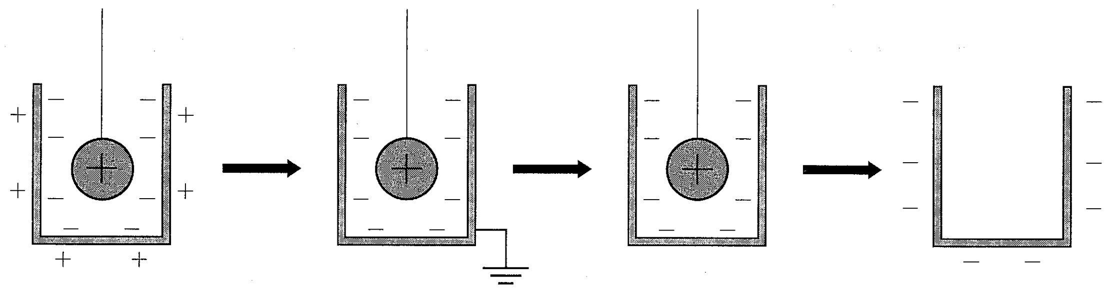
Electrostatics

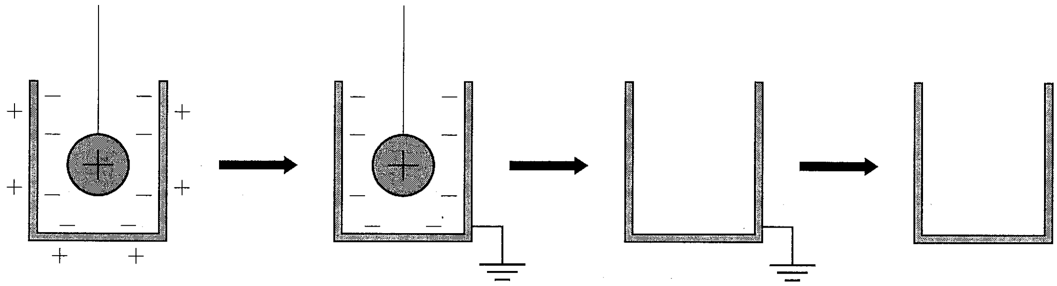
CHAPTER 18

1.

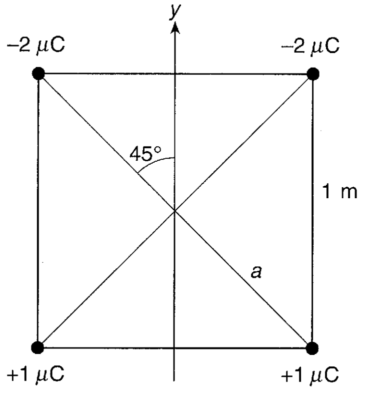


The induced charge on the can is nearly equal to that on the ball (i.e. Q) in magnitude if the opening is small.

2.



3.



$a = 1 / \sqrt{a}$ m (from the figure)

By symmetry, the electric field must be along the y-direction.

$E_y = (1 / 4\pi\varepsilon_0)[2(2 \times 10^{-6}) \cos 45° / a^2 + 2(1 \times 10^{-6}) \cos 45° / a^2]$

$= 7.63 \times 10^4 F \text{ m}^{-1}$

$U = (1 / 4\pi\varepsilon_0)[2(-2 \times 10^{-6}) / a + 2(1 \times 10^{-6}) / a]$

$= -2.54 \times 10^4$ V

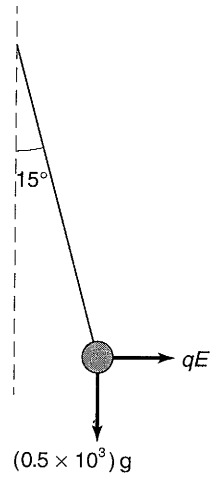
Tip: Remember that the electric field strength E is a vector while potential is a scalar. Vectorial change must be considered when calculating E.

4. (a) $(1 / 4\pi\varepsilon_0)q / r = 2 \times 10^3$

$q = 3.3 \times 10^{-9}$ C

(b) $qE / [0.5 \times 10^{-3})(10)] = \tan 15°$ (from the figure)

$E = 4 \times 10^5 F \text{ m}^{-1}$



Tip: In (a), the ball must be assumed to be far from all earthed bodies. Also, the negative terminal of the EHT is earthed.

5. (a) If Q is constant, then E remains constant.

Since $\Delta V = \int_0^d E dx = Ed$, doubling d doubles V.

(b) ΔV = constant and $\Delta V = Ed$

$\therefore E \propto 1 / d$

Doubling d decreases E by half.

6. (a) Since $E = Q / A$ and $E = V / d$, doubling V doubles E and so doubles Q.

(b) d is halved, therefore E and Q are doubled.

7. (a) $F = (1.6 \times 10^{-19})(2 \times 10^4)$

$= 3.2 \times 10^{-15}$ N

The force is towards the positively charged plate.

(b) Acceleration $= 3.2 \times 10^{-15} / 9.1 \times 10^{-31} = 3.5 \times 10^{15} \text{ m s}^{-2}$

Time required $= [2(10 \times 10^{-2}) / 3.5 \times 10^{15}]^{1/2} = 7.6 \times 10^{-9}$ s

(c) Velocity $= (3.5 \times 10^{15})(7.6 \times 10^{-9}) = 2.66 \times 10^7 \text{ m s}^{-1}$

(d) Since g/(acceleration due to the electric field) $= 2.9 \times 10^{-15}$, the effect of g is negligible.

8. Energy gained by the particle when it reaches the sphere

$= (Q_1 Q_2 / 4\pi\varepsilon_0)[1 / r_1 - 1 / r_2] = 8.09 \times 10^{-4}$ J

$\therefore$ KE of the particle $= 8.09 \times 10^{-4}$ J

Velocity of the particle $= [2(8.09 \times 10^{-4}) / (10^{-5})]^{1/2} = 12.7 \text{ m s}^{-1}$

Tip: Here we have assumed that the mass of the sphere is much larger than that of the particle.

9. (a) KE = PE loss = PE $= (-1.6 \times 10^{-19})(-1) = 1.6 \times 10^{-19}$ J

(b) KE gained $= (1.6 \times 10^{-19})(2 \times 10^3) = 3.2 \times 10^{-16}$ J

$\therefore v = \sqrt{[2(3.2 \times 10^{-16}) / (9.1 \times 10^{-31})]}$

$= 2.65 \times 10^7 \text{ m s}^{-1}$

10. Weight of the oil drop $= 4\pi(10^{-6})^3(900)(10) / 3$

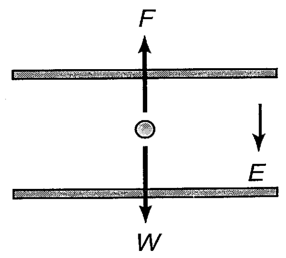
$= 3.77 \times 10^{-14}$ N

At 385 V, the electric field $= 385 / 5 \times 10^{-3}$

$= 7.7 \times 10^4 \text{ N C}^{-1}$

$\therefore$ The charge on the oil drop $= 3.77 \times 10^{-14} / 7.7 \times 10^4$

$= 4.9 \times 10^{-19}$ C



From the diagram, F is opposite to E, thus the charge is negative.

11. For a 100 V potential difference,

$E = 100 / 5 \times 10^{-3} = 2 \times 10^{4}\ \mathrm{N\ C^{-1}}$

Net force on the oil drop $= 3.77 \times 10^{-14} - (4.9 \times 10^{-19})(2 \times 10^{4})$

$= 2.8 \times 10^{-14}\ \mathrm{N}$

Acceleration $= 2.8 \times 10^{-14} / 3.77 \times 10^{-15} = 7.4\ \mathrm{m\ s^{-2}}$

$\therefore$ Final velocity $= \sqrt{[2(7.4)(5 \times 10^{-3})]}$

$= 0.272\ \mathrm{m\ s^{-1}}$

12. (a) By $mv^2 / r = (1 / 4\pi\varepsilon_0) q_p q_e / r^2$,

$v = \{(9 \times 10^{9})(1.6 \times 10^{-19})^2 / [(0.05 \times 10^{-9})(9.1 \times 10^{-31})]\}^{1/2}$

$= 2.25 \times 10^{6}\ \mathrm{m\ s^{-1}}$

$\therefore T = 2\pi r / v = 1.4 \times 10^{-16}\ \mathrm{s}$

(b) $\mathrm{KE} = (9.1 \times 10^{-31})(2.25 \times 10^{6})^2 / 2 = 2.3 \times 10^{-18}\ \mathrm{J}$

$\mathrm{PE} = -(1/4\pi\varepsilon_0) q_p q_e / r = -4.6 \times 10^{-18}\ \mathrm{J}$

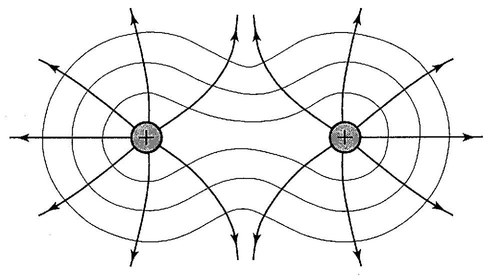
$\mathrm{TE} = -2.3 \times 10^{-18}\ \mathrm{J}$

(c) Energy required $= 2.3 \times 10^{-18}\ \mathrm{J}$

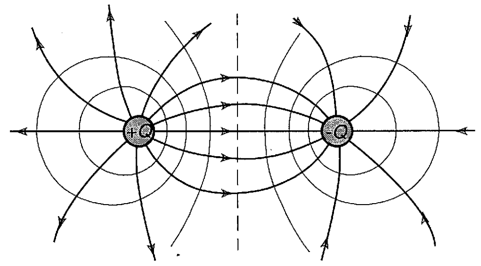
Tip: (i) The minimum kinetic energy at infinity is zero.
(ii) For a particle in a circular orbit that is acted upon by a force obeying the inverse-square-law, KE = −PE / 2 = −TE.

13. The potential at the position of the electron $= -\mathrm{PE} / q_e = +28.75\ \mathrm{V}$

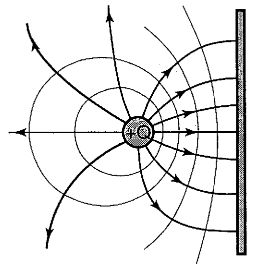
14.



15. (a)

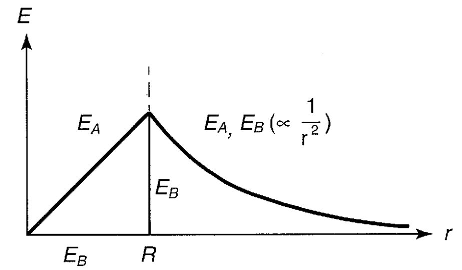


(b)

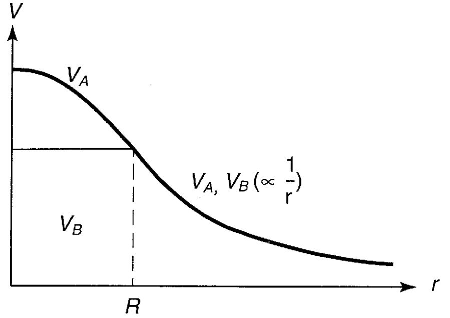


Tip: (i) Equipotentials are perpendicular to field lines.
(ii) Each equipotential should be labelled with the value of its potential. They are not labelled here because the values are not available.
(iii) The metal surface is an equipotential surface.

16. (a)



(b)



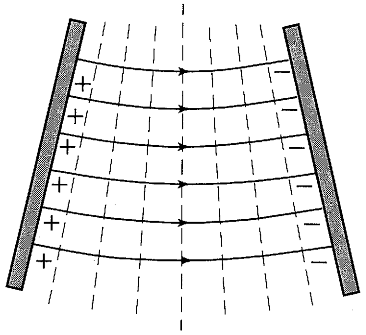
(c) The charge only resides on the surface for a metallic sphere (i.e. sphere B).

Tip: (i) The charge is assumed to be positive.

(ii)
- Both spheres behave in the same way when viewed from the outside. The graphs beyond R are the same.
- For A, $E_x \propto (\rho x^3 / x)$, $E_x = x$ for $x < R$
- For B, $E_x = 0$ for $x < R$ since there is no charge with a sphere of radius x.

(iii)
- For A, slope $= \mathrm{d}V / \mathrm{d}x$, $-E \propto -x$ for $x < R$
- For B, $E = 0$, $V =$ constant for $x < R$

17. The electric field decreases downward.



Tip: (i) Note that $E = -\Delta v / \Delta x$ is smaller at the bottom than at the top.

(ii) The electric field is not uniform across the gap.

(iii) The equipotentials are not equally spaced even along a field line.

18. (a) At A, E_Q and E_{-Q} are both $(1 / 4\pi\varepsilon_0)Q / [r^2 + (d/2)^2]$ in the direction shown.

From geometry, $\cos\alpha = (d/2) / [r^2 + (d/2)^2]^{1/2}$

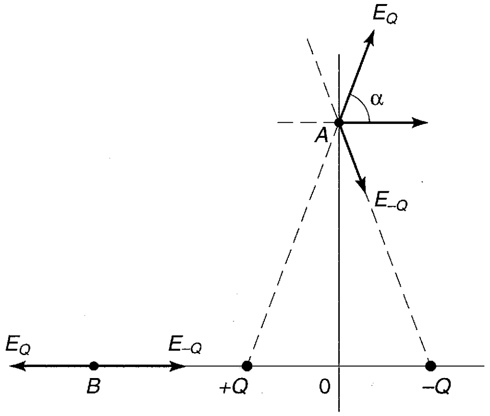
Resultant field $= (2 / 4\pi\varepsilon_0)Q / [r^2 + (d/2)^2] \cos\alpha$

$= 2Qd / [\pi\varepsilon_0(4r^2 + d^2)]^{3/2}$ to the right

At B, $E_Q = (1 / 4\pi\varepsilon_0)Q / [r - (d/2)]^2$ and

$E_{-Q} = (1 / 4\pi\varepsilon_0)Q / [r + (d/2)]^2$

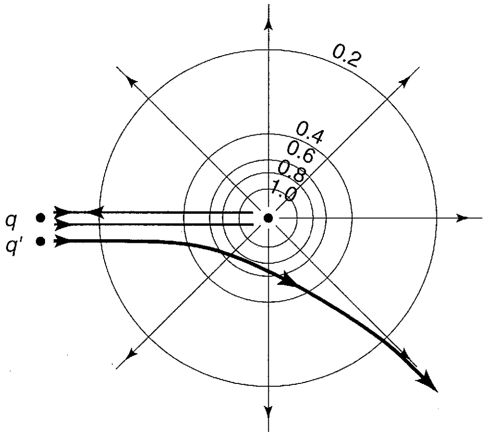
$\therefore$ Resultant field $= 8Qrd / [\pi\varepsilon_0(4r^2 - d^2)]^2$ to the left



(b) Here, $\mathrm{d}V / \mathrm{d}r = 0$ only refers to points along AO. Nothing has been said about points along other directions.

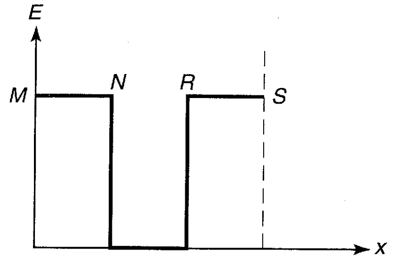
Tip: V_A obtained in Example 5 is the value of V at A. $V_A = 0$ only means that $V(x, y) = 0$ for all points along $x = 0$. It is correct to say that the component of the electric field along AO is zero and hence AO is an equipotential.

19. (a)



(b) A positive charge occurs at the centre of the circles.

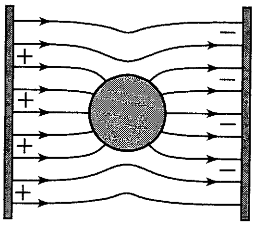
20.



Tip: (i) There are only three regions of uniform field. The fields in MN and RS are of the same intensity. The field in NR is zero.

(ii)
- Both cases result in the same sketch, except for different electric field strengths in MN and RS.
- For isolated charges, the electric field intensity is proportional to the charges.
- For a constant voltage, charges can flow into or away from the plates to keep the voltage, and thus the electric field, at the fixed value.

21. (a)



(b) When the sphere is very small, its electrical effect is negligible and the field remains uniform.

22. (a) The field in the metallic region is zero. The field inside is the same. The field outside is due to the charge Q_1 on a solid metallic sphere. The potential inside the shell is lower than it would be without the shell.

(b) The field inside is the same as if the shell is absent. The field outside and the potential outside are zero. The potential inside is lower than that without the shell, although the dependence on r is the same.

(c) The graphs are the same as in (b) since to earth the shell is to charge the shell with the amount of charge it carries but of the opposite polarity.