Supplemenatry Exercise on Capacitor(PPC)

Level I

1. (a) $R_1 = 120\ \text{k}\Omega$

   $R_2 = 200\ \text{k}\Omega$

   (b) $C_2 = 72.13\ \mu\text{F}$

2. (a) 3.077 μC, 4.923 μC

   (b) both are at 5.54×10^5 V

   (c) 0.124 5 J

3. 0.05 C, 25 J

4. Charge on capacitor A = 0.036 C

   Charge on capacitor B = 0.072 C

   Equivalent capacitance = 900 μF

5. Equivalent capacitance = 1.5 μF

   Charge on capacitor A = 1.5 mC

   Charge on capacitor B = 1.5 mC

6. 8 μF

7. (a) 9.6×10^{-3} J

   (b) (i) 53.3 V

   (ii) 8.53×10^{-3} J

8. 4 μF

Level II

1. (a) Since the capacitor has been disconnected and isolated, the charge does not change.

$$\therefore\ Q = CV$$
$$= (950\ \text{pF})(400\ \text{V})$$
$$= \underline{3.8 \times 10^{-7}\ \text{C}}$$

(b)
$$Q = C'V'$$
$$\therefore V' = \frac{Q}{V'}$$
$$= \frac{3.8\times10^{-7}}{50\times10^{-12}}$$
$$= \underline{7\ 600\ \text{V}}$$

(c)
$$\text{Final energy} = \frac{1}{2}QV'$$
$$= \frac{1}{2}(3.8\times10^{-7})(7\ 600)$$
$$= \underline{1.444\times10^{-3}\ \text{J}}$$

(d)
$$\text{Initial energy} = \frac{1}{2}QV$$
$$= \frac{1}{2}(3.8\times10^{-7})(400)$$
$$= 7.6\times10^{-5}\ \text{J}$$

Work done required = final energy − initial energy

$$= \underline{1.368\times10^{-3}\ \text{J}}$$

2. (a) Combined capacitance of 15-μF and 30-μF capacitors = 10 μF

∴ Combined capacitance between Z and B = 10 + 40

= 50 μF

$$\therefore\ \text{p.d. between } Z \text{ and } B = (240)\left(\frac{50}{50+50}\right)$$
$$= \underline{120\ \text{V}}$$

(b) For capacitors in series,

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$
$$\therefore\ \text{p.d. between } X \text{ and } Y = (120\ \text{V})\left(\frac{15}{15+30}\right)$$
$$= \underline{40\ \text{V}}$$

(c) If the 50-μF capacitor is shorted,

$$V_{XY} = (240 \text{ V})\left(\frac{15}{15+30}\right)$$
$$= \underline{80 \text{ V}}$$

(d) We know from part (c) that it will not be the 50-μF capacitor.

If the 30-μF capacitor is shorted, $V_{XY} = 0$.

If the 40-μF capacitor is shorted, $V_{XY} = 0$.

If the 15-μF capacitor is shorted,

combined capacitance between X and Y = 40 + 30

= 70 μF

$$\therefore \quad V_{XY} = (240 \text{ V})\left(\frac{50}{50+70}\right)$$
$$= 100 \text{ V}$$

∴ the 15-μF capacitor is shorted.

3(a) Equivalent capacitance $= \dfrac{1}{\frac{1}{3}+\frac{1}{2}+\frac{1}{3}}$

$= \dfrac{6}{7}$ μF

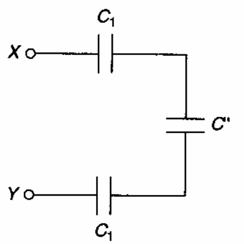
(b) The equivalent capacitance between X and Y, C_{XY}, can be calculated by using the following equations.

$$\begin{cases} C = \sum_i C_i & \text{for capacitors in parallel} \\ C = \dfrac{1}{\sum_i \dfrac{1}{C_i}} & \text{for capacitors in series} \end{cases}$$

It is found that $C_{XY} = \dfrac{546}{547}$ μF

$= 0.998\ 2$ μF

(c) The network can be considered as consisting of three capacitors, as shown in the following figure, of which C' has a capacitance equivalent to the remaining six capacitors in Figure 2.29.



Since the three capacitors are connected in series, the charge on each of the capacitors is the same.

$$Q = C_{XY}V_{XY}$$
$$= (0.998\ 2 \times 10^{-6})(20)$$
$$= 1.996 \times 10^{-5} \text{ C}$$

4(a) (i) $Q = CV$

$= (120 \times 10^{-12})(100)$

$= 1.2 \times 10^{-8}$ C

(ii) $E = \dfrac{1}{2}CV^2$

$= \underline{6 \times 10^{-7} \text{ J}}$

(b) (i) Let C_1 be the capacitance of the air-filled capacitor and C_2 be the capacitance of the insulator-filled capacitor.

$$\because \quad C = \frac{\varepsilon A}{d}$$

$$\therefore \quad C_1 = \frac{\varepsilon_0(A/2)}{d}$$
$$= \frac{(\varepsilon/6)(A/2)}{d}$$
$$= \frac{1}{12}\left(\frac{\varepsilon A}{d}\right)$$
$$= \frac{120 \text{ pF}}{12}$$
$$= 10 \text{ pF}$$

and
$$C_2 = \frac{\varepsilon(A/2)}{d}$$
$$= \frac{1}{2}\left(\frac{\varepsilon A}{d}\right)$$
$$= \frac{120 \text{ pF}}{2}$$
$$= 60 \text{ pF}$$

∴ combined capacitance = 10 pF + 60 pF

= 70 pF

(ii) Since the capacitor is isolated, the total charge is constant.

$Q = 1.2 \times 10^{-8}$ C

$$\text{Energy} = \frac{Q^2}{2C}$$
$$= \frac{(1.2 \times 10^{-8})^2}{2(70 \times 10^{-12})}$$
$$= \underline{1.03 \times 10^{-6} \text{ J}}$$

5. (a) (i) $Q = CV$

$= (6 \times 10^{-6})(24)$

$= \underline{144\ \mu C}$

(ii) Let q_1 and q_2 be the charges on the 6-μF and 4-μF capacitors respectively.

$q_1 + q_2 = 144$ ......(1)

$\frac{q_1}{c_1} = \frac{q_2}{c_2}$

$\therefore\ 2q_1 = 3q_2$ ......(2)

Solving equations (1) and (2),

$q_1 = 144\ \mu C \times \frac{3}{5}$

$= \underline{86.4\ \mu C}$

$q_2 = 144\ \mu C \times \frac{2}{5}$

$= \underline{57.6\ \mu C}$

(iii) $V = \frac{q_2}{C_2}$

$= \frac{57.6 \times 10^{-6}}{4 \times 10^{-6}}$

$= \underline{14.4\ V}$

(b) Let the new charges on the 6-μF and 2-μF capacitors be Q_1 and Q_2 respectively.

$Q_1 + Q_2 = 86.4\ \mu C - 57.6\ \mu C$

$\therefore\ Q_1 + Q_2 = 28.8\ \mu C$ ......(3)

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$2Q_1 = 3Q_2$ ......(4)

Solving equations (3) and (4), we get

$Q_1 = \underline{17.28\ \mu C}$

$Q_2 = \underline{11.52\ \mu C}$