 = \frac{Q}{C} + \frac{2Q}{C} = \frac{3Q}{C}$$

Since C_4, C_5 and C_6 are identical, they share V equally. Thus,

$$V_4 = V_5 = V_6 = \frac{Q}{C}$$

For C_4, C_5 and C_6 : the stored charge are

$$Q_4 = Q_5 = Q_6 = \frac{Q}{C}\times C = \underline{\underline{Q}}$$

(b) The total charge on the upper branch is

$$Q_U = Q_{12} = Q_3 = 2Q$$

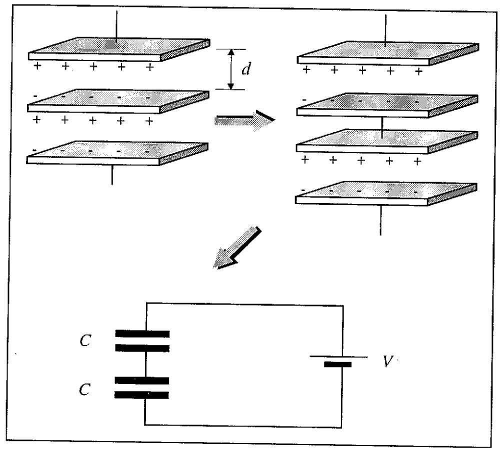
The total charge on the lower branch is

$$Q_L = Q_4 = Q_5 = Q_6 = Q$$

Thus, the total charge stored is

$$Q_0 = Q_U + Q_L = 2Q + Q = \underline{\underline{3Q}}$$

28. Let's analyze the system of plates.



Since the middle plate has equal but opposite charges induced on the upper and lower plates, it can be regarded as two plates joined by a wire. The whole arrangement is equivalent to two capacitors connected in series as shown in the last diagram.

(a) The capacitance of each capacitor is

$$C = \frac{\varepsilon_0 A}{d}.$$

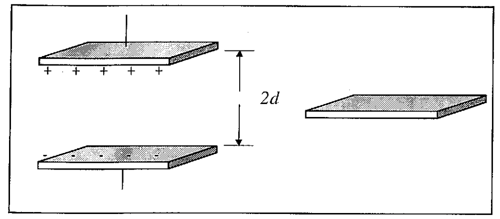
Thus, the overall capacitance is

$$C' = \tfrac{1}{2}C = \frac{\varepsilon_0 A}{2d}$$

The charge stored in the system of plates is

$$Q = C'V = \underline{\underline{\frac{\varepsilon_0 AV}{2d}}}$$

(b) If the middle plate is removed, the separation of the plates is 2d.



The capacitance of the plate system is

$$C'' = \frac{\varepsilon_0 A}{2d}.$$

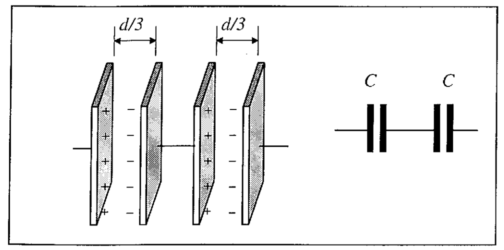
The charge stored in the system of plates is

$$Q = C''V = \underline{\underline{\frac{\varepsilon_0 AV}{2d}}}$$

Effectively, removing the middle plate would not cause any changes to the system, including energy.

N.B. We need to assume that the plates are thin.

29. (a) Using the same idea as in the previous question, the system of plates is equivalent to joining two capacitors each of plate-separation $d' = \frac{1}{3}d$ in series.



The capacitance of each capacitor is

$$C = \frac{\varepsilon_0 A}{\frac{1}{3}d} = \frac{3\varepsilon_0 A}{d}$$

Thus, the final capacitance is

$$C' = \frac{C}{2} = \underline{\underline{\frac{3\varepsilon_0 A}{2d}}}$$

(b) Points to note:

1. The spaces that form the two capacitors share the applied p.d. V_0 equally. This is because the capacitances are equal.

2. The potential decreases linearly with distance in the spaces between the metal surfaces.

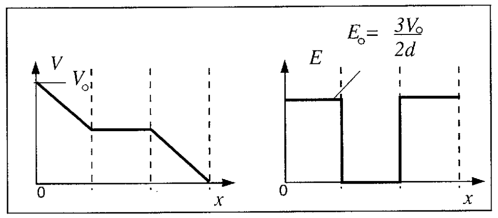
3. The potential along the metal block is constant and is at $V_0/2$ relative to the earthed plate.

4. The electric field strength is constant in the spaces between the metal surfaces. This is because the electric field is uniform between two parallel plates.

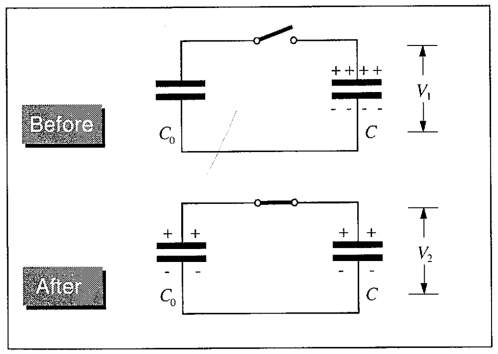
5. The electric field strengths in the two spaces are the same because the charge densities are the same. In fact, the field strength is

$$E = \frac{V}{\frac{1}{3}d} = \frac{3V_0}{2}$$

6. The field strength in the metal block is zero.



30. Both voltmeter readings give the voltage across the unknown capacitor C. V_1 : before C is connected across C_0. V_2 : after C is connected across C_0.



The initial charge stored in C is

$$Q_0 = CV_1 \quad ....(1)$$

After closing the switch, the two capacitors are effectively joined in parallel, and their capacitance is

$$C' = C + C_0 \quad ....(2)$$

As charge is not destroyed in this process, the final charge in the system remains the same as before. Thus, the final voltage is

$$V_2 = \frac{Q_0}{C'} \quad ....(3)$$

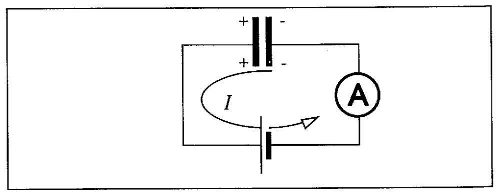
Combining the three equations, we have

$$V_2 = \frac{Q_0}{C'} = \frac{CV_1}{C + C_0}$$
$$\Rightarrow CV_2 + C_0V_2 = CV_1$$
$$\Rightarrow CV_1 - CV_2 = C_0V_2$$
$$\therefore \quad C = \frac{C_0V_2}{V_1 - V_2}$$

31. Since the capacitor has been connected to the 1.5 V cell, we can assume that it has been fully charged. Thus, the current in the circuit is zero. The current remains zero when the resistance is changed (either decrease or increase). The p.d. across the capacitor is constant at 1.5 V and the charge stored in the capacitor remains unaffected.

32. Note that in all actions, the p.d. V across the capacitor remains unchanged.

(a) When the plate separation is increased, by $C = \frac{\varepsilon_0 A}{d}$, the capacitance decreases. Since V is constant, by $Q = CV$, the amount of charge stored decreases. This is only possible when +ve charge moves towards the +ve terminal of the cell and -ve charge moves towards the -ve terminal of the cell. Hence, current flows in the anti-clockwise direction, or up through the ammeter.



(b) When the overlapping area A is increased, by $C = \frac{\varepsilon_0 A}{d}$, the capacitance decreases.

Since V is constant, by $Q = CV$, the amount of charge stored decreases. This is only possible if current flows in the anti-clockwise direction, or up through the ammeter.

(c) When a thin insulated metal plate is inserted into the space, the overall capacitance is unchanged (please refer to question 28 for detailed explanation).

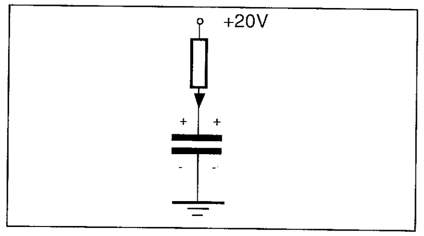
Since both C and V are unchanged, the charge stored is unchanged. Thus, no current is detected in the ammeter.

(d) When a dielectric material ($\varepsilon_r > 1$) is inserted, by $C = \frac{\varepsilon_r \varepsilon_0 A}{d}$, the capacitance increases. Since V is constant, by $Q = CV$, the amount of charge stored increases.

This is only possible when some extra +ve charge is supplied from the cell through the +ve terminal. Similarly, for -ve charge. Hence, current flows in the clockwise direction, or down through the ammeter.

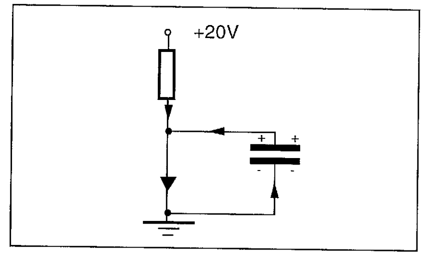
33. The switch is initially closed. It means that the capacitor is initially uncharged. Although there is current through the resistor and the switch, there is no deflection on the ammeter.

(a) When the switch is opened, the capacitor would be charged up -- both its charge and its p.d. increase gradually. The upper plate of C carries +ve charge, the lower plate -ve charge. Thus, the current flows downward:



The current is large initially and then decreases gradually. Thus, the meter deflects to the right quickly and then gradually returns to the zero position.

(b) When the switch is closed again, the capacitor discharges:

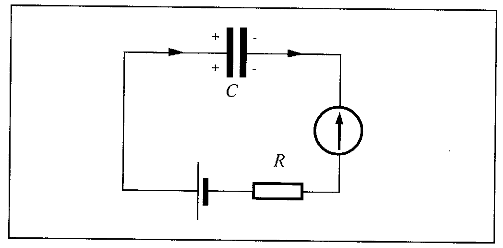


The capacitor is discharged very quickly because the resistance is zero. Thus, the discharging current is very large but it flows for a very short while. As a result, the meter deflects to the left by a large value and then quickly returns to the zero position.

N.B. As the switch is opened, the current through R decreases gradually until it is zero. When the switch is closed again, the current through R resumes its steady value. This current would not flow into the ammeter causing any change in deflection. This is because the conducting path between R and the earth is non-resistive. The current through R will not flow into the capacitor but downward directly. The discharging current merely comes from the stored charge in C.

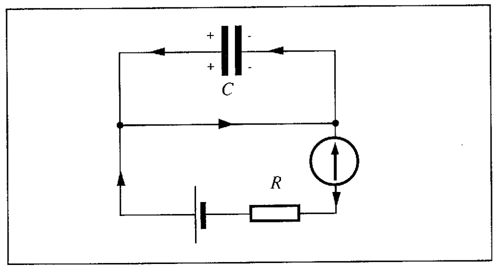
34. The switch is initially closed. It means that the capacitor is initially uncharged.

(a) When the switch is opened, the circuit is effectively as shown below:



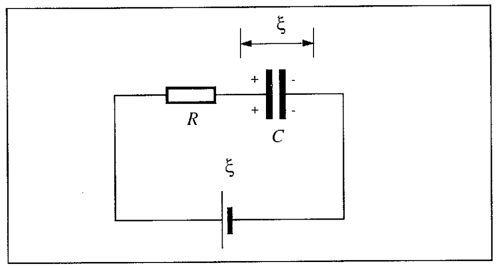
The capacitor is being charged. As the stored charge increases, its p.d. increases. Then, the current falls because the p.d. across R decreases. In conclusion, the meter deflects to the right and then gradually returns to the zero position.

(b) When the switched is closed again, the capacitor discharges rapidly. The discharging current flows in the upper loop anti-clockwise.



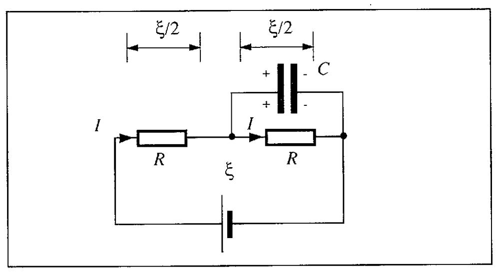
The current through the ammeter depends on the e.m.f. of the cell and the total resistance of the circuit. It is a constant value and is unaffected by the discharging process. Thus, the ammeter gives a constant deflection to the right after the switch is closed.

35. When the switch is open, the circuit is the same as follows:



In steady state, no current flows in the circuit and the capacitor is fully charged. The p.d. across C is ξ. Thus, the charge stored is $Q_0 = C\xi$.

When the switch is closed, the circuit is equivalent to the following:



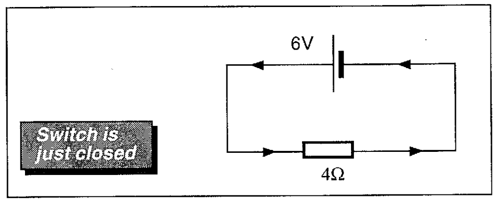
In steady state, the capacitor is fully charged. The same current flows through the two resistors. Thus, the p.d. across the resistors are both $V = \frac{1}{2}\xi$. Since no current flows into the capacitor, its p.d. is also $\frac{1}{2}\xi$. Hence, the charge stored in C is

$$Q' = CV = \tfrac{1}{2}C\xi = \tfrac{1}{2}Q_0$$

N.B. As the switch is closed, the charge in C decreases from Q_0 to $0.5\,Q_0$. The discharge current flows in a loop that consists of C and R. This current is temporary. When the charge falls to $0.5\,Q_0$, the discharge current stops and the current through both resistors is constant.

36. **Circuit A**

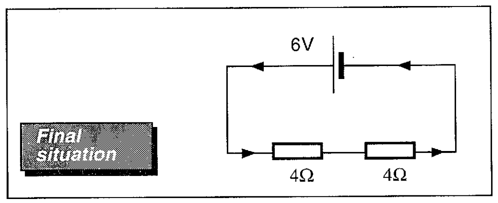
(a) When the switch is closed, the p.d. across C (and the resistor in parallel to it) is zero. All current passes through C, none through the resistor in parallel with C. The circuit is equivalent to the following:



Thus, the current is $I_0 = \frac{\xi}{R} = \frac{6}{4} = 1.5\text{ A}$

This current flows through C, charging C gradually.

(b) After the switched has been closed for a long time, C is fully charged. The current through the resistors is steady and no current flows through C. It is equivalent to the following:

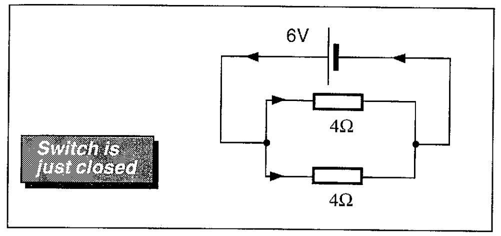


Thus, the current drawn from the cell is

$$I' = \frac{\xi}{2R} = \frac{6}{8} = 0.75\text{ A}$$

Circuit B

(a) When the switch is closed, the p.d. across C (and the resistor in parallel to it) is zero. The same current flows through the two resistors. The circuit is equivalent to the following:



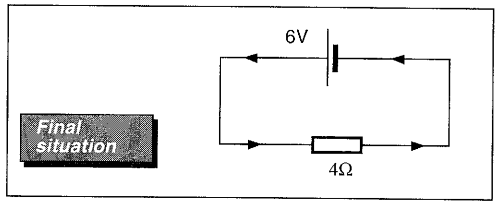
The equivalent resistance is $R' = \frac{1}{2}R = 2\,\Omega$.

Thus, the current through the cell is

$$I_0 = \frac{\xi}{R'} = \frac{6}{2} = 3\text{ A}$$

This current flows through C, charging C gradually.

(b) After a long time while , C is fully charged. The current through the upper branch is zero. A steady current flows through the lower branch. The circuit is equivalent to the following:



Thus, the current drawn from the cell is

$$I' = \frac{\xi}{R} = \frac{6}{4} = 1.5\text{ A}$$

37. (a) Just after the switch is closed, the p.d. across the capacitor is zero. Thus, the p.d. across the resistor is 12 V.

The initial current is

$$I_0 = \frac{\xi}{R} = \frac{12}{200 \times 10^3} = 6.0 \times 10^{-5}\text{ A}$$

(b) When current falls to one third,

$$I = \tfrac{1}{3}I_0 = \tfrac{1}{3} \times 6 = 2 \times 10^{-5}\text{ A}$$

The p.d. across the resistor is therefore

$$V_R = IR = 2 \times 10^{-5} \times 200 \times 10^3 = 4.0\text{ V}$$

Hence, the p.d. across the capacitor is

$$V_C = \xi - V_R = 12 - 4 = 8\text{ V}$$

By $Q = CV$, the charge stored is

$$Q = 50 \times 10^{-6} \times 8 = 4.0 \times 10^{-4}\text{ C}$$

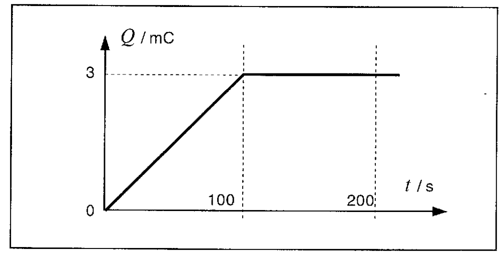
38. (a) As current is flowing, the p.d. across C increases. So, the p.d. across R decreases. In order to maintain constant current, the value of R must decrease continuously.

(b) Points to note:

1. Since the current is constant, the charge increases linearly with time during $0 \le t \le 100\text{ s}$.

2. By $Q = It$, the maximum charge stored in the capacitor is $Q_0 = 30 \times 10^{-6} \times 100 = 3 \times 10^{-3}\text{ C}$

3. When $t > 100\text{ s}$, the current stops and the charge remains at the maximum value Q_0.



(c) When the current stops, the capacitor is fully charged and its p.d. is exactly 6 V.
By $Q = CV$, the capacitance is

$$C = \frac{Q_0}{\xi} = \frac{3 \times 10^{-3}}{6} = 5 \times 10^{-4}\text{ F}$$

(d) At t = 50 s, the charge stored in C is

$$Q_1 = It_1 = 30\times10^{-6}\times50 = 1.5\times10^{-3}\text{ C}$$

The p.d. across C is

$$V_1 = \frac{Q_1}{C} = \frac{1.5\times10^{-3}}{5\times10^{-4}} = 3.0\text{ V}$$

Therefore, the p.d. across R is

$$V_R = \xi - V_1 = 6-3 = 3.0\text{ V}$$

The resistance of R must have been

$$R = \frac{V_R}{I} = \frac{3}{30\times10^{-6}} = 1.0\times10^5\ \Omega$$

As the switch K is pressed, C is completely discharged. When K is released again, the p.d. across C is zero and the p.d. across R is 6 V. Thus, the current immediately afterwards is

$$I' = \frac{\xi}{R} = \frac{6}{1\times10^5} = 6\times10^{-5} = \underline{\underline{60\ \mu\text{A}}}$$

39. From the given data, the capacitor is fully charged and fully discharged in each cycle.

The maximum charge stored in C is

$$Q_0 = CV_0 = 12\times10^{-6}\times12 = 1.44\times10^{-4}\text{ C}$$

In each cycle, this amount of charge will flow through the ammeter. In each second, 200 such charge will flow through the ammeter. Thus, the average current is

$$\bar{I} = Q_0 f = 1.44\times10^{-4}\times200 = 2.88\times10^{-2} \approx \underline{\underline{29\text{ mA}}}$$

40. The terminal voltage of the cell is equal to the p.d. across the 10 Ω resistor or the 10 μF capacitor.

(a) Since $V_c = 1\text{ V}$, by $Q = CV$, the charge stored in the capacitor is

$$Q = 10\times10^{-6}\times1 = \underline{\underline{10\ \mu\text{C}}}$$

(b) Since $V_R' = 1\text{ V}$, the current drawn from the cell is

$$I = \frac{V_R}{R} = \frac{1}{10} = \underline{\underline{0.10\text{ A}}}$$

(c) From (b), the e.m.f. of the cell is

$$\xi = I\cdot(R+r) = 0.1\times(10+5) = 1.5\text{ V}$$

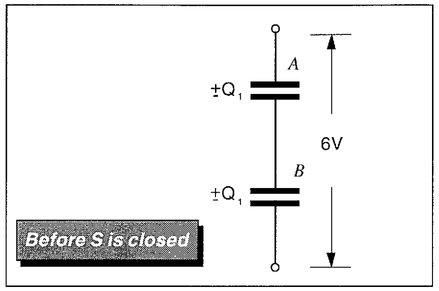
If the capacitor is connected in series with the cell and R, its maximum p.d. is equal to the e.m.f. of the cell. This is because the current drawn from the cell would be zero. The p.d. across R would be zero.

Thus, $V'_C = \xi = 1.5\text{ V}$.

By $Q = CV$, the maximum stored charge is

$$Q' = 10\times10^{-6}\times1.5 = \underline{\underline{15\ \mu\text{C}}}$$

41. (a)



i) Before the switch S is closed, the two capacitors are connected in series and they carry the same quantity of charges. The p.d. across the combination is 6 V.

The equivalent capacitance is

$$C_1 = \frac{C_A C_B}{C_A + C_B} = \frac{1\times2}{1+2} = 0.67\ \mu\text{F}$$

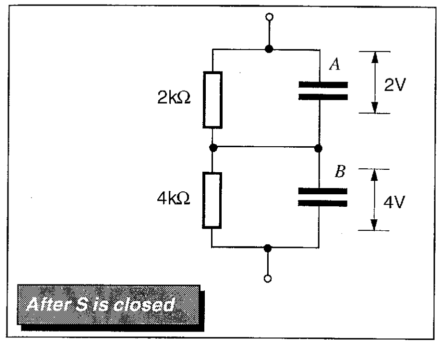
The total charge stored in the combination

$$Q_1 = C_1 V = 0.67\times6 = 4.0\ \mu\text{C}$$

This is also the charge stored in A and B.

Thus, $Q_A = Q_B = Q_1 = \underline{\underline{4.0\ \mu\text{C}}}$

ii)



After S is closed, the p.d. across each capacitor is the same as that across the resistor in parallel with it.

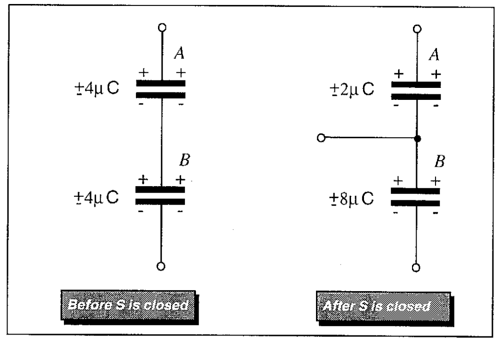
For A, $V_A = \frac{2}{2+4}\times V = 2\text{ V}$

$$Q_A = C_A V_A = 1\times10^{-6}\times2 = \underline{\underline{2.0\ \mu\text{C}}}$$

For B, $V_B = \frac{4}{2+4}\times V = 4\text{ V}$

$$Q_B = C_B V_B = 2\times10^{-6}\times4 = \underline{\underline{8.0\ \mu\text{C}}}$$

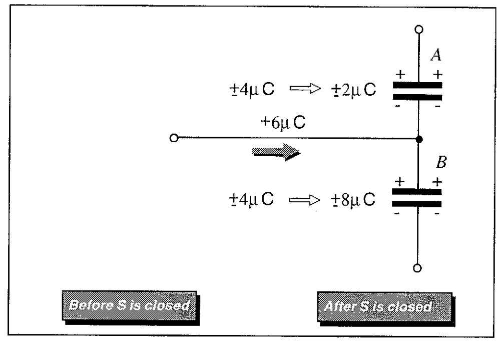
(c) The initial and final charge on the capacitors are schematically shown below:



On the lower plate of A, the charge changes from -4 μC to -2 μC. Thus, +2 μC of charge is supplied to it.

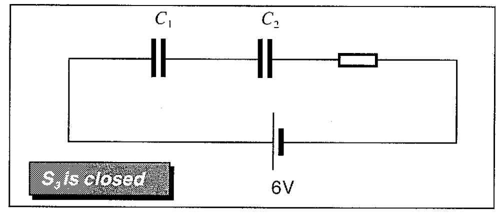
On the upper plate of B, the charge changes from +4 μC to +8 μC. Thus, +4 μC of charge is supplied to it.

Altogether, there are +6 μC of charge supplied to the middle plates of the capacitor. This constitutes the initial current that flows from left to right.



N.B. The charges that move as shown above do **not** come from the cell. They come from the upper plate of capacitor A and the lower plate of capacitor B. Thus, it is actually a re-distribution of charge.

42. (a) When S_3 alone is closed, the circuit is equivalent to the following:



Here, no current flows. The capacitors are in series. The equivalent capacitance is

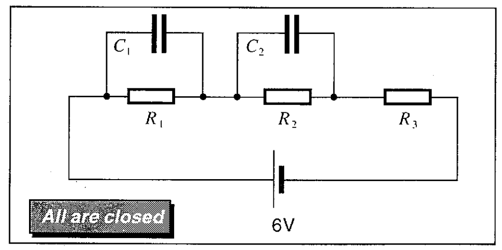
$$C = \frac{C_1C_2}{C_1+C_2} = \frac{4\times6}{4+6} = 2.4\ \mu\text{F}$$

As the p.d. across the combination is 6 V, the total charge stored is

$$Q = CV = 2.4\times6 = 14.4\ \mu\text{C}$$

As C_1 and C_2 are in series, their stored charges are $Q_1 = Q_2 = Q = \underline{\underline{14.4\ \mu\text{C}}}$

(b) When S_1 and S_2 are also closed, the circuit is equivalent to the following:



Now, a steady current flows through the circuit and is independent of the capacitors. The p.d. across each capacitor is the same as that across the resistor in parallel to it.

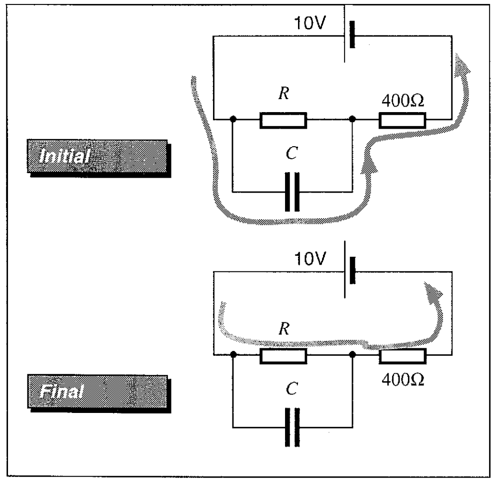
For C_1: $V_1 = \frac{100}{100+50+100} \times 6 = 2.4\text{ V}$

Thus, $Q_1 = C_1V_1 = 4 \times 2.4 = \underline{\underline{9.6\ \mu\text{C}}}$

For C_2: $V_2 = \frac{50}{100+50+100} \times 6 = 1.2\text{ V}$

Thus, $Q_2 = C_2V_2 = 6 \times 1.2 = \underline{\underline{7.2\ \mu\text{C}}}$

43. The circuit can be re-drawn as follows. C is an ideal capacitor and R is the *resistance* of the leaky capacitor.



Initially, the capacitor is uncharged. Its p.d. is zero. Current does not flow through R. Thus, the effective resistance of the circuit is $R_1 = 400 + r$, where r is the internal resistance of the cell.

Since the initial current is 0.025 A, we have

$\xi = I_1R_1$

$\Rightarrow 10 = 0.025 \times (400 + r)$

$\therefore\ r = 0\ \Omega$

This confirms that the internal resistance of the cell is zero which is given.

Long after the switch is closed, the capacitor is fully charged and all current flows through R. Then, we have

$\xi = I_2R_2$

$\Rightarrow 10 = 0.02 \times (400 + R)$

$\therefore\ \underline{\underline{R = 100\ \Omega}}$

44. (a) In the given circuit, the capacitor is fully charged and does not have current. Currents through the two resistors are equal. The overall resistance is $R = 4 + 5 = 9\ \Omega$. Thus, the voltage across the 5 Ω resistor is

$V_1 = \frac{5}{R} \times 9 = \underline{\underline{5.0\text{ V}}}$

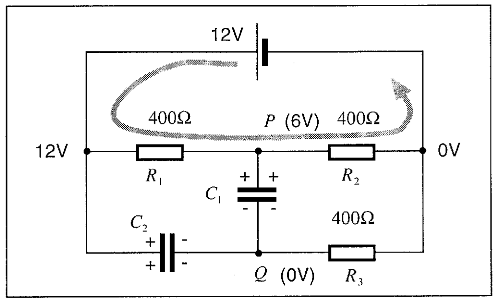
(b) **If a cell is ideal, any device connecting across the cell should have p.d. exactly equal to the e.m.f. of the cell.** Now, the terminal voltage is less than the e.m.f., we say that the cell must have internal resistance.

If the capacitor is ideal, current would not pass through it except during charging or discharging.

Hence, an ideal capacitor connected across any cell (whether the cell is ideal or not) should have a p.d. exactly equal to the e.m.f. of the cell.

Now, at steady state, the terminal voltage is less than the e.m.f., a current must be drawn from the cell. That is the capacitor allows current to flow even it is fully charged. We conclude that the capacitor leaks. i.e. There is a finite resistance in the capacitor.

45. The current in the circuit is shown below:



(a) Current flows through R_1 and R_2 only. There is no current through R_3. Clearly, the potential at P is 6 V and that at Q is 0 V. Thus, the p.d. across C_1 is $\underline{6\text{ V}}$ and that across C_2 is $\underline{12\text{ V}}$.

(b) The charges on the capacitors are

for C_1 : $Q_1 = C_1V_1 = 1 \times 6 = \underline{\underline{6.0\ \mu\text{F}}}$

for C_2 : $Q_2 = C_2V_2 = 1 \times 12 = \underline{\underline{12\ \mu\text{F}}}$

46. (a) By $U = \frac{1}{2}CV^2$, the maximum stored energy is

$U = \frac{1}{2} \times 22000 \times 10^{-6} \times 50^2 = \underline{\underline{27.5\text{ J}}}$

(b) The amount of charge stored in the capacitor is

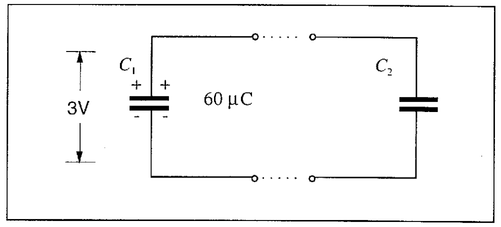
$Q = CV = 22000 \times 10^{-6} \times 50 = 1.1\text{ C}$

This amount of charge passes through the power supply. Thus, the energy supplied from the power supply is

$U' = QV = 1.1 \times 50 = \underline{\underline{55\text{ J}}}$

N.B. The difference between answers in (a) and (b) is due to the energy loss in the resistance of the circuit as the charging current flows through the wire and the cell (which may have internal resistance).

47.



(a) Before connection, the energy stored is

$U_0 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} \times \frac{(60 \times 10^{-6})^2}{20 \times 10^{-6}}$

$= \underline{\underline{9.0 \times 10^{-5}\text{ J}}}$

(b) Effectively, the capacitors are connected in parallel (as like charges are directly electrically connected). Their equivalent capacitance is

$C_{12} = C_1 + C_2 = 20 + 10 = 30\ \mu\text{F}$

The total charge is unaffected by the connection. As their voltages are the same, we have

$V = \frac{Q}{C_{12}} = \frac{60}{30} = 2.0\text{ V}$

Thus, the charges on the capacitors are

for C_1 : $Q_1 = C_1V = 20 \times 2 = \underline{\underline{40\ \mu\text{C}}}$

for C_2 : $Q_2 = C_2V = 10 \times 2 = \underline{\underline{20\ \mu\text{C}}}$

(c) After connection, the energy stored is

$U' = \frac{1}{2}\left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2}\right)$

$= \frac{1}{2} \times \left(\frac{(40 \times 10^{-6})^2}{20 \times 10^{-6}} + \frac{(20 \times 10^{-6})^2}{10 \times 10^{-6}}\right)$

$= 6.0 \times 10^{-5}\text{ J}$

Thus, the energy change is

$\Delta U = U' - U_0 = 6 - 9 = \underline{\underline{-3 \times 10^{-5}\text{ J}}}$

where the -ve sign represents a loss.

N.B. The electrical p.e. is dissipated in the resistive component of the circuit and appears in the form of internal energy.

48. Initially, the stored energy is

$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0} = 20\text{ J}$ ....(1)

By $C = \frac{\varepsilon_0 A}{d}$, the capacitance after halving the plate separation is

$C' = \frac{\varepsilon_0 A}{\frac{1}{2}d} = \frac{2\varepsilon_0 A}{d} = 2C_0$

Since the capacitor is disconnected from the cell before the change in plate separation, the stored charge Q_0 remains unchanged. The final stored energy is

$U' = \frac{1}{2}\frac{Q_0^2}{C'} = \frac{1}{2}\frac{Q_0^2}{2C_0} = \frac{1}{2}U_0 = \frac{1}{2} \times 20$

$= \underline{\underline{10\text{ J}}}$

N.B. Negative work is done on the capacitor. **Alternatively**, we say that the electric field has done work onto the external world (the agent that reduces the separation). This is reasonable because the plates tend to attract each other. As a result, the electrical p.e. is reduced.

49. As connection is made to the power supply, the voltage across the capacitor remains constant.

(a) By $Q=CV$, to double the charge while keeping V constant requires C to be doubled.

By $C=\frac{\varepsilon_0 A}{d}$, to double C, we can either

1. double the overlapping area A,
2. halve the separation, or
3. insert a dielectric sheet of relative permittivity $\varepsilon_r=2$.

(b) As V is constant and C is doubled, by $U=\frac{1}{2}CV^2$, the energy stored is also doubled.

N.B. The increased energy comes from the power supply. This is because additional charge has passed through the cell, retrieving electrical energy from the supply. The external agent, on the other hand, may take away some of the energy from the system in doubling the capacitance e.g. by reducing the plate separation (see question 48).

50. (a) The total initial energy stored in the system is

$$U_0=\frac{1}{2}C_1V_1^2+\frac{1}{2}C_2V_2^2$$
$$=\frac{1}{2}\times\left(4\times10^{-6}\times100^2+2\times10^{-6}\times400^2\right)$$
$$=\underline{\underline{0.18\text{ J}}}$$

(b) The total charge of the system is conserved in the connection. It is given by

$$Q_0=Q_1+Q_2=C_1V_1+C_2V_2$$
$$=4\times100+2\times400=1200\ \mu\text{C}$$

After connection, the capacitors are in parallel and the equivalent capacitance is

$$C'=C_1+C_2=4+2=6\ \mu\text{F}$$

By $U=\frac{1}{2}\frac{Q^2}{C}$, the final energy stored is

$$U'=\frac{1}{2}\frac{Q_0^2}{C'}=\frac{1}{2}\times\frac{\left(1200\times10^{-6}\right)^2}{6\times10^{-6}}$$
$$=\underline{\underline{0.12\text{ J}}}$$

(c) The loss in electrical p.e. is

$$\Delta U=U_0-U'=0.06\text{ J}$$

In the process of connection, charges re-distribute themselves among the capacitors. This constitutes current flow.

In practice, the resistive component in the circuit would deprive electrical energy as current flows and convert into internal energy. However, in case of superconductors, electrons are found to oscillate between the two plates. The acceleration of the charges would cause EM waves to produce.

51. (a) As the capacitors are connected in series, the equivalent capacitance is

$$C=\frac{C_1C_2}{C_1+C_2}=\frac{2\times3}{2+3}=1.2\ \mu\text{F}$$

By $U=\frac{1}{2}CV^2$, the stored energy in the system is

$$U_0=\frac{1}{2}\times1.2\times10^{-6}\times10^2$$
$$=6.0\times10^{-5}=\underline{\underline{60\ \mu\text{J}}}$$

(b) Before re-connection, the charge stored in the system is

$$Q_0=CV=1.2\times10^{-6}\times10=12\ \mu\text{C}$$

This is also the charge stored in each capacitor. i.e. $Q_1=Q_2=Q_0=12\ \mu\text{C}$.

In the re-connection, the total charge of the system is $Q'=Q_1+Q_2=24\ \mu\text{C}$.

As the capacitors are connected in parallel, their equivalent capacitance is

$$C'=C_1+C_2=2+3=5\ \mu\text{F}$$

i) By $U=\frac{1}{2}\frac{Q^2}{C}$, the final energy stored is

$$U'=\frac{1}{2}\frac{Q'^2}{C'}=\frac{1}{2}\times\frac{\left(24\times10^{-6}\right)^2}{5\times10^{-6}}$$
$$=5.76\times10^{-5}\approx\underline{\underline{58\ \mu\text{J}}}$$

ii) The loss in electrical p.e. is

$$\Delta U=U_0-U'=60-58=2\ \mu\text{J}$$

In the process of re-connection, charges re-distribute themselves among the capacitors. This constitutes a current flow which converts electrical energy into internal energy. (In case of superconductors, the energy is dissipated in the form of EM waves).

52. The charging current is $100\ \mu\text{A}$.

(a) By $\Delta Q=I\,\Delta t$, the amount of charge stored in the capacitor is

$$Q=100\times10^{-6}\times50=5.0\times10^{-3}\text{ C}$$

By $U=\frac{1}{2}\frac{Q^2}{C}$, the energy stored in the capacitor is

$$U=\frac{1}{2}\times\frac{\left(5\times10^{-3}\right)^2}{100\times10^{-6}}=\underline{\underline{0.125\text{ J}}}$$

(b) Since a constant current is drawn from the power supply, by $E=Pt$ and $P=VI$ the energy supplied is

$$E=VIt=50\times100\times10^{-6}\times50=0.25\text{ J}$$

Thus, the power dissipated during the charging process is

$$\Delta U=E-U=0.25-0.125=\underline{\underline{0.125\text{ J}}}$$

N.B. This energy is dissipated as internal energy by the resistance in the circuit.

(c) Since all the energy of the capacitor would be converted into internal energy by the resistance in the discharging circuit, the energy dissipation is $\underline{\underline{0.125\text{ J}}}$.

N.B. In discharging, the power supply is not involved.

N.B. The final energy stored in the capacitor is zero. As the cell has supplied 0.25 J of energy, it is correct to find that the total energy dissipation in (b) and (c) to be 0.125+0.125=0.25 J.

N.B. The energy dissipated in discharging is independent of the size of current flow. In other words, if the current falls exponentially, the result is the same.

53. As the capacitor is disconnected from the power supply, the stored charge Q is conserved. i.e. **Q is constant.**

In order to find the voltage change, we should apply $V=\frac{Q}{C}$ ....(1)

In order to find the change in E-field, we should apply $E=\frac{\sigma}{\varepsilon}$ ....(2)

In order to find the energy change, we should apply $U=\frac{1}{2}\frac{Q^2}{C}$ ....(3)

For action A, plate separation is increased.

By $C=\frac{\varepsilon_0 A}{d}$, the capacitance decreases as d increases.

(a) From (1), as Q is constant and C decreases, V would increase.

(b) As the area and the stored charge are unchanged, $\sigma=\frac{Q}{A}$ is unchanged. Thus, E remains unchanged.

(c) From (3), as Q is constant and C decreases, U would increase.

For action B, overlapping area is decreased.

By $C=\frac{\varepsilon_0 A}{d}$, the capacitance decreases as A decreases.

(a) From (1), as Q is constant and C decreases, V would increase.

(b) As the stored charge is unchanged and A decreases, $\sigma=\frac{Q}{A}$ would increases. Thus, from (2), E increases.

(c) From (3), as Q is constant and C decreases, U would increase.

For action C, dielectric sheet is inserted.

As dielectric sheet has relative permittivity $\varepsilon_r > 1$.

By $C = \dfrac{\varepsilon_r \varepsilon_0 A}{d}$, the capacitance increases.

(a) From (1), as Q is constant and C increases, V would decrease.

(b) As the area and the stored charge are unchanged, $\sigma = \dfrac{Q}{A}$ is unchanged. However, from (2), as ε increases, E decreases.

(c) From (3), as Q is constant and C increases, U would decrease.

54. As the capacitor is maintained at constant voltage by the power supply, **V is constant.**

In order to find the change in amount of charge, we should apply $Q = CV$ ....(1)

In order to find the change in E-field, we should apply $E = \dfrac{V}{d}$ ....(2)

In order to find the energy change, we should apply $U = \frac{1}{2}CV^2$ ....(3)

For action A, plate separation is increased.

By $C = \dfrac{\varepsilon_0 A}{d}$, the capacitance decreases as d increases.

(a) From (1), as V is constant and C decreases, Q would decrease.

(b) As d increases, from (2), E decreases.

(c) From (3), as V is constant and C decreases, U would decrease.

For action B, overlapping area is decreased.

By $C = \dfrac{\varepsilon_0 A}{d}$, the capacitance decreases as A decreases.

(a) From (1), as V is constant and C decreases, Q would decrease.

(b) From (2), as both V and d are constant E remains constant.

(c) From (3), as V is constant and C decreases, U would decrease.

For action C, dielectric sheet is inserted.

As dielectric sheet has relative permittivity $\varepsilon_r > 1$.

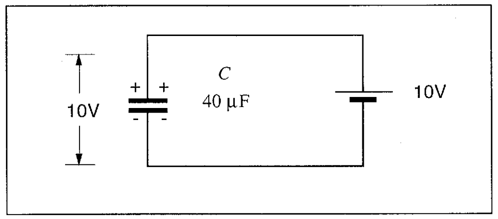
By $C = \dfrac{\varepsilon_r \varepsilon_0 A}{d}$, the capacitance increases.

(a) From (1), as V is constant and C increases, Q would increase.

(b) From (2), as both V and d are constant E remains constant.

(c) From (3), as V is constant and C increases, U would increase.

55.



(a) In steady state, the current is zero. The p.d. across the capacitor is always equal to the e.m.f. i.e. $V_c = 10\text{ V}$

(b) Before connection, the charge in the capacitor is

$Q_0 = C\,V_0 = 40 \times 20 = 800\ \mu\text{C}$

After connection, the charge in the capacitor is

$Q' = C\,V_c = 40 \times 10 = 400\ \mu\text{C}$

The decrease in charge storage is

$\Delta Q = Q_0 - Q' = 400\ \mu\text{C}$

Here, 400 μC of charge from the +ve plate has passed through the cell and reached the -ve plate.

(c) The change in stored energy is

$$\Delta U = U' - U_0 = \frac{1}{2}\frac{Q'^2}{C} - \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2C}\left(Q'^2 - Q_0^2\right)$$

$$= \frac{1}{2 \times 40 \times 10^{-6}} \times \left[\left(400 \times 10^{-6}\right)^2 - \left(800 \times 10^{-6}\right)^2\right]$$

$$= -6.0 \times 10^{-3}\text{ J}$$

-ve sign represents loss in electrical p.e.

(d) During the connection, +ve charge moves into the cell through the +ve terminal and out through the -ve terminal. The cell is recharged.

By $U = QV$, the energy gained by the second cell is

$$U = 400 \times 10^{-6} \times 10 = 4.0 \times 10^{-3}\text{ J}$$

(e) The energy lost by the capacitor is greater than the energy gain by the cell. This is because during the charges re-distribute, they form a current. Some electrical p.e. is then converted into internal energy by the resistance of the circuit.

56. (a) The charged capacitor and the resistor are connected in parallel. When the switch is closed, the p.d.s across C and R are the same.

As the initial voltage of R is 100 V, the initial discharging current is

$$I_0 = \frac{V_0}{R} = \frac{100}{1 \times 10^3} = 0.10\text{ A}$$

(b) The time constant of the circuit is

$$CR = 2000 \times 10^{-6} \times 1 \times 10^3 = 2.0\text{ s}$$

By $I = I_0\, e^{-\frac{t}{CR}}$, the current at t = 3 s is

$$I = 0.1 \times e^{-\frac{3}{2}} = 0.0223\text{ A} \approx 22\text{ mA}$$

N.B. To compute $e^{-1.5}$ using a CASIO calculator, first enter -1.5, then key the function e^x.

57. The time constant of the circuit is

$$CR = 400 \times 10^{-6} \times 20 \times 10^3 = 8.0\text{ s}$$

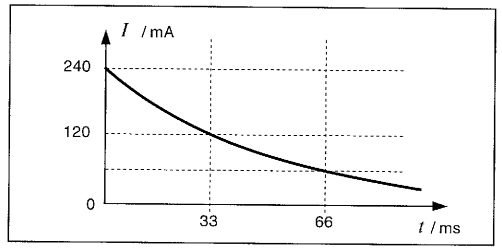
By $t_{\frac{1}{2}} = CR \ln 2$, the time required for the p.d. to fall to half is

$$T = 8 \times \ln 2 = 5.545 \approx 5.5\text{ s}$$

N.B. The half-life is the same for the stored charge to fall to half. It also has the same value for the current.

58. (a) Points to note:

1. The time constant of the circuit is $CR = 47 \times 10^{-6} \times 1 \times 10^3 = 0.047\text{ s}$
2. The half-life of the current is $t_{1/2} = 0.047 \times \ln 2 \approx 33\text{ ms}$
3. When K is just closed, the p.d. across the capacitor is zero. The initial current is $I_0 = \dfrac{V}{R} = \dfrac{240}{1 \times 10^3} = 240\text{ mA}$
4. After 33 ms, the current is 120 mA.



(b) When the capacitor is fully charged, its p.d. is the same as the e.m.f. of the cell. i.e. $V_c = 240\text{ V}$.

By $U = \frac{1}{2}CV^2$, the energy stored in C is

$U = \frac{1}{2} \times 47 \times 10^{-6} \times 240^2 = 1.354$

$\approx 1.4\text{ J}$

(c) The discharging process transfers all the electrical p.e. of the capacitor into energy of other form, like light and internal energy.

By $P = \dfrac{E}{t}$, the average power of the flash-lamp is

$$P = \frac{1.354}{1.6 \times 10^{-3}} = 846\text{ W} \approx 0.85\text{ kW}$$

59. From the given V-t graph, the time required for $V = 80$ V to 40 V is 1.5 minutes; that for $V = 40$ V to 20 V is also 1.5 minutes.

i.e. $t_{\frac{1}{2}} = 1.5 \times 60 = 90$ s

By $t_{\frac{1}{2}} = CR \ln 2$, the resistance of R is given by

$$5.5 = 300 \times 10^{-6} \times R \times \ln 2$$

$$\therefore \quad R = 4.328 \times 10^5\ \Omega \approx 430\ \text{k}\Omega$$

60. (a) The capacitor is being charged. If the capacitor is fully charged, its p.d. is $V_0 = 12$ V.

When $t = 1.3\ \mu$s, the p.d. across the capacitor is $V_C = 5$ V.

By $V_C = V_0 (1 - e^{-\frac{t}{CR}})$, the time constant is given by

$$5 = 12 \times (1 - e^{-\frac{1.3\times10^{-6}}{CR}})$$

$$\Rightarrow \quad e^{-\frac{1.3\times10^{-6}}{CR}} = \frac{7}{12}$$

Taking natural log on both sides, we have

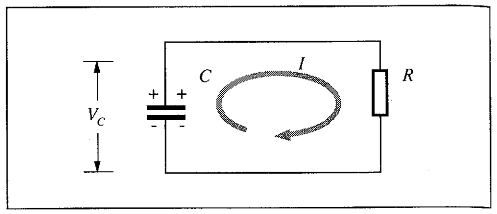
$$-\frac{1.3 \times 10^{-6}}{CR} = \ln\frac{7}{12} = -0.539$$

$$\therefore \quad CR = 2.41 \times 10^{-6} \approx 2.4\ \mu\text{s}$$

(b) Since R = 15 kΩ, the capacitance is

$$C = \frac{CR}{R} = \frac{2.41 \times 10^{-6}}{15 \times 10^3} = 1.6 \times 10^{-10}\ \text{F}$$

61. The charged leaky capacitor left alone would discharge by itself. The equivalent circuit is shown below. C is an ideal capacitor and R is the resistance of the leaky capacitor.



The voltage across the combination varies with time according to $V = V_0\, e^{-\frac{t}{CR}}$.

When the p.d. falls to 1/4th, then $\frac{V}{V_0} = \frac{1}{4}$.

Hence, the time constant is given by

$$\frac{1}{4} = e^{-\frac{2}{CR}}$$

$$\Rightarrow \quad \ln\frac{1}{4} = -\frac{2}{CR}$$

$$\therefore \quad CR = 1.443\ \text{s}$$

Since C = 2 μF, the equivalent resistance is

$$R = \frac{CR}{C} = \frac{1.443}{2 \times 10^{-6}} \approx 7.2 \times 10^5\ \Omega$$

62. (a) By $V = V_0\, e^{-\frac{t}{CR}}$, the time constant of the circuit is given by

$$10 = 100 \times e^{-\frac{10}{CR}}$$

$$\Rightarrow \quad \frac{1}{10} = e^{-\frac{10}{CR}}$$

$$\Rightarrow \quad \ln\frac{1}{10} = -\frac{10}{CR}$$

$$\therefore \quad CR = 4.34\ \text{s}$$

(b) By $V = V_0\, e^{-\frac{t}{CR}}$, the voltmeter reading at $t = 17$ s is

$$V = 100 \times e^{-\frac{17}{4.34}}$$

$$= 1.995 \approx 2.0\ \text{V}$$

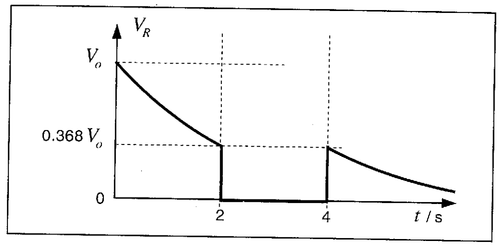
63. Points to note:

1. Time constant is the time for the voltage of a discharging capacitor to fall to $\frac{V_0}{e}$ or $0.368\,V_0$.

2. In the discharging ciruit, the current decreases exponentially. However, it stops flowing between $2.0\ \text{s} \le t \le 4.0\ \text{s}$.

3. The voltage across R has similar variation as the current in the circuit.

4. Since charge is conserved, when the switch is closed again, the quantity of charge is the same as that just after opened.



64. (a) The voltage of the capacitor when fully charged is $V_0 = 100$ V. From the graph, when $V_C = 50$ V, $t = 14$ s.

By $V_C = V_0 (1 - e^{-\frac{t}{CR}})$, the time constant of the circuit is given by

$$50 = 100 \times (1 - e^{-\frac{14}{CR}})$$

$$\Rightarrow \quad e^{-\frac{14}{CR}} = \frac{1}{2}$$

$$\Rightarrow \quad -\frac{14}{CR} = \ln\frac{1}{2} = -0.693$$

$$\therefore \quad CR = 20.19 \approx 20\ \text{s}$$

Alternatively, at $t_{1/2} = 14$ s, $V_C = \frac{1}{2} V_0$.

In charging circuit, $V_C = V_0 (1 - e^{-\frac{t}{CR}})$.

Thus,

$$\tfrac{1}{2} V_0 = V_0 (1 - e^{-\frac{t_{1/2}}{CR}})$$

$$\Rightarrow \quad e^{-\frac{t_{1/2}}{CR}} = \tfrac{1}{2}$$

$$\therefore \quad t_{1/2} = CR \ln 2$$

Hence, $CR = \frac{t_{1/2}}{\ln 2} = \frac{14}{0.693} \approx 20$ s

N.B. The expression $t_{1/2} = CR \ln 2$ is true for **both charging and discharging circuits.**

(b) Points to note:

1. The maximum voltage across the capacitor is now $V_{C0} = 200$ V.

2. Since $t_{1/2} = CR \ln 2$, the time for the voltage to reach half of V_0 is independent of the value of V_0. Thus, the time for $V_C = 100$ V is also 14 s.

3. The shapes of the two graphs are similar.