# Capacitors and Capacitance

CHAPTER 20

1. Charge delivered = $CV = (10^{-8})(0.18) = 1.8 \times 10^{-9}$ C
If this amount is all of the charge in the spoon, then its capacitance = $1.8 \times 10^{-9}/10^{3} = 1.8$ pF.

> Tip: The capacitance of the spoon is only 1/5000 that of the built-in capacitor. Therefore 99.98% of the charge is delivered.

2. (a) Current = 59 $\mu$A

(b) Charge in the capacitor = $(59 \times 10^{-6})(45) = 2.66 \times 10^{-3}$ C
Capacitance = $2.66 \times 10^{-3} / 5.6 = 470\ \mu$A

(c) Current leaking through the voltmeter increases. When it is 59 $\mu$A, the voltage becomes steady.
$V/20 \times 10^{3} = 59 \times 10^{-6}$
$V = 1.18$ V

3. (a) Charge on the capacitor at 1 s = $(59 \times 10^{-6})(1) = 59 \times 10^{-6}$ C
Voltage = $59 \times 10^{-6} / 470 \times 10^{-6} = 0.126$ V
Vertical deflection = 0.126 div

(b) Charge stored at 20 s = $(59 \times 10^{-6})(20) = 1.18 \times 10^{-3}$ C

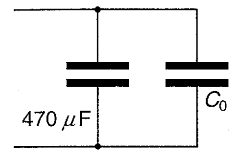
(c) Voltage of capacitor = (0.126)(20) = 2.52 V
$R = (6 - 2.52) / 59 \times 10^{-6} = 5.9 \times 10^{4}\ \Omega$

(d) When the voltage of the capacitor reaches 6 V, a steady current cannot be sustained.
Charge in the capacitor at 6 V = $(6)(470 \times 10^{-6}) = 2.82 \times 10^{-3}$ C
Time of charging = $2.82 \times 10^{-3} / 59 \times 10^{-6} = 47.8$ s



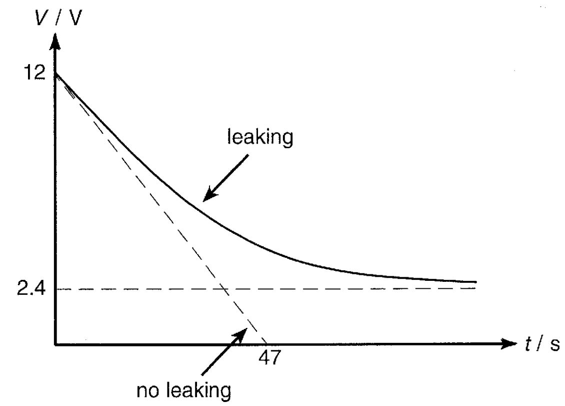


(e)




Equivalent capacitance = $470 \times 10^{-6} + C_0$ (from the diagram)
If the measured voltage is $V$, then $Q$ is $470 \times 10^{-6}$ V, but in fact $Q$ should be $(470 \times 10^{-6} + C_0)V$.
Thus, a factor of $C_0 / 470 \times 10^{-6}$ should be added to the observed voltage.

4. (a) If there is no leakage, the relationship between $R$ and $t$ is given by $12 - Q/C = IR$ with $Q = It$ and $V = IR$.
$V = 12 - (120 \times 10^{-6} / 470 \times 10^{-6})t = 12 - (12/47)t$
At $t = 47$ s, $V$ should be zero.




(b) Final voltage of capacitor = 12 – 2.4 = 9.6 V
Final charge stored = $(9.6)(470 \times 10^{-6}) = 4.5 \times 10^{-3}$ C

(c) The charge in the capacitor remains constant when the voltage across the capacitor remains constant. All of the 120 $\mu$A current leaks through the capacitor.
∴ Leakage resistance = $9.6 / 120 \times 10^{-6} = 80$ kΩ

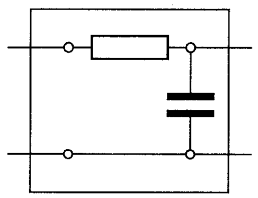
5. (a) Since $C = \varepsilon_0 A / d$, $A$ decreases as $C$ decreases. $V (= Q / C)$ will increase.
The capacitor discharges and the pointer 'kicks' in the opposite direction.

(b) Since $C = \varepsilon\varepsilon_0 A / d$, inserting a dielectric sheet increases $C$. $V$ then decreases and more charge flows into the capacitor. The pointer 'kicks' in the original direction.

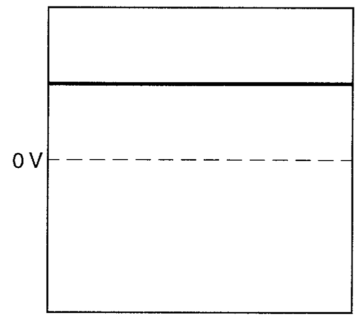
6. $C = \varepsilon\varepsilon_0 A / d = (6)(8.85 \times 10^{-12})(100 \times 10^{-4}) / 0.1 \times 10^{-3}$
$= 5.3 \times 10^3$ pF

Tip: The answer is seldom written as $5.3 \times 10^{-9}$ F. The two commonly used units of capacitance are $\mu$F or pF, thus write it as $5.3 \times 10^{-3}$ $\mu$F or $5.3 \times 10^3$ pF.

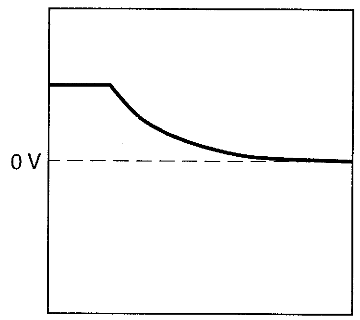
7. (a)



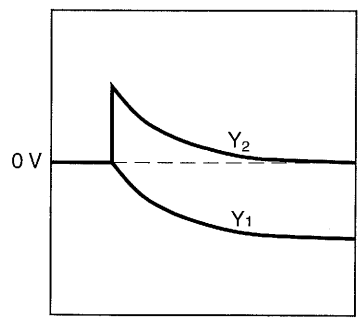
(b)




(c)



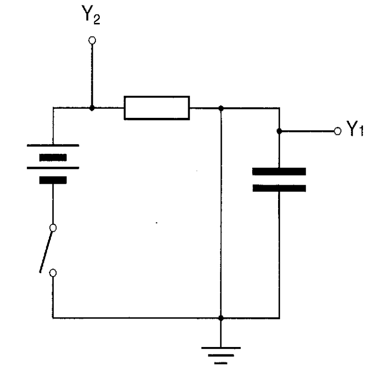

8. (a)








(b) If $Y_1$ and E are interchanged, the capacitance is shorted and the circuit is as shown in the diagram.




$Y_1 = 0$ and $Y_2 =$ voltage of the battery

Tip: Thus, there are restrictions on the connections of $Y_1$, $Y_2$ and E when using a double-beam CRO.

9. (a) Initial charge $= CV_0 = (470 \times 10^{-6})(4.7) = 2.2 \times 10^{-3}$ C

(b) From the figure, $Q$ drops to $1/e$ of its original value in 28 s.
$RC = 28$
$R = 28 / 470 \times 10^{-6} = 5.96 \times 10^4\ \Omega$

10. (a) Increase $Q_0$. $I_0 = Q_0 C / R$

(b) Increase $C$. $I_0 = V_0 / R = V_0 C^2 \sqrt{2} / (RC\sqrt{2}) = V_0 C^2 \sqrt{2} / t_{1/2}$

11. (a) Time of discharge $= (1 / 400) / 2 = 1.25 \times 10^{-3}$ s
Time constant of the $RC$ circuit $= (580 \times 10^{-12})(10 \times 10^6)$
$= 5.8 \times 10^{-3}$ s
$V = V_0 \exp(-t / RC) = 20.15$ V

(b) Current registered = charge passed through in 1 s
$= (400)(25 - 20.15)(580 \times 10^{-12})$
$= 1.13\ \mu$A

(c) $\exp(-t / RC) = (V_0 - V)/V_0 = 1 - 0.99$
$t / RC = 4.61$ with $t = 1.25 \times 10^{-3}$
$R = 470$ k$\Omega$

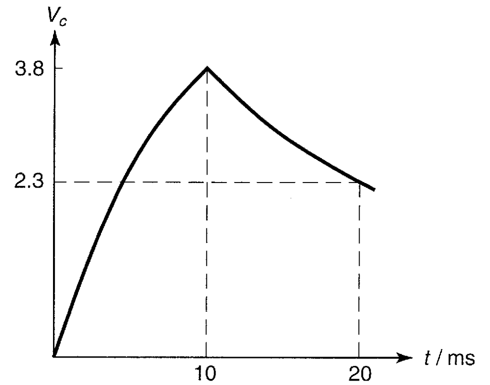
12. Time of discharge = $0.5 \times 10^{-3}$ s
Minimum value of $RC = (400 \times 10^3)(580 \times 10^{-12}) = 2.32 \times 10^{-4}$ s
$\therefore$ Time of discharge = 2.2 $RC$

Fraction of charge left = exp (–2.2) = 0.11
A switching frequency of 1 kHz is not practical since only 89% of the charge would be discharged.

13. (a) Time of charging = (1 / 50) / 2 = 10 ms
Time constant of the charging circuit = $(2 \times 10^{-6})(5 \times 10^3)$ = 10 ms
$V = 6 \exp[(1 - (1/e)] = 3.8$ V

(b) Time constant of the discharging circuit = $(2 \times 10^{-6})(10^4)$ = 20 ms
$V = 3.8 \exp(-10/20) = 2.3$ V

(c)




14. (a) Time constant of the circuit = $(4700 \times 10^{-6})(10^4)$ = 47 s
$Q = (4700 \times 10^{-6})(9) \exp(-20/47)$
$= 2.76 \times 10^{-2}$ C

(b) $V = Q / C = 5.88$ V

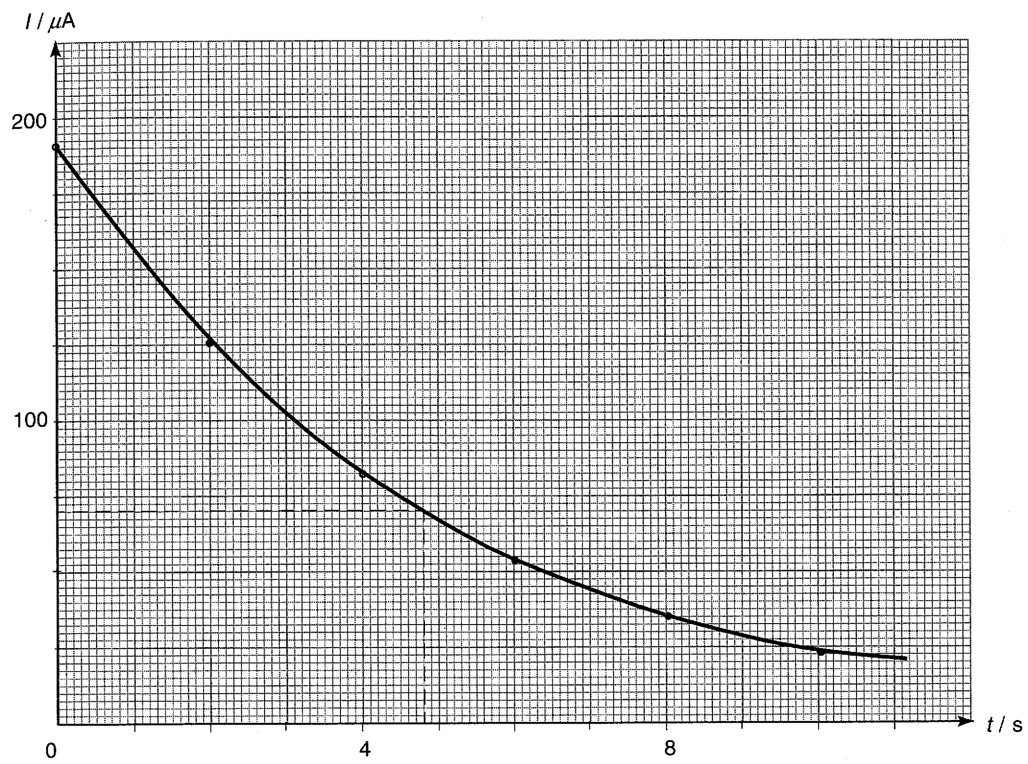
(c) $V_R = V_C = 5.88$ V

(d) $I = 5.88 / (10 \times 10^3) = 588$ $\mu$A





15. (a)




(b) The time for the current to fall to 1/$e$ of the initial value,
i.e. when $I = 0.37 \times 190 = 70$ $\mu$A, is 4.8 s.
$\therefore C = 4.8 / R = 102$ $\mu$F

(c) $R$ must be ten-times larger for the time constant to be measurable.

16. (a) Initial current = 25 / 500 = 50 mA

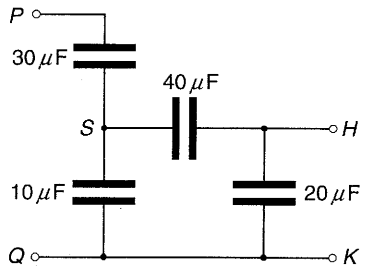
(b) Time for the initial current to drop to 10 mA is $t$, where $t$ is given by
$10 = 50 \exp(-t / RC)$.
$t = (1.61)(500)(4700 \times 10^{-6})$
$= 3.8$ s

17. (a) Equivalent capacitance of the 100 $\mu$F and the 220 $\mu$F capacitors
= 100 + 220 = 320 $\mu$F
Equivalent capacitance of the 320 $\mu$F capacitor and the
470 $\mu$F capacitor = $[(320 \times 10^{-6})^{-1} + (470 \times 10^{-6})^{-1}] = 190$ $\mu$F

(b) Voltage ratio of the 470 and 320 $\mu$F capacitors = 320 / 470 .
Sum of the two voltages = 6 V
Voltage across the 470 $\mu$F capacitor = (6)(320)/(320 + 470) = 2.43 V
Voltage across the 220 $\mu$F capacitor = 3.57 V

Energy = $CV^2/2$, thus the energy stored in the 100, 220 and 470 $\mu$F capacitors is respectively 0.64 mJ, 1.40 mJ and 1.39 mJ.

18.

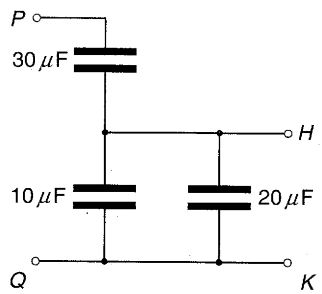



(a) Equivalent capacitance of the 40 and 20 $\mu$F capacitors
$= [(40 \times 10^{-6})^{-1} + (20 \times 10^{-6})^{-1}] = 13.3\ \mu$F
Equivalent capacitance between $SQ = 13.3 + 10 = 23.3\ \mu$F

$V_{PS} / V_{SQ} = 23.3 / 30$ and $V_{PQ} = 600$ V
$V_{SQ} = 338$ V
$V_{SH} / V_{HK} = 20 / 40$ and $V_{SK} = 338$ V
$V_{HK} = 225.3$ V

(b) The 40 $\mu$F capacitor.

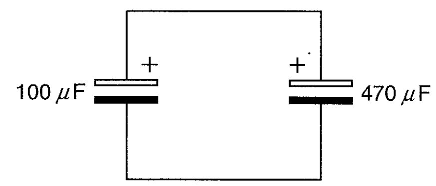
Tip: In (b), the equivalent circuit is shown.
Verify that $V_{HK} = 300$ V.








19. (a)




(b) Total charge = $(100 \times 10^{-6})(9) = 900\ \mu$C
Final charge on the 100 $\mu$F capacitor
$= (100 / 570)(900 \times 10^{-6})$
$= 158\ \mu$C
Final charge on the 470 $\mu$F capacitor
$= 900 - 158$
$= 742\ \mu$C

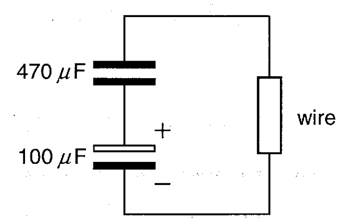
(c) P.d. across the capacitor
$= 158 \times 10^{-6} / 100 \times 10^{-6}$
$= 1.58$ V

(d) Energy stored in the 100 $\mu$F capacitor
$= (158 \times 10^{-6})(1.58)$
$= 125\ \mu$J
Energy stored in the 470 $\mu$F capacitor
$= (742 \times 10^{-6})^2 / 470 \times 10^{-6}$
$= 586\ \mu$J

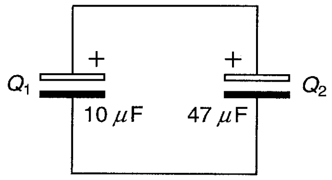
(e) Equivalent capacitance = $[(100 \times 10^{-6})^{-1} + (470 \times 10^{-6})^{-1}] = 82.5\ \mu$F
Time constant = $(0.1)(82.5 \times 10^{-6}) = 8.25 \times 10^{-6}$ s
It takes about $5RC$ to complete the discharge, i.e. $\sim 40 \times 10^{-3}$ s

(f) Some energy is dissipated in resistive heating in the wire.

Tip: The figure shows the equivalent circuit in (e).



20. (a)




$Q_{10} = (10 \times 10^{-6})(9) = 90\ \mu C$

$Q_{20} = (47 \times 10^{-6})(6) = 282\ \mu C$

Total charge = $372\ \mu C$

At equilibrium, $Q_1 / Q_2 = 10/47$

$Q_1 = 65\ \mu C$, $Q_2 = 307\ \mu C$

$V_1 = V_2 = 65 \times 10^{-6} / 10 \times 10^{-6} = 6.5$ V

Initial energy = $[90 \times 10^{-6})(9) + (282 \times 10^{-6})(6)] / 2$
$= 1.251 \times 10^{-3}$ J

Final energy $= (372 \times 10^{-6})(6.5)/2$
$= 1.209 \times 10^{-3}$ J

Energy dissipated = $42\ \mu J$

(b) Total charge = $282 - 90 = 192\ \mu C$

$Q_1 = (10/57)(192) = 33.7\ \mu C$

$Q_2 = 192 - 33.7 = 158.3\ \mu C$

$V_1 = V_2 = 33.7 \times 10^{-6} / 10 \times 10^{-6} = 3.37$ V

Final energy $= (192 \times 10^{-6})(3.37)/2$
$= 0.3235 \times 10^{-3}$ J

Energy dissipated = $930\ \mu J$

21. (a) When it is fully charged, the voltage across it is 9 V.

Energy stored $= \int V dQ = \int (Q/C) dQ = Q^2/2C = CV^2/2$
$= (4700 \times 10^{-6})(9)^2/2 = 0.19$ J

(b) Energy dissipated in resistor $= R\int I^2 dt = (V^2/R)\int \exp(-2t/RC)\, dt$
$= V^2 C/2 - 0.19$ J

(c) Total energy output $= QV = QC^2 = 0.38$ J

> Tip: Note the relationships among the results of the three parts.