14 Capacitors

Self Evaluation Exercise 14.2 (p. 63)

1. B

2. D

3. The charge stored in the capacitor is:

$$Q = CV = (5 \times 10^{-6})(12) = 6 \times 10^{-5}\ \text{C} = 60\ \mu\text{C}$$

4. (a) Current is the rate of flow of charge per unit time. Therefore, the charge stored by a constant current is:

$$Q = It = (2 \times 10^{-6}) \times 20 = 4 \times 10^{-5}\ \text{C} = 40\ \mu\text{C}$$

(b) The capacitance of a capacitor is defined as the charge stored per unit voltage across it. Therefore, the capacitance is:

$$C = \frac{Q}{V} = \frac{4 \times 10^{-5}}{20} = 2 \times 10^{-6}\ \text{F} = 2\ \mu\text{F}$$

5. To create 30 V potential difference across the capacitor, certain amount of charge has to accumulate on the capacitor. The amount of charge required is:

$$Q = CV = (20 \times 10^{-6}) \times 30 = 6 \times 10^{-4}\ \text{C}$$

And the time needed to accumulate the charges by constant current of 10 mA is:

$$t = \frac{Q}{I} = \frac{6 \times 10^{-4}}{10 \times 10^{-3}} = 0.06\ \text{s}$$

Self Evaluation Exercise 14.3 (p. 68)

1. A

2. (a) For two parallel plates with air as medium between them, the capacitance is:

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}) \times \left(\frac{20}{100^2}\right)}{0.01} = 1.77 \times 10^{-12}\ \text{F} = 1.77\ \text{pF}$$

(b) For two parallel plates with dielectric of dielectric constant (ε_r) equals to 5, the capacitance becomes:

$$C' = \frac{\varepsilon_0 \varepsilon_r A}{d} = \varepsilon_r C = 5 \times (1.77 \times 10^{-12}) = 8.85 \times 10^{-12}\ \text{F} = 8.85\ \text{pF}$$

3. The area of metal strip overlapped for storage of charge is:

$$A = 0.02 \times 0.4 = 8 \times 10^{-3}\ \text{m}^2$$

The capacitance for the paper capacitor is:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{(8.85 \times 10^{-12}) \times 2 \times (8 \times 10^{-3})}{0.002 \div 100} = 7.08 \times 10^{-9}\ \text{F} = 7.08\ \text{nF}$$

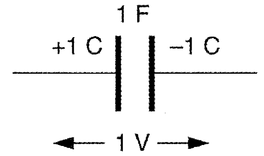
Self Evaluation Exercise 14.6 (p. 75)

1. B

2. (a) Equivalent capacitance (C) = $C_1 + C_2$ = 100 + 50 = 150 μF

(b) We have $Q = CV$.
For the 100 μF capacitor:
Charge stored = $(100 \times 10^{-6}) \times 12 = 1.2 \times 10^{-3}$ C
For the 50 μF capacitor:
Charge stored = $(50 \times 10^{-6}) \times 12 = 6.0 \times 10^{-4}$ C

3. (a) (i) The capacitance of a conductor is defined as the charge stored per unit voltage applied across the capacitor, i.e. capacitance = $\frac{Q}{V}$.
The unit for capacitance is farad (F). One farad is the capacitance of a capacitor if the charge on each of the plates of the capacitor is 1 C when a p.d. of 1 volt is applied across the capacitor.



(ii) Charges cannot flow into or out of a body made of insulating material. This means it cannot be used to store charges or supply charges when required. Thus, the concept of capacitance is inappropriate for a charged body made of insulating material.

(b) (i) The capacitance of a metal sheet increases when an earthed conductor is placed near it. The presence of the earthed conductor reduces the electric potential V of the metal sheet. Hence, from the equation:

$$C = \frac{Q}{V}$$

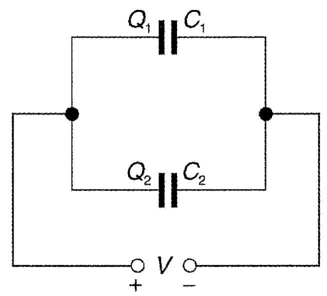
when V decreases, the capacitance increases.

(ii) The insertion of a dielectric between the metal sheets further increases the capacitance. From the equation:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

where A = area of the metal sheets,
d = separation between the metal sheets,
ε_r = relative permittivity of the dielectric.
Since $\varepsilon_r > 1$, the capacitance C of the capacitor increases.

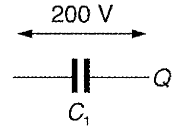
(c)

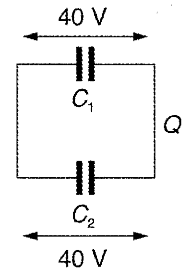


The p.d. across C_1 and C_2 are the same, i.e. V.
Charge on C_1 (Q_1) = $C_1 V$
Charge on C_2 (Q_2) = $C_2 V$

$$\therefore \frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{C_1}{C_2}$$

4. By conservation of charge, the amount of charge stored in the first capacitor is equal to that stored in first and second capacitors after connection.
Since they are connected in parallel, the voltages across the first and second capacitors are the same (40 V). And the equivalent capacitance is equal to the sum of the two capacitances.





$$Q = C_1 V_1 = (C_1 + C_2)\,V_2$$
$$(2 \times 10^{-6}) \times 200 = (2 \times 10^{-6} + C_2) \times 40$$
$$C_2 = 8 \times 10^{-6}\ \text{F} = 8\ \mu\text{F}$$

Self Evaluation Exercise 14.7 (p. 79)

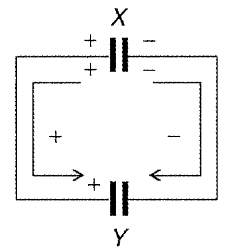
1. A
Since the equivalent capacitance is less than the original one, the additional capacitor has to be connected in series with the original capacitor.
Choice B, D are incorrect.
Choice A (correct)

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{2}$$
$$C = 1\ \mu\text{F}$$

Choice C (incorrect)

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{0.5}$$
$$C = 0.4\ \mu\text{F}$$

2. D
Because the system is closed, after they are connected, the total charge in the system must be unchanged.
And when the capacitor Y is connected to the capacitor X, charge on capacitor X redistributes and a certain amount of charge moves to capacitor Y according to the capacitances.



The p.d. across X is $V = \frac{Q}{C}$.

After connection, same charge moves to capacitor Y, the charge stored on capacitor X decreases, therefore p.d. across capacitor X also decreases.

$$V = \frac{Q}{C}$$

Capacitance is constant, $V \propto Q$, if $Q \downarrow$, $V \downarrow$
Therefore, the total charge on both capacitors remains unchanged but the p.d. across X decreases.

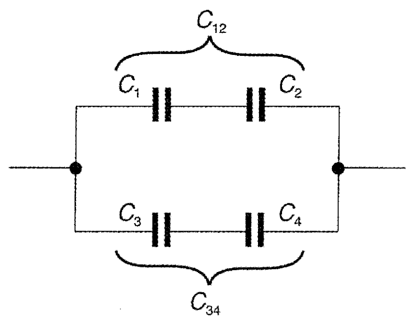
3. (a) The capacitors are connected in series. Therefore, the equivalent capacitance is:

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}$$
$$=\frac{1}{2\times10^{-6}}+\frac{1}{2\times10^{-6}}+\frac{1}{2\times10^{-6}}+\frac{1}{2\times10^{-6}}$$
$$=\frac{4}{2\times10^{-6}}$$
$$C=5\times10^{-7}\text{ F}=0.5\ \mu\text{F}$$

(b) The capacitors are connected in parallel, the equivalent capacitance is:

$$C=C_1+C_2+C_3+C_4$$
$$=2\times10^{-6}+2\times10^{-6}+2\times10^{-6}+2\times10^{-6}$$
$$=4(2\times10^{-6})$$
$$=8\times10^{-6}\text{ F}$$
$$=8\ \mu\text{F}$$

(c) The arrangement is equivalent to two capacitors C_{12} and C_{34} connected in parallel.

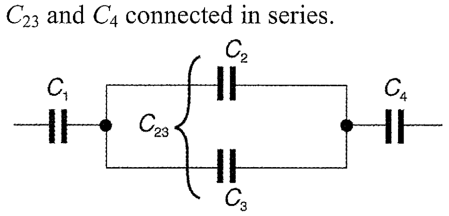


$$C_{12}=C_{34}$$
$$\frac{1}{C_{12}}=\frac{1}{C_{34}}=\left(\frac{1}{2\times10^{-6}}+\frac{1}{2\times10^{-6}}\right)$$
$$C_{12}=C_{34}=1\times10^{-6}\text{ F}$$
$$=1\ \mu\text{F}$$

∴ The equivalent capacitance is:

$$C=C_{12}+C_{34}=1\times10^{-6}+1\times10^{-6}$$
$$=2\times10^{-6}\text{ F}$$
$$=2\ \mu\text{F}$$

(d) The arrangement is equivalent to 3 capacitors, C_1, C_{23} and C_4 connected in series.

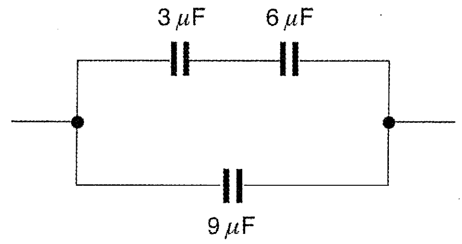


$$C_{23}=C_2+C_3$$
$$=2\times10^{-6}+2\times10^{-6}$$
$$=4\times10^{-6}\text{ F}=4\ \mu\text{F}$$

The equivalent capacitance is:

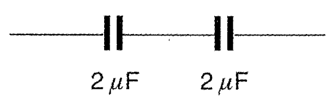
$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_{23}}+\frac{1}{C_4}$$
$$=\frac{1}{2\times10^{-6}}+\frac{1}{4\times10^{-6}}+\frac{1}{2\times10^{-6}}$$
$$=1.25\times10^{6}$$
$$C=0.8\times10^{-6}\text{ F}=0.8\ \mu\text{F}$$

4.



$$C=9+\left(\frac{1}{3}+\frac{1}{6}\right)^{-1}$$
$$=9+2$$
$$=11\ \mu\text{F}$$

5. (a)

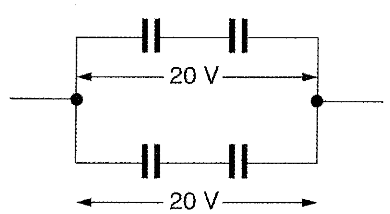


For two identical capacitors connected in series, the equivalent capacitance is halved.

$$C=1\ \mu\text{F}$$

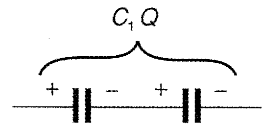
And the charge stored in each capacitor is the same. As the capacitor has the same capacitance, the voltage across each capacitor is 10 V and the total voltage is the sum of two voltages, which is 20 V.

(b)



The arrangement is equivalent to two series of capacitors connected in parallel. As we calculated in the above part, each series of capacitors has equivalent capacitance of 1 μF. These two series are connected in parallel, the equivalent capacitance of the whole system is 1 μF + 1 μF = 2 μF.
And the voltage across the capacitors connected in parallel is the same. As we calculated before, the upper series and lower series both use up 20 V. As they are connected in parallel, the equivalent voltage across the whole system is 20 V.

6. (a)



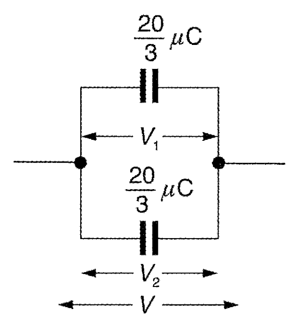
The equivalent capacitance is:

$$\frac{1}{C}=\frac{1}{0.10\times10^{-6}}+\frac{1}{0.20\times10^{-6}}$$
$$C=\frac{1}{15}\ \mu\text{F}$$

As the capacitors are connected in series, the charge stored on each capacitor is the same.

$$Q=CV$$
$$=\left(\frac{1}{15}\times10^{-6}\right)\times100$$
$$=\frac{20}{3}\ \mu\text{C}$$

(b)



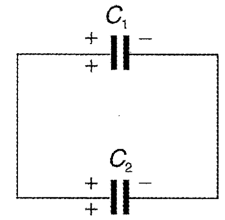
After they are connected in parallel, they store 20/3 μC.
Because they are connected in parallel, the voltage across each capacitor is the same. Besides, the equivalent capacitance is equal to the addition of capacitance of two capacitors,
$C=0.10\ \mu\text{F}+0.20\ \mu\text{F}=0.3\ \mu\text{F}$.
And by conservation of charge, the total charge stored on the equivalent capacitor is 2 × 20/3 μC.
Therefore, the potential difference across the capacitors is:

$$V=\frac{Q}{C}$$
$$=\frac{2\times\frac{20}{3}\times10^{-6}}{0.30\times10^{-6}}$$
$$=44.4\text{ V}$$

Self Evaluation Exercise 14.9 (p. 86)

1. D

If the two positive plates are connected and the two negative plates are connected, there will be no charge cancellation. The total charge remains unchanged and only redistribution of charge occurs. The connection is in parallel.



There is redistribution of charge to make the p.d. across each capacitor equal to each other. When there is a flow of charge, energy is required to overcome the repulsion of like charge and resistance of wire.
Therefore, there is a decrease in energy but the charge remains unchanged.

2. B

3. C

4. (a) The energy stored in the capacitor is:

$$E_c=\frac{1}{2}CV^2$$
$$=\frac{1}{2}(10.0\times10^{-6})(500)^2$$
$$=1.25\text{ J}$$

(b) To charge a capacitor to a p.d. of 500 V, the e.m.f. of battery should be 500 V. The charge supplied by the battery is equal to the charge stored on the capacitor, which is:

$$Q=CV$$
$$=(10.0\times10^{-6})\times500$$
$$=5\times10^{-3}\text{ C}$$

(c) The energy provided by the battery is:

$$E_b=QV$$
$$=(5\times10^{3})\times500$$
$$=2.5\text{ J}$$

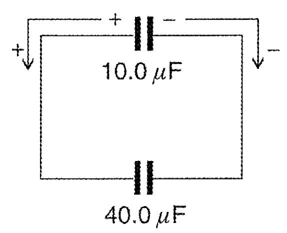
(d) The total heat dissipated in the resistance of the connecting wire and battery is equal to the difference between energy stored in capacitor and the energy provided by battery.

$$E_h=E_b-E_c$$
$$=2.5-1.25$$
$$=1.25\text{ J}$$

5. (a) The charge stored on the 10.0 μF capacitor is:

$$Q=CV$$
$$=(10.0\times10^{-6})\times500$$
$$=5\times10^{-3}\text{ C}$$

When it is connected in parallel with a 40.0 μF capacitor which is initially uncharged, there is redistribution of charge from 10.0 μF capacitor to 40.0 μF.



The equivalent capacitance is:
$C = 10.0\ \mu F + 40.0\ \mu F$
$= 50.0\ \mu F$
The energy stored in capacitors before connection is:

$$\frac{1}{2}CV^2$$
$$= \frac{1}{2} \times (10.0 \times 10^{-6}) \times (500)^2 = 1.25\ J$$

(b) In the redistribution of charge, the p.d. across capacitor changes, so energy cannot be calculated by equation $\frac{1}{2}CV^2$,
Only when the charge remains unchanged, the total energy stored in the capacitor after connection is:

$$\frac{1}{2} \cdot \frac{Q^2}{C}$$
$$= \frac{1}{2} \cdot \frac{(5\times10^{-3})^2}{50.0\times10^{-6}}$$
$$= 0.25\ J$$

Although the total energy stored in capacitors before and after connection are different, the energy in the process still conserved. The difference in energy, 1.00 J, is dissipated as heat in connecting wires because wires has resistance.

6. (a) Because the capacitor is charged by connecting it to a battery of e.m.f. 200 V, the final p.d. across the capacitor is also 200 V.
The charge on the capacitor is:
$Q = CV$
$= (5 \times 10^{-10}) \times 200$
$= 1 \times 10^{-7}$ C

(b) (i) As there is no charge loss, the amount of charge remains unchanged. Therefore, by conservation of charge, the p.d. across the capacitor is:
$Q = CV = C'V'$
$1 \times 10^{-7} = 1 \times 10^{-10}\ V'$
$V' = 1\ 000$ V

(ii) The work done against electric field is the difference between energy stored before and after the adjustment.

$$W = \frac{1}{2}CV^2 - \frac{1}{2}C'V'^2$$
$$= \frac{1}{2}(5 \times 10^{-10} \times 200^2 - 1 \times 10^{-10} \times 1\ 000^2)$$
$$= 4 \times 10^{-5}\ J$$

7. (a) When switch S is closed, the two capacitors are being charged. They are connected in parallel, therefore the p.d. across them are the same, and it is equal to V_0. The total energy stored is:

$$\frac{1}{2}C_0V_0^2 + \frac{1}{2}C_0V_0^2$$
$$= C_0V_0^2$$

(b) (i) When the switch is opened, the two capacitors are isolated from the battery. Thus, the total charge on the capacitors remains unchanged. Therefore, the resulting p.d. V' across the capacitors is:

$$Q = 2C_0V_0 = \left(C_0 + \frac{1}{4}C_0\right)V'$$
$$2C_0V_0 = \frac{5}{4}C_0V'$$
$$V' = \frac{8}{5}V_0$$
$$= 1.60\ V_0$$

(ii) The work done is equal to the difference in energy stored before and after the adjustment.
Before reducing capacitance:

$$\text{Energy} = \frac{1}{2}CV^2 = \frac{1}{2}(2C_0)V_0^2 = C_0V_0^2$$

After reducing capacitance:

$$\text{Energy} = \frac{1}{2}C'V'^2$$
$$= \frac{1}{2}\left(\frac{5}{4}C_0\right)\left(\frac{8}{5}V_0\right)^2$$
$$= 1.6\ C_0V_0^2$$

The work done is:
$W = 1.6\ C_0V_0^2 - C_0V_0^2$
$= 0.6\ C_0V_0^2$

8. (a) (i) Charge stored on one plate of the capacitor,
$Q = CV$
$= (200 \times 10^{-6})(30) = 0.006\ 0$ C

(ii) Energy stored by the capacitor,

$$E = \frac{1}{2}QV = \frac{1}{2}(0.006\ 0)(30) = 0.090\ J$$

(b) (i) The total charge on the capacitors will have the same value before and after connection.
(ii) The p.d. across each capacitor will be the same after the connection.

(c) Total capacitance of capacitors in parallel
$C = 200 \times 10^{-6} + 100 \times 10^{-6} = 300 \times 10^{-6}$ F
P.d. across capacitors,

$$V = \frac{Q}{C} = \frac{0.006\ 0}{300\times10^{-6}} = 20\ V$$

Total energy stored,

$$E = \frac{1}{2}QV = \frac{1}{2}(0.006\ 0)(20) = 0.060\ J$$

9. (a) The electric potential at a point in an electric field is defined as the work done in bringing a unit positive charge from infinity to the point.

(b) (i) $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

(ii) $C = \dfrac{Q}{V}$

(iii) Sub **(i)** into **(ii)**:

$$C = Q\frac{4\pi\varepsilon_0 r}{Q} = 4\pi\varepsilon_0 r$$

(c) (i) Capacitance of the sphere,
$C = 4\pi\varepsilon_0 r = 4\pi(8.85 \times 10^{-12})(0.15)$
$= 1.67 \times 10^{-5}\ \mu F$

(ii) Energy stored on the sphere,

$$E = \frac{1}{2} \cdot \frac{Q^2}{C}$$
$$= \frac{1}{2} \cdot \frac{(2.0\times10^{-6})^2}{1.668\times10^{-11}}$$
$$= 0.120\ J$$

Self Evaluation Exercise 14.10 (p. 91)

1. D
At the beginning of charging, the capacitor has no charge on it. It can be treated as a conductor. Thus, the current in the circuit is the highest: $I_0 = \dfrac{V_0}{R}$.
And as time goes by, charge accumulates on the capacitor. Potential is built up across capacitor and against the e.m.f. of the battery. Therefore, current starts to decrease as follows:

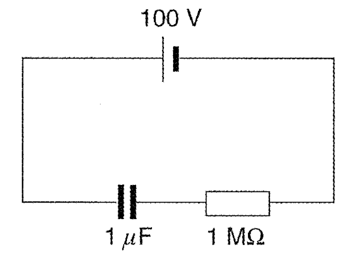
$$I = I_0\ e^{-\frac{t}{CR}}$$

It is an exponential decay. Only graph of choice D shows this characteristics.

2. B
When the switch is closed, we can treat the capacitor as a conductor and the initial current in the circuit is:

$$I_0 = \frac{V_0}{R}$$
$$= \frac{100}{25\times10^3}$$
$$= 4 \times 10^{-3}\ A$$
$$= 4\ mA$$

3. C



Just after the switch is closed, we can treat the capacitor as a part of wire. So the current is:

$$I = \frac{V}{R}$$
$$= \frac{100}{1\times10^6} = 1 \times 10^{-4}\ A$$

4. B
The time constant is equal to:
$\tau = CR$
And the equivalent resistance

$$\frac{1}{R} = \left(\frac{1}{3\times10^6} + \frac{1}{3\times10^6}\right)$$
$$R = 2.1 \times 10^6\ \Omega$$

Therefore, the time constant is:
$\tau = (5 \times 10^{-6}) \times (2.1 \times 10^6)$
$= 10.5$ s

5. B
The capacitor is charged by a constant current. Thus, the charge on the capacitor with respect to time is equal to
$Q = It$
(not $Q = Q_0\left(1 - e^{-\frac{t}{CR}}\right)$, because the e.m.f. of battery is not fixed, it increases with time to keep constant current in the circuit.)
To make the p.d. across the capacitor reach 200 V, the charge on capacitor is:
$Q = CV$
$= (20 \times 10^{-6}) \times 300 = 6 \times 10^{-3}$ C
Therefore, the time taken is:

$$t = \frac{Q}{I} = \frac{6\times10^{-3}}{10\times10^{-3}}$$
$$= 0.6\ s$$

6. C

7. A

8. D

9. **(a)** **(i)** 'A capacitance of 100 μF' meant that a charge of 100 μC will be stored per unit volt of p.d. applied across the capacitor.

(ii) Charge stored with p.d. of 20 V

$$Q = CV$$
$$= (100 \times 10^{-6})(20) = 0.002\ 0\ \text{C}$$

(iii) Maximum charge which may be stored
= charge stored at marked 20 V
= 0.002 0 C

(iv) Energy stored in the capacitor

$$E = \frac{1}{2}QV$$
$$= \frac{1}{2}(0.002\ 0)(20) = 0.020\ \text{J}$$

(b) If a p.d. greater than the marked voltage is applied, insulation between the plates may breakdown and allow current to flow.

(c) **(i)** Immediately after the switch is closed, voltage across capacitor is zero, and
V_{out} = supply voltage = 6 V

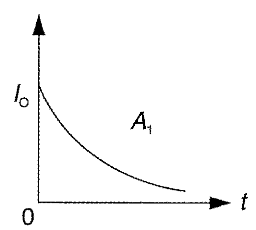
(ii) After a long time, the capacitor would be fully charged, current would be zero, and $V_{out} = 0$.

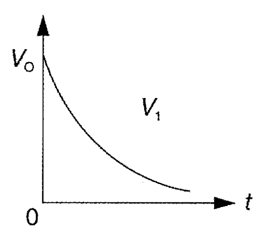
Self Evaluation Exercise 14.11 (p. 101)

1. D

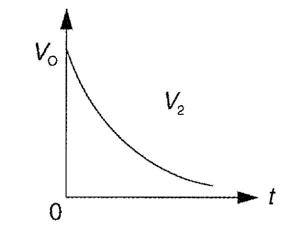
The reading θ is proportional to the magnitude of parameter it measured.
In Circuit 1, the capacitor is initially charged. When the switch is closed, the capacitor starts to discharge. $\mathbf{A}_1$ measures the current in the circuit, $\mathbf{V}_1$ measures the voltage across the capacitor, $\mathbf{V}_2$ measures the voltage across the resistor.
For discharging through a constant resistor:

$$I = I_0\, e^{-\frac{t}{CR}} \qquad V_c = V_0 e^{-\frac{t}{CR}}$$



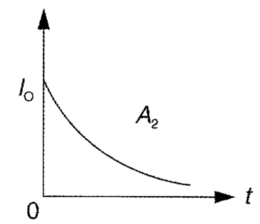


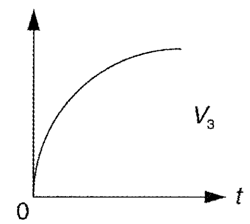
$$V_R = IR$$
$$= I_0 R\, e^{-\frac{t}{CR}}$$
$$= V_0 e^{-\frac{t}{CR}}$$



In circuit 2, the capacitor is being charged, $\mathbf{A}_2$ measures current in the circuit, V_2 measures voltage across capacitor. For charging via a constant resistance:

$$I = I_0\, e^{-\frac{t}{CR}} \qquad V_c = V_0(1 - e^{-\frac{t}{CR}})$$





Therefore, only the graph for $\mathbf{V}_3$ matches the graph shown in question.

2. D

The time constant of a circuit is the time equal to CR.
And when a capacitor discharges, the charge Q on the capacitor follows:

$$Q = Q_0\, e^{-\frac{t}{CR}}$$

When one time constant passes, $Q = Q_0\, e^{-\frac{CR}{CR}} = Q_0 e^{-1}$

$$Q = Q_0 \cdot \frac{1}{e}$$

Therefore, the time constant is the time after $\frac{1}{e}$ of initial charge is left on the capacitor.

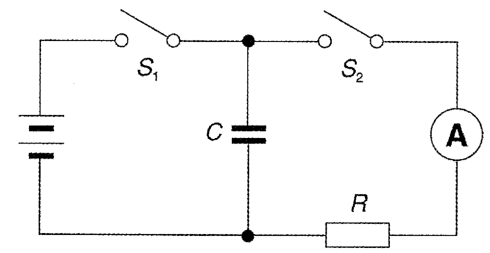
3. A

The time taken for p.d. across a charged capacitor to decrease from V_0 to $\frac{V_0}{e^2}$ is two time constants.

$$V = V_0 e^{-\frac{t}{CR}}$$
$$V_0 e^{-2} = V_0 e^{-\frac{t}{CR}}$$
$$\therefore \quad \frac{t}{CR} = 2$$
$$t = 2CR$$
$$10 = (2 \times 2 \times 10^6)\, C$$
$$C = 2.5 \times 10^{-6}\ \text{F}$$
$$= 2.5\ \mu\text{F}$$

4. A

5. Time constant of a circuit is the time needed for the current to fall from I_0 to $\frac{I_0}{e}$ (0.369 I_0) during discharging. Therefore, by measuring current with respect to time, we can find the time constant.
First we set up the apparatus as follows:



Second, close switch S_1 to charge the capacitor. Then open S_1 and close S_2 for discharging. Because the time constant is $CR = (10 \times 10^{-6}) \times (10 \times 10^6) = 100$ s. We choose to record the ammeter reading every 15 s.
Finally, plot the graph of I against t. The time constant is the time for I to reach 0.369 I_0.

Review Exercise 14 (p. 105)

A. Structured Questions

1. The capacitance of a plate capacitor is:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$
$$= \frac{(2.5)(8.85 \times 10^{-12})(0.02 \times 0.4)}{0.003 \times 10^{-2}}$$
$$= 5.9 \times 10^{-9}\ \text{F}$$

2. The energy provided by discharge of capacitor is:
Energy = Mean power × Time
= 2 000 × 0.040
= 80 J
And it is equal to the energy stored in the capacitor.

$$\text{Energy} = \frac{1}{2}CV^2$$
$$80 = \frac{1}{2}C(1\ 000)^2$$
$$C = 1.6 \times 10^{-4}\ \text{F}$$
$$= 160\ \mu\text{F}$$

3. **(a)** **(i)** The charge stored on the capacitor is:

$$Q = CV$$
$$= (3.0 \times 10^{-6}) \times 800$$
$$= 2.4 \times 10^{-3}\ \text{C}$$

The energy supplied by the battery is the total energy of charge Q.

$$\text{Energy} = QV$$
$$= (2.4 \times 10^{-3}) \times 800$$
$$= 1.92\ \text{J}$$

(ii) The energy stored in the capacitor is:

$$\text{Energy} = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(3.0 \times 10^{-6})(800)^2$$
$$= 0.96\ \text{J}$$

(iii) The difference in energy is:

$$\Delta E = 1.92 - 0.96$$
$$= 0.96\ \text{J}$$

It is used to overcome the resistance of the wire when a current flows through the wire. The energy is dissipated as heat in the wire.

(b) **(i)** The charge still stored on the capacitor at 200 V is:

$$Q = CV$$
$$= (3.0 \times 10^{-6}) \times 200$$
$$= 6 \times 10^{-4}\ \text{C}$$

And the charge flows through the tube is equal to the net charge stored at 800 V and 200 V.

$$\Delta Q = 2.4 \times 10^{-3} - 6 \times 10^{-4}$$
$$= 1.8 \times 10^{-3}\ \text{C}$$

(ii) The energy stored in capacitors at 200 V is:

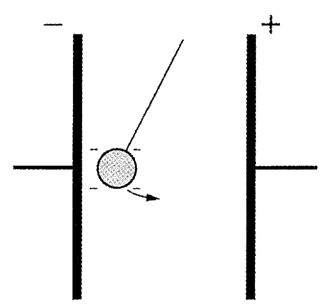
$$\text{Energy} = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(3.0 \times 10^{-6})(200)^2$$
$$= 0.06\ \text{J}$$

Therefore, the energy dissipated by the capacitor is equal to the difference of energy before and after connecting to the tube.

$$\Delta\text{Energy} = 0.96 - 0.06$$
$$= 0.90\ \text{J}$$

4. For each swing of the bob, 10% of the total charge remains on the plates of the capacitor.
Therefore, after five swings, the charge on capacitor is:

$$Q = (0.9)^5\, Q_0$$
$$= 0.590\ 49\ Q_0 \quad (Q_0 - \text{initial total charge on capacitor})$$



The p.d. across the capacitor after five swings is:

$$V = \frac{Q}{C} = \frac{0.590\,49 Q_0}{C} \qquad \left(\frac{Q_0}{C} = V_0 = 20\text{ V}\right)$$

$= 0.590\,49\ V_0$
$= 0.590\,49 \times 20$
$= 11.809\,8$ V

5. **(a)** Assume that the capacitor is fully charged, the p.d. across the capacitor should be equal to the e.m.f. of battery, which is 10 V.

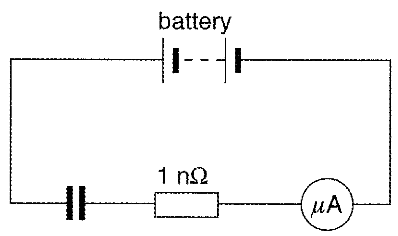
$Q = CV$
$= (2 \times 10^{-6}) \times 10$
$= 2 \times 10^{-5}$ C
$= 20\ \mu$C

(b) This is discharging via a constant resistor. The charge on the capacitor follows this equation. After 8 s,

$$Q = Q_0 e^{-\frac{t}{CR}}$$

$= (2 \times 10^{-5}) \times e^{-\frac{8}{(2\times10^{-6})\times(2\times10^{6})}}$
$= 2.71 \times 10^{-6}$ C
$= 2.71\ \mu$C

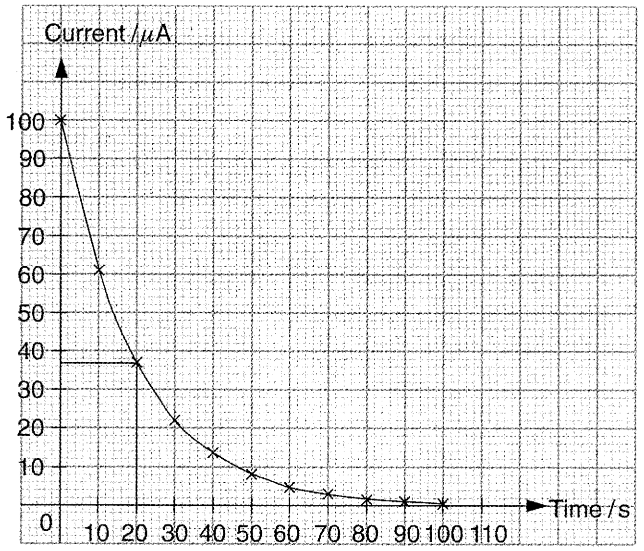
6. **(a)**



At time = 0, the capacitor can be treated as part of the wire. The p.d. of the battery is:

$V = IR$
$= (100 \times 10^{-6}) \times (1 \times 10^{6})$
$= 100$ V

(b)



Time constant τ is the time taken for current in the circuit to fall from I_0 (100 μA) to $\frac{1}{e} I_0$ (0.369 I_0)

$= 36.9\ \mu$A

From the graph, the time is about 20 s.

As $\tau = CR$
$20 = C(1 \times 10^{6})$
$C = 2 \times 10^{-5}$ F
$= 20\ \mu$F

7. Because the current was kept constant, the charge passed through the circuit is:

$Q = It$
$= (40 \times 10^{-6}) \times 40$
$= 1.6 \times 10^{-3}$ C

And it is equal to the charge left from the capacitor. The p.d. across the capacitor fell is then:

$$V = \frac{Q}{C} = \frac{1.6\times10^{-3}}{10\times10^{-6}} = 160\text{ V}$$

8. **(a)** **(i)** The capacitance of a capacitor is the charge Q stored on the capacitor per unit voltage across it.

$$C = \frac{Q}{V}$$

(ii) The capacitance of a conductor is the ratio of charge Q on the conductor to the potential of the conductor.

$$C = \frac{Q}{V}$$

(b) The capacitance is:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

(c) The resistivity of an insulator is ρ, then the resistance is:

$$R = \rho\frac{\ell}{A}$$

The time constant is defined as $\tau = CR$. Because the process is the self-discharge of the capacitor, the components of time constant are the resistance and capacitance of the capacitor. Thus, area A in the two terms are the same and $\ell = d$,

$$\therefore \quad CR = \frac{\varepsilon_0 \varepsilon_r A}{d} \cdot \left(\rho \cdot \frac{\ell}{A}\right) = \varepsilon_0\ \varepsilon_r \rho$$

(d) After disconnection, the paper capacitor would undergo self-discharge. By the expression obtained in part **(c)**, the time constant for paper conductor is:

$\tau = \varepsilon_0\ \varepsilon_r \rho$
$= (8.85 \times 10^{-12}) \times 3.7 \times (1.0 \times 10^{10})$
$= 0.327\,45$ s

If the time equals a time constant, charge decreases from initial value of Q_0 to $\frac{1}{e} Q_0$, which is equal to $0.369\ Q_0$.

(i) After 0.33 s,

$t = \tau$

$\therefore$ The fraction of charge remains on the capacitor is 0.369.

(ii) After 2.0 s,

$Q = Q_0\ e^{-\frac{t}{\tau}}$
$= Q_0\ e^{-\frac{2}{0.33}}$
$= 2.33 \times 10^{-3}\ Q_0$

$\therefore$ The fraction of charge remained is 2.33×10^{-3}.

9. A gold leaf electroscope can be used to compare the potential of different systems. The deflected angle θ is proportional to the potential. So θ increases if potential increases.

(a) If the plates are placed closer, the capacitance of the capacitor increases:

$$C = \frac{\varepsilon_0 A}{d}$$

As $d\downarrow$, $C\uparrow$

And the charge on the plates of capacitor remains unchanged. So the voltage across the capacitor decreases as capacitance increases.

$$V = \frac{Q}{C}$$

As $C\uparrow$, $V\downarrow$

Therefore, the deflected angle θ decreases.

(b) When a piece of insulator is inserted between the plates, the permittivity of the medium between the plates increases. And

$$C' = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

As $\varepsilon > \varepsilon_0$, $C' > C$
$\Rightarrow C\uparrow$

The charge on the capacitor remains unchanged. As the capacitance increases, the voltage across the capacitance decreases.

$$V = \frac{Q}{C}$$

As $C\uparrow$, $V\downarrow$

Therefore, the deflected angle θ decreases.

(c) When a metal plate is inserted between the plates but not touching any plate, it will be served as part of the capacitor.

The inserted metal plate has negligible thickness, so the capacitance after insertion is:

$$C'' = \frac{\varepsilon_0 (2A)}{d} = \frac{2\varepsilon_0 A}{d} = 2C$$

$C'' > C$

The charge on the capacitor remains unchanged. Then, the voltage across the capacitor decreases as the capacitance increases.

$$V = \frac{Q}{C}$$

As $C\uparrow$, $V\downarrow$

Therefore, the deflected angle θ decreases.

10. **(a)** **(i)** Capacitance is a measure of how much charge must be put on the capacitor to produce a certain potential difference across it. In the other words, the capacitance of a capacitor is the charge stored per unit voltage across it.

$$C = \frac{Q}{V}$$

(ii) Farad is the SI unit of capacitance. It is coulomb per volt.

(iii) Dielectric constant is a characteristic of material. If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance is increased by a factor ε_r, it is the dielectric constant.

(b) The energy dE stored in a capacitor by storing a small amount of charge dq on the plates of a capacitor at voltage V is: $dE = Vdq$

And the p.d. V is $\frac{q}{C}$

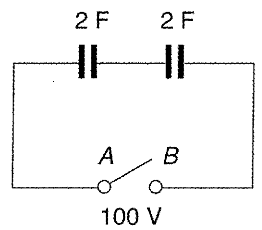
$\therefore \quad dE = \frac{q}{C}\,dq$

Total energy E stored by total of Q charge is:

$$E = \int dE = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\cdot\frac{Q^2}{C}$$

(c) **(i) & (ii)** Because there are four 100 V batteries, we can verify the e.m.f. of batteries to make the p.d. across A and B be the value we want.
We only need to consider the combination of capacitors to make equivalent capacitance as we want and prevent the voltage across capacitor to exceed its max.

1.



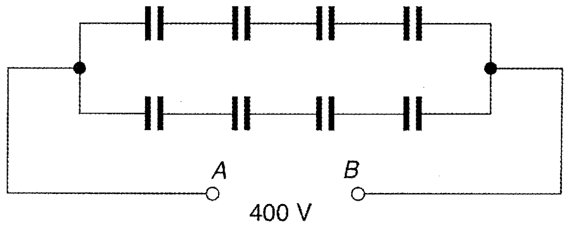
The equivalent capacitance is:

$$\frac{1}{C} = \left(\frac{1}{2}+\frac{1}{2}\right) = 1\text{ F}$$

$$\text{Energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}(1)(100)^2 = 5\,000\text{ J}$$

Voltage across each capacitor is 50 V.

2.



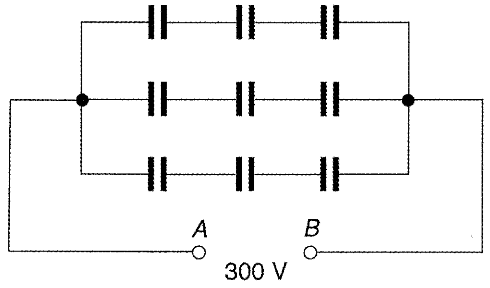
The equivalent capacitance is:

$$C = 2\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)^{-1} = 1\text{ F}$$

$$\text{Energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}(1)(400)^2 = 8\times10^4\text{ J}$$

Voltage across each capacitor is 100 V.

3.



The equivalent capacitance is:

$$C = 3\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)^{-1} = 2\text{ F}$$

$$\text{Energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}(2)(300)^2 = 9\times10^4\text{ J}$$

Voltage across each capacitor is 100 V.

11. **(a)** The total energy stored in the 10 capacitors is:

$$\text{Energy} = 10\times\frac{1}{2}CV^2$$

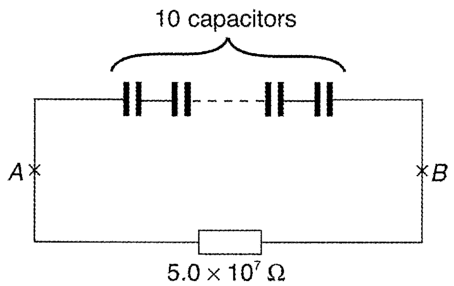
$$= 10\times\frac{1}{2}\times(500\times10^{-6})\times(150)^2$$

$$= 56.25\text{ J}$$

The total charge supplied by the battery is equal to the total charge stored on the 10 capacitors.

$Q = 10\,CV$
$= 10\times(500\times10^{-6})\times150$
$= 0.75\text{ C}$

(b)



The p.d. across each capacitor is 150 V. As the 10 capacitors are connected in series, the p.d. across A and B is the sum of p.d. across the 10 capacitors.

$V = 10\times150 = 1\,500\text{ V}$

The equivalent capacitance is:

$$C = \left(10\times\frac{1}{500\times10^{-6}}\right)^{-1}$$

$$= 5\times10^{-5}\text{ F}$$

(i) The initial current is simply:

$$I = \frac{V}{R} = \frac{1\,500}{5.0\times10^7} = 3\times10^{-5}\text{ A}$$

(ii) The total charge through the resistor is equal to the total charge stored on the capacitors. Every charge passes through the resistor to discharge.
Total charge through resistor is:

$Q = CV$
$= (5\times10^{-5})\times1500$
$= 0.075\text{ C}$
$= 7.5\times10^{-2}\text{ C}$

(iii) By the conservation of energy, the total heat produced in the resistor is equal to the energy stored in the 10 capacitors.
The total heat is:

$$\text{Energy} = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(5\times10^{-5})(1\,500)^2$$

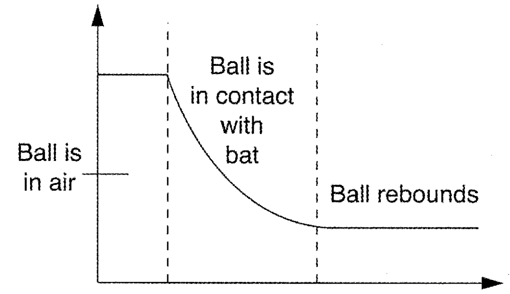
$$= 56.25\text{ J}$$

(iv) The energy is conserved in this case. The electric energy stored in the electric field between the plates of capacitors is converted into heat. It is because there is resistance, when a current passes through, work done is needed to overcome resistance and heat is then produced.

12. **(a)** The trace shown on screen of CRO represents the p.d. across the capacitor.
When the switch S is opened and the ball is released, the ball is moving towards the bat but is still in the air. The circuit is not closed, therefore no discharge occurs. The p.d. remains unchanged and is at 6.0 V. This explains the shape of the first part of the trace.
When the ball is in contact with the bat, the circuit is closed. The capacitor discharges through the resistor. The p.d. across the capacitor follows:

$$V = V_0 e^{-\frac{t}{CR}}$$

The p.d. decays exponentially with time. This explains the exponential decay curve of the trace. When the ball rebounces, the circuit is opened again. The discharge stops and therefore the p.d. across the capacitor remains uncharged.



(b) **(i)** In the initial stage, the reading of CRO is 6 divisions. As the sensitivity is 1.0 V per division, the initial p.d. across the capacitor $= 6\times1.0 = 6\text{ V}$
Similarly, in the final stage, the reading is 1.75 divisions. Therefore, the final p.d. across the capacitor is 1.75 V.
The rate is:

$$\frac{1.75}{6.00} = 0.292$$

(ii) By the relation between initial p.d. and final p.d. across the capacitor during discharge, we can calculate the time of contact as follows:

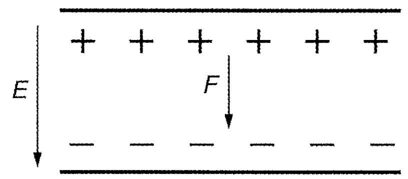
$$V = V_0\,e^{-\frac{t}{CR}}$$

$$1.75 = 6\,e^{-\frac{t}{(1.2\times10^{-6})\times(2.0\times10^3)}}$$

$$\ln 0.292 = -\frac{t}{2.4\times10^{-3}}$$

$$t = 2.96\times10^{-3}\text{ s}$$

13. **(a)** When the potential is increased, the top plate is charged positively and the bottom plate is charged negatively.



There is electric field between the two plates. And the attractive force between positive charges and negative charges decreases the separation between the plates.

(b) By the definition of capacitance:

$$C = \frac{Q}{V}$$

$$Q = CV$$

And the capacitance of the parallel plates capacitor is equal to:

$$C = \frac{\varepsilon_0 A}{d}$$

Therefore, the force F is:

$$F = \frac{QV}{2d}$$
$$= \frac{CV^2}{2d} \qquad (Q = CV)$$
$$= \frac{\varepsilon_0 A V^2}{2d} \qquad (C = \frac{\varepsilon_0 A}{d})$$

Because the copper plate is held by four springs, the attraction force is shared by four springs. The force acting on each spring is:

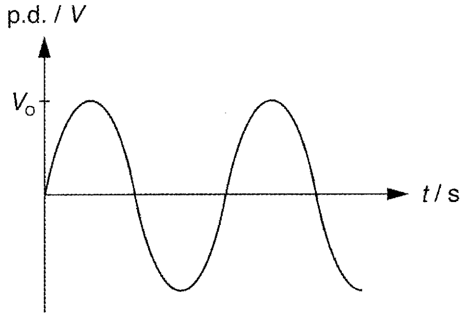
$$\frac{F}{4} = \frac{\varepsilon_0 A V^2}{8d}$$

By Hooke's Law, the extra extension of each spring is:

$$F = kx$$
$$\frac{\varepsilon_0 A V^2}{8d} = kx$$
$$x = \frac{\varepsilon_0 A V^2}{8kd}$$

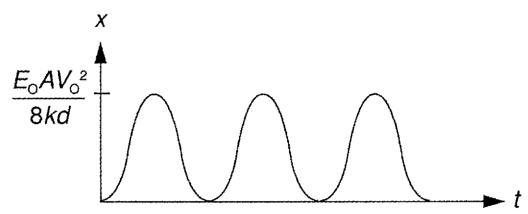
(c) **(i)** The potential difference between plates has a sinusoidal waveform because the p.d. supplied across the plates is:

$$V = V_0 \sin \omega t$$



(ii) Because the frequency of a.c. is much less than the natural frequency, the oscillation is a damped oscillation by a driving force. The extension will not increase continuously. There is a limit of extra extension.
Based on the answer we obtained in part **(b)**, the extra extension is:

$$x = \frac{\varepsilon_0 A V^2}{8kd}$$
$$\propto V^2$$
$$\propto V_0^2 \sin^2 \omega t$$



B. Overseas & HKALE Questions

14. **(a)** The capacitance of a capacitor is the ratio of the electric charge stored on it to the potential difference across it.

(b) **(i)** The graph shown represents the charging of the capacitor.

(ii) From the graph, e.m.f. $E = 9.0$ V

15. **(a)** The capacitance of a capacitor is the ratio of the electric charge stored on it to the potential difference across it.
The farad is a unit of capacitance and is defined as coulomb per volt.

(b) **(i)** From Fig. (b),
(1) at $t = 10.0$ s, $I_{10} = 1.46$ mA
(2) at $t = 30.0$ s, $I_{30} = 0.78$ mA

(ii) Since there is no external e.m.f., p.d. across capacitor = p.d. across R

(1) $V_{10} = I_{10} R$
$= (1.46 \times 10^{-3})(20 \times 10^3) = 29.2$ V

(2) $V_{30} = I_{30} R$
$= (0.78 \times 10^{-3})(20 \times 10^3) = 15.6$ V

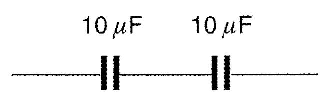
(iii) For a variable current I, the charge flowed,
ΔQ = average current × time interval

$$\approx \frac{1}{2}(I_{10} + I_{30})(30.0 - 10.0)$$
$$= \frac{1}{2}(1.46 + 0.78) \times 10^{-3}\ (20.0)$$
$$= 0.022\,4 \text{ C} = 22.4 \text{ mC}$$

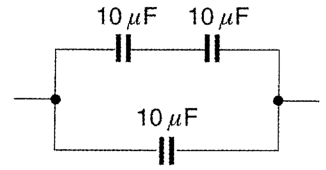
(iv) Estimated capacitance,

$$C \approx \frac{\Delta Q}{(V_{10} - V_{30})}$$
$$= \frac{0.022\,4}{(29.2 - 15.6)}$$
$$= 0.001\,647 \text{ F} = 1\,650\ \mu\text{F}$$

16. **(a)** **(i)** Sketch 2 capacitors connected in series.



(ii) Sketch 2 in-series capacitors connected in parallel with a single capacitor.



(b) **(i)** $C = \frac{Q}{V}$

(ii) $C = \frac{Q}{V} \equiv Q = CV$

$$\therefore \quad W = \frac{1}{2}QV = \frac{1}{2}(CV)V = \frac{1}{2}CV^2$$

(c) Let W_0 = energy of fully charged capacitor
W = minimum energy of capacitor
V_0 = voltage of fully charged capacitor
V = minimum voltage of capacitor

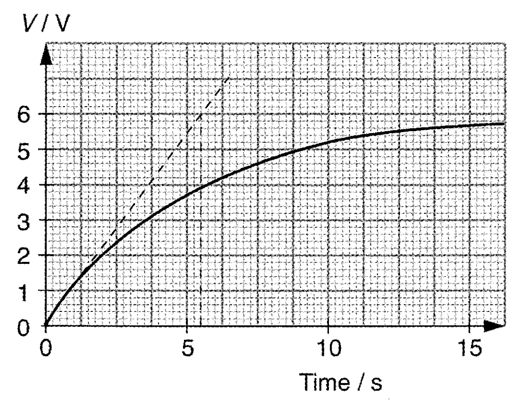
$$W = \frac{1}{2}CV^2 \equiv W \propto V^2$$
$$\frac{W}{W_0} = \left(\frac{V}{V_0}\right)^2 \quad \text{i.e. } 0.80 = \left(\frac{V}{V_0}\right)^2$$
$$\therefore \frac{V}{V_0} = \sqrt{0.80} = 0.894$$

17. **(a)** Charge on capacitor
$Q = CV$
$= 220 \times 10^{-6} \times 6.0$
$= 1.32 \times 10^{-3} = 1.3 \times 10^{-3}$ C

(b) Energy stored on capacitor

$$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 220 \times 10^{-6} \times (6.0)^2$$
$$= 3.96 \times 10^{-3} = 4.0 \times 10^{-3} \text{ J}$$

(c) **(i)** Draw tangent at origin as dashed line on Fig. (a).

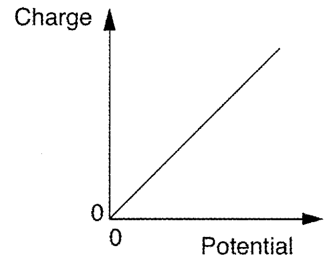


(ii) Initial rate ≡ tangent at origin
time constant ≡ t at $V = 6.0$ V on tangent
From the tangent on Fig. (a),
at $V = 6.0$ V, $t = 5.5$ s

∴ Time constant = 5.5 s

(iii) Time constant = CR
$5.5 = 220 \times 10^{-6} \times R$
∴ $R = 2.5 \times 10^4 = 25$ kΩ

18. **(a)** **(i)** Sketch variation of charge with potential:



(ii) **1.** The capacitance of a capacitor is the ratio of the electric charge stored on it to the potential difference across it.

2. The energy E stored in the capacitor is the work done W to store the charge Q on the capacitor at the applied p.d. V, and is given by:
E = work done W to store charge on capacitor
= average applied p.d. V × charge Q moved
≡ area under V-Q graph
As the V-Q graph is a straight line through the origin, the area under the graph is a right-angle triangle (area = $\frac{1}{2}QV$) and the energy E stored is given by:

$$E = W = \frac{1}{2}QV$$

Since $Q = CV$, it is equivalent to:

$$E = W = \frac{1}{2}(CV)V = \frac{1}{2}CV^2$$

(b) **(i)** **1.** Capacitance of the arrangements

$$= \frac{1}{\frac{1}{C} + \frac{1}{C}} + \frac{1}{\frac{1}{C} + \frac{1}{C}}$$
$$= \frac{1}{\frac{1}{50} + \frac{1}{50}} + \frac{1}{\frac{1}{50} + \frac{1}{50}} = 50\ \mu\text{F}$$

2. One advantage of this arrangement is that the potential difference across each capacitor would be lower, rendering them safer to handle.

(ii) The magnitude of the force on a nucleus will be much bigger than the force on an electron. The force on a nucleus is in the direction of the electric field while the force on an electron is in the opposite direction.

(iii) When the potential difference across the tube is sufficiently large, the electric field created will produce forces strong enough to ionise the xenon gas atoms, i.e. separate some electrons from the gas atoms. The movement of the resulting charge carriers to opposite electrodes constitute the current.

(iv) 1. Energy dissipated

$= 63\%$ of energy stored in all capacitors

$= 0.63 \times \frac{1}{2} CV^2$

$= 0.63 \times \frac{1}{2}(50 \times 10^{-6})(540)^2$

$= 4.593 = 4.6$ J

2. Let V_1 = p.d. across each capacitor immediately after the flash of light

Energy in each capacitor,

$E_1 = \frac{1}{2} CV_1^2$

It is also given by:

$E_1 = \frac{1}{4} \times 37\%$ of energy stored in all capacitors

$= \frac{1}{4}(0.37)\frac{1}{2} CV^2$

Equating E_1:

$\frac{1}{2} CV_1^2 = \frac{1}{4}(0.37)\frac{1}{2} CV^2$

$V_1^2 = \frac{1}{4}(0.37)V^2 = \frac{1}{4}(0.37)(540)^2$

$= 26\,973$

$\therefore \quad V_1 = \sqrt{269\,73} = 164.2 = 160$ V

19. (a) (i) The capacitance C of a capacitor is the ratio of the electric charge Q stored on it to the potential difference V across it.

(ii) The resistance R of a resistor is the ratio of the potential difference V across it to the current I flowing through it.

(b) (i) When the switch is closed, there is no charge on the capacitor and the applied voltage causes charges to be stored on the capacitor. The flow of charges to the capacitor constitutes the current in the circuit and the resistor.

(ii) 1. Kirchhoff's 2nd law $\Rightarrow E = V_C + V_R$

2. As t increases, the amount of charges on the capacitor increases and so the p.d. V_C across the capacitor increases.

From the equation $E = V_C + V_R$, a constant E and an increasing V_C implies a decreasing V_R and so the current in the resistor and the circuit decreases.

(c) (i) $V_R = IR$

$= (1.8 \times 10^{-6})(2.0 \times 10^{6}) = 3.6$ V

(ii) $E = V_C + V_R$

$V_C = E - V_R = 6.0 - 3.6 = 2.4$ V

(iii) $q = CV$

$= (14 \times 10^{-6})(2.4)$

$= 3.36 \times 10^{-5} = 3.4 \times 10^{-5}$ C

(iv) $E_C = \frac{1}{2} qV$

$= \frac{1}{2}(3.36 \times 10^{-5})(2.4)$

$= 4.032 \times 10^{-5} = 4.0 \times 10^{-5}$ J

(d) (i) 1. Fully charged, charge on capacitor,

$Q = CV = (14 \times 10^{-6})(6.0) = 8.4 \times 10^{-5}$ C

Assuming that charge Q remains consta[nt] during the reduction in capacitance, new p.d. across the 5.0 μF capacitor,

$V_{new} = \frac{Q}{C} = \frac{(8.4 \times 10^{-5})}{(5.0 \times 10^{-6})}$

$= 16.8 = 17$ V

2. New energy stored,

$E_{new} = \frac{1}{2} QV_{new} = \frac{1}{2}(8.4 \times 10^{-5})(16.8)$

$= 7.06 \times 10^{-4} = 7.1 \times 10^{-4}$ J

(ii) With the same charge (Q), the capacitance o[f] the capacitor ($C = \frac{Q}{V}$) can only be reduced b[y] increasing the p.d. (V) across it.

P.d. between 2 points is the amount of electrical energy converted into other forms [of] energy per unit charge moved across the points. Therefore, energy must be supplied t[o] the capacitor to increase the p.d. across it. Conservation of energy implies that the energ[y] supplied to increase the p.d. must be converte[d] into energy stored on the capacitor.

20. – 22. HKALE Questions