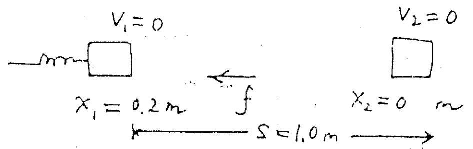
EXERCISE (Energy and Momentum)

1. (a) Elastic potential energy stored

$= \frac{1}{2}kx_1^2 = \frac{1}{2}(100)(0.2)^2 = 2\ J$

(b)



$W' = \left(\frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2\right) - \left(\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2\right)$

$-fs = 0 + 0 - 0 - 2$

$-f(1) = -2$

$f = 2\ N$

$f = \mu R = \mu mg$

$2 = \mu(1)(10) \Rightarrow \mu = 0.2$

2. (a) $W_1 = \Delta P.E = mgh$

$= 800(10)(10)$

$= 8\times10^4\ J$

(b) $W_2 = \Delta K.E = \frac{1}{2}mv^2 - 0$

$= \frac{1}{2}(800)(20)^2$

$= 1.6\times10^5\ J$

(c) $P = \frac{\Delta W}{\Delta t} = \frac{W_1 + W_2}{\Delta t}$

$= \frac{(8\times10^4) + (1.6\times10^5)}{60}$

$= 4000\ W$

3. (a) At highest point $v = 0$

$\Rightarrow t = 1.6s$

At C, collision between the ball and the ground occurs (v changes sign) $\Rightarrow t = 3.6s$

(b) There exists air resistance which absorbs energy from the ball

(c) The ball is at ground level when at A, C

$\Delta E = \Delta K.E = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$= \frac{1}{2}(0.1)(-14)^2 - \frac{1}{2}(0.1)(20)^2$

$= -10.2\ J$

$\Rightarrow$ 10.2 J energy was lost

(d) (i) $F = \frac{\Delta p}{\Delta t} = \frac{mV - mV_0}{\Delta t}$

$= \frac{0.1(0) - 0.1(20)}{1.6} = -1.25\ N$ (downward)

(ii) $F = \frac{0.1(-14) - 0.1(0)}{3.6 - 1.6}$

$= -0.7\ N$

(e) At E, $v = 8\ ms^{-1}$ (after collision)

$\Rightarrow \Delta p = mV - mV_0$

$= 0.1(8) - 0.1(-14)$

$= 2.2\ kg\ ms^{-1}$

$\Delta E = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$= \frac{1}{2}(0.1)(8)^2 - \frac{1}{2}(0.1)(-14)^2$

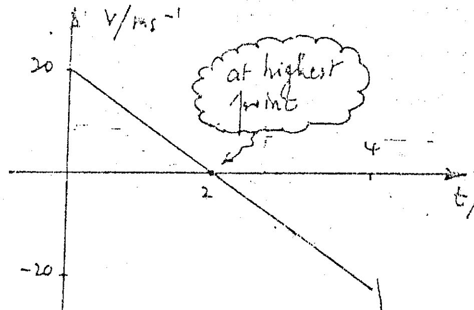
$= -6.6\ J$

energy loss = 6.6 J

(f) In vacuum, no air resistance

$\Rightarrow a = -g$ (upward is +ve)

$V = V_0 - gt = 20 - 10t$



4 (a) $p = mv$

before collision $22 = 2u_A$

$u_A = 11 \text{ kg m s}^{-1}$

after collision $-10 = 2v_A$

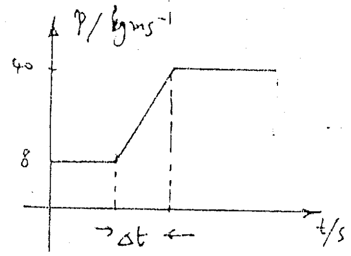
$v_A = -5 \text{ m s}^{-1}$

(b) Before collision

Total momentum $= 22 + 8 = 30 \text{ kg m s}^{-1}$

After collision

$p_B + (-10) = 30 \Rightarrow p_B = 40 \text{ kg m s}^{-1}$



(c) Since A moves backward (rebounces) after collision, $m_B > m_A$.

(d) Before collision $u_A = 11 \text{ m s}^{-1}$

$u_B = \frac{8}{8} = 1 \text{ m s}^{-1}$

After collision $v_A = -5 \text{ m s}^{-1}$

$v_B = \frac{40}{8} = 5 \text{ m s}^{-1}$

$K.E._i = \frac{1}{2}(2)(11)^2 + \frac{1}{2}(8)(1)^2 = 125 \text{ J}$

$K.E._f = \frac{1}{2}(2)(-5)^2 + \frac{1}{2}(8)(5)^2 = 125 \text{ J}$

$\because K.E._i = K.E._f$

$\Rightarrow$ elastic collision

(e) For A:

$F_A = \frac{m_A v_A - m_A u_A}{\Delta t} = \frac{2(-5) - 2(11)}{0.05}$

$= -640 \text{ N}$ (backward)

$F_B = 640 \text{ N}$ (forward)

(f) $\overrightarrow{u_A = 11 \text{ ms}^{-1}}$ $\overrightarrow{u_B = 1 \text{ ms}^{-1}}$

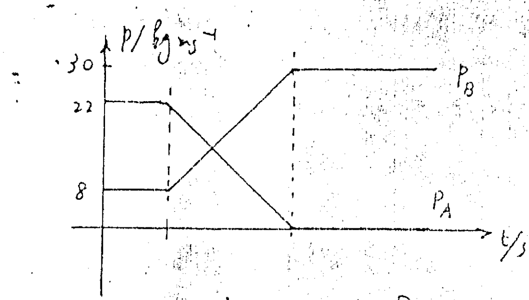
$\Rightarrow$ $v_A = 0$ $\overrightarrow{v_B}$

$P_{A_i} + P_{B_i} = P_{A_f} + P_{B_f}$

$30 = 0 + P_{B_f}$

$\Rightarrow P_{B_f} = 30 \text{ kg m s}^{-1}$

$v_B = \frac{30}{8} = 3.75 \text{ m s}^{-1}$



$\Delta K.E. = \frac{1}{2}(8)(3.75)^2 - \left[\frac{1}{2}(2)(11)^2 + \frac{1}{2}(8)(1)^2\right]$

$= -68.75 \text{ J}$

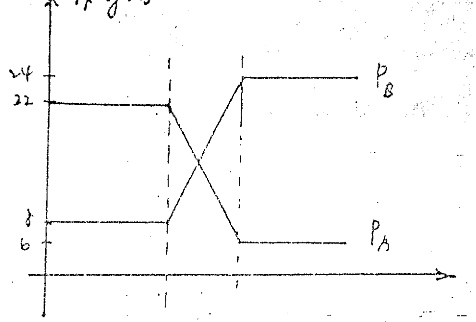
(g) $\overrightarrow{u_A = 11 \text{ ms}^{-1}}$ $\overrightarrow{u_B = 1 \text{ ms}^{-1}}$ $\Rightarrow$ $\overrightarrow{V}$

$m_A u_A + m_B u_B = (m_A + m_B)V$

$V = \frac{22 + 8}{2 + 8} = 3 \text{ m s}^{-1}$

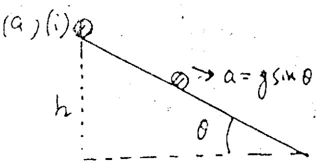
$\therefore P_{A_f} = m_A V_A = 2(3) = 6 \text{ kg m s}^{-1}$

$P_{B_f} = m_B V_B = 8(3) = 24 \text{ kg m s}^{-1}$



5.

(a)(i)



along the plane
$v_0 = 0$
$a = g\sin\theta$
$s = \frac{h}{\sin\theta}$

$\therefore \frac{h}{\sin\theta} = 0 + \frac{1}{2} g\sin\theta\, t^2$

$\Rightarrow t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

(ii) $mgh + \frac{1}{2}mv_{top}^2 = 0 + \frac{1}{2}mV_{bottom}^2$

$mgh + 0 = \frac{1}{2}mV_{bottom}^2$

$\Rightarrow K.E. = \frac{1}{2}mV_{bottom}^2 = mgh$

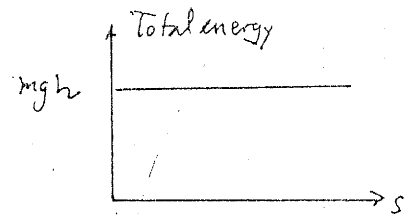
$V_{bottom} = \sqrt{2gh}$ (downplane)

(iii) $F\Delta t = mv - mv_0$

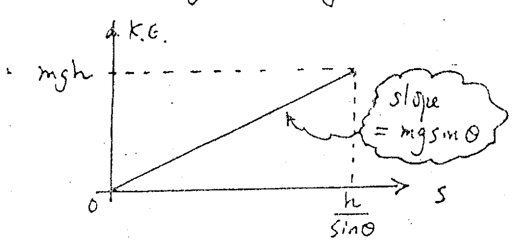
$mg\sin\theta \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}} = mV_{bottom} - 0$

$\Rightarrow V_{bottom} = \sqrt{2gh}$

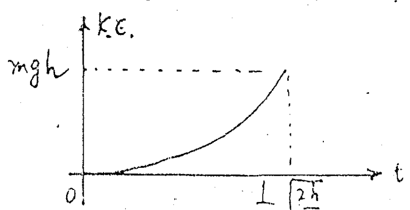
(iv) Total energy = M.E. = P.E. + K.E.
$= mgh$ = constant



$K.E. = mg\Delta h = mgs\sin\theta$

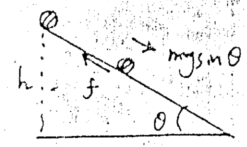


$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m(g\sin\theta\, t)^2$



$\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

(b)



when the plane is rough
net force downplane
$= mg\sin\theta - f$
$\Rightarrow a = \frac{mg\sin\theta - f}{m}$

(i) $\frac{h}{\sin\theta} = 0 + \frac{1}{2}\frac{mg\sin\theta - f}{m}t^2$

$\Rightarrow t = \sqrt{\frac{2hm}{\sin\theta(mg\sin\theta - f)}}$

(ii) $W' = \left(\frac{1}{2}mV_{bottom}^2 + 0\right) - (0 + mgh)$

$-f\left(\frac{h}{\sin\theta}\right) = \frac{1}{2}mV_{bottom}^2 - mgh$

$K.E._{bottom} = \frac{1}{2}mV_{bottom}^2 = mgh - \frac{fh}{\sin\theta}$

$V_{bottom} = \sqrt{\frac{2}{m}\left(mgh - \frac{fh}{\sin\theta}\right)}$

$= \sqrt{2gh - \frac{2fh}{m\sin\theta}}$

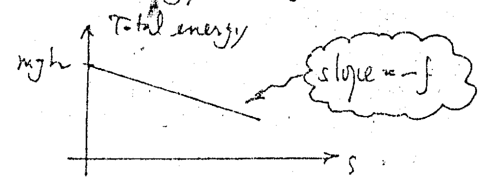
(iii) $F\Delta t = mV_{bottom} - mV_0$

$(mg\sin\theta - f)\sqrt{\frac{2hm}{\sin\theta(mg\sin\theta - f)}} = mV_{bottom} - 0$

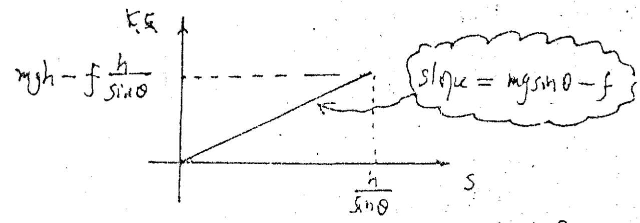
$V_{bottom} = \frac{1}{m}\sqrt{\frac{2hm(mg\sin\theta - f)}{\sin\theta}}$

$= \sqrt{2gh - \frac{2fh}{m\sin\theta}}$

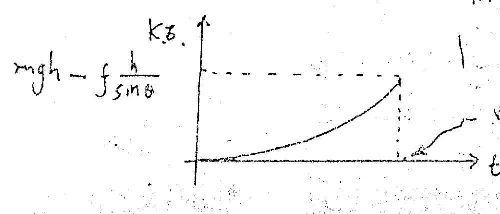
(iv) Total energy $= mgh - fs$



$K.E. = mg\Delta h - f\frac{\Delta h}{\sin\theta} = mgs(\sin\theta) - fs$



$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{mg\sin\theta - f}{m}t\right)^2$



$\sqrt{\frac{2hm}{\sin\theta(mg\sin\theta - f)}}$