5 Circular Motion

Self Evaluation Exercise 5.1 (p.169)

1. A

Angular velocity $= \dfrac{\text{Angle covered}}{\text{Time taken}}$

$= \dfrac{\frac{\ell}{r}}{t}$

$= \dfrac{\ell}{rt}$

2. A

Let ω be the constant angular velocity of the disc, we obtain

$\dfrac{v}{r} = \omega = \text{constant}$

$\therefore \quad v \propto r$

3. B

The minute hand completes one revolution in 60 minutes. Hence, its mean angular speed is

$\omega = \dfrac{2\pi}{t}$

$= \dfrac{2\pi}{60 \times 60}$

$\approx 1.7 \times 10^{-3}$ rad s^{-1}

Self Evaluation Exercise 5.2 (p.176)

1. C

Centripetal acceleration

$a = r\omega^2$

$= r\left(\dfrac{2\pi}{T}\right)^2$

$= 2 \times \left(\dfrac{2\pi}{2}\right)^2$

$= 2\pi^2$ m s^{-2}

2. C

Resultant force, $F = mr\omega^2$

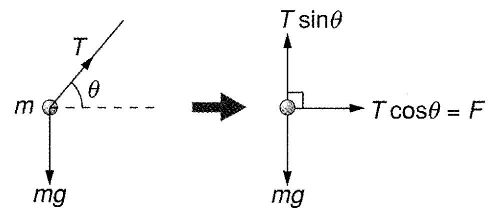
$= mr\left(\dfrac{2\pi}{T}\right)^2$

$= 5 \times 2 \times \left(\dfrac{2\pi}{3}\right)^2$

$= \dfrac{40\pi^2}{9}$ N

3. D

The forces acting on the mass is:



where T is the tension in the string,
m is the mass of the body,
g is the acceleration of gravity.

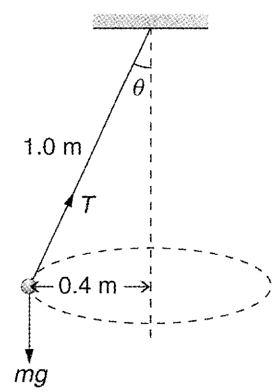
Hence, vertical forces are balanced. $F = T\cos\theta$ is the resultant centripetal force to keep the body in the circular motion in the horizontal plane.

4. C

5. C

6. A

7. **(a)**



The vertical component of the tension T balances the weight of the mass. The horizontal component of the tension T provides the centripetal acceleration.

$T\cos\theta = mg$ ...... (1)

$T\sin\theta = \dfrac{mv^2}{r}$ ...... (2)

$\sin\theta = \dfrac{0.40}{1.0}$, $\theta = 23.58°$

Thus, the centripetal force F_c is

$F_c = \dfrac{mv^2}{r} = T\sin\theta$

$= \dfrac{mg}{\cos\theta} \times \sin\theta$ (from (1))

$= mg\tan\theta$

$= 0.50 \times 10 \times \tan 23.58°$

$= 2.18$ N

(b) As the centripetal force F_c is 2.2 N, the angular speed ω of the mass is

$F_c = mr\omega^2$

$\omega = \sqrt{\dfrac{F_c}{mr}}$

$= \sqrt{\dfrac{2.18}{0.50 \times 0.40}}$

$= 3.30$ rad s^{-1}

(c) The tension T of the string is

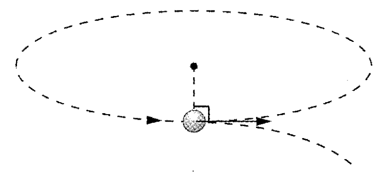
$T\sin\theta = mr\omega^2 = F_c$

$T = \dfrac{F_c}{\sin\theta}$

$= \dfrac{2.2}{\sin 23.58°}$

$= 5.5$ N

(d) If the string suddenly breaks, the centipetal force F_c provided by the tension T disappears. The mass will move away tangentially and then fall with a projectile motion under the effect of gravity.



8. **(a)** The angular speed ω is

$\omega = \dfrac{15 \times 2\pi}{60}$

$= 1.57$ rad s^{-1}

(b) The relation between linear speed v and angular speed ω is

$v = r\omega$

Thus, the linear speed v of the child seated 12 m from the centre is

$v = r\omega$

$= 1.2 \times 1.57$

$= 1.88$ m s^{-1}

(c) The acceleration of the child is

$a = r\omega^2$

$= 1.2 \times 1.57^2$

$= 2.96$ m s^{-1}

(d) The centripetal force F acting on the child is

$F = ma$

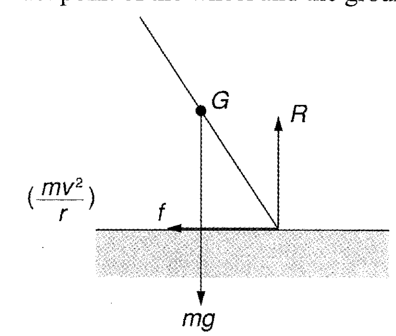
$= 30 \times 2.96$

$= 88.8$ N

Self Evaluation Exercise 5.3 (p.182)

1. A

Only the weight mg, normal reaction R and friction f act on the system. The weight mg always acts on the centre of mass G. And the friction f and normal reaction R act at the contact point of the wheel and the ground.



The magnitude of f is equal to $\dfrac{mv^2}{r}$.

2. A

3. **(a)** The frictional force f depends on the coefficient of friction μ of the surface and the normal reaction R. Also, the magnitude of R is equal to the weight.

$f = \mu R$

$= \mu(mg)$

Only the frictional force f contributes the centripetal acceleration of the car. Thus, the maximum speed of the car can take without sliding is

$f = \dfrac{mv_{max}^2}{r}$

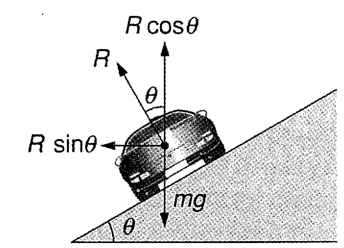
$\mu mg = \dfrac{mv_{max}^2}{r}$

$v_{max} = \sqrt{\mu gr}$

$= \sqrt{0.87 \times 10 \times 230}$

$= 44.73$ m s^{-1}

(b)



In this case, the car turns safely even if there is no friction. Thus, only the horizontal component of the normal reaction R contributes the centripetal acceleration.

$R\sin\theta = \dfrac{mv^2}{r}$

The vertical component of R is equal to the weight mg.

$$R\cos\theta = mg$$

Thus, the relation of angle θ and velocity v becomes

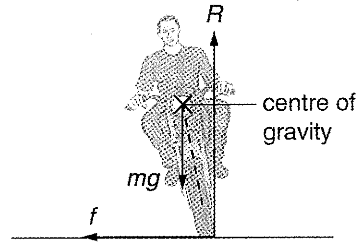
$$\tan\theta = \frac{v^2}{rg}$$

If the speed of the car is 25 m s^{-1}, the banking angle θ should be

$$\tan\theta = \frac{25^2}{230\times10}$$

$$\theta = 15.20°$$

4. (a)



R = normal reaction
mg = weight of the cyclist and the bicycle
f = frictional force

(b) The normal reaction acts vertically upwards and balances the weight. The frictional force acts horizontally to provide centripetal acceleration.

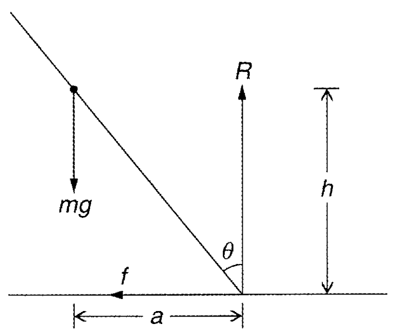
$$R = mg \quad .... (1)$$

$$f = \frac{mv^2}{r} \quad .... (2)$$

The frictional force f exterted by the road when the cyclist rounds a corner of 17.0 m with speed at 11 m s^{-1} is

By (2), $$f = \frac{mv^2}{r} = \frac{35\times11^2}{17.0} = 249.12 \text{ N}$$

(c)



To prevent toppling, the total moment about the centre of gravity should be zero. Thus

$$fh = Ra$$

Thus, the angle to the vertical, θ should be

$$\tan\theta = \frac{a}{h}$$

$$\tan\theta = \frac{f}{R}$$

$$= \frac{\frac{mv^2}{r}}{mg} \quad \text{(By (1) and (2))}$$

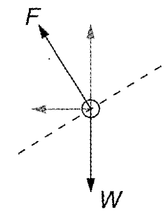
$$= \frac{v^2}{gr}$$

$$= \frac{11^2}{10\times17.0}$$

$$\theta = 35.44°$$

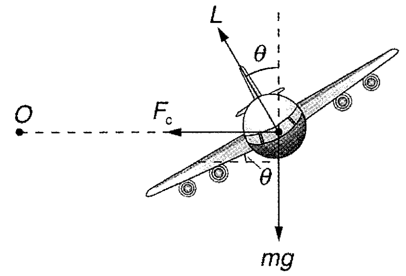
Self Evaluation Exercise 5.4 (p.184)

1. C



The vertical component of force F is used to balance the weight of the aircraft. And the horizontal component of the force F is used as a centripetal force to change the direction of the aircraft. Then the aircraft can travel around the centre Q.

2. (a)



The lifting force L balances the weight mg and provides centripetal acceleration.
The relations are

$$L\cos\theta = mg \quad .... (1)$$

$$L\sin\theta = F_c = \frac{mv^2}{r} \quad ....(2)$$

The angle θ from the horizontal is

$$\frac{(2)}{(1)}: \quad \frac{L\sin\theta}{L\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{gr}$$

$$= \frac{(\frac{280\times1\,000}{60\times60})^2}{10\times1\,200}$$

$$= 0.50$$

$$\theta = 26.75°$$

(b) From the answer obtained in part **(a)**, the angle θ depends on velocity v and turning radius r, i.e.

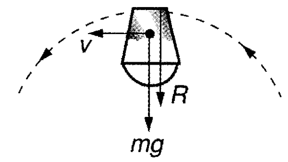
$$\tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}(\frac{v^2}{gr})$$

Thus, it is independent of the weight of the aircraft.

Self Evaluation Exercise 5.5 (p.188)

1. (a)



When the bucket is whirling, the weight of water and the normal reaction on water provide the centripetal acceleration of water.

$$mg + R = \frac{mv^2}{r}$$

If $R = 0$, only the weight of water provides its centripetal acceleration. Thus, the speed of the bucket is the minimum at the highest point where the water does not spill out.

$$mg = \frac{mv^2}{r}$$

The speed v of bucket is

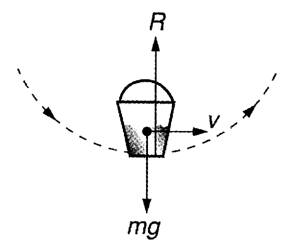
$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr}$$

$$= \sqrt{10\times1.2}$$

$$= 3.46 \text{ m s}^{-1}$$

(b)



When the bucket is at the lowest position, the normal reaction R balances the weight and provides the centripetal acceleration.

$$R = mg + \frac{mv^2}{r}$$

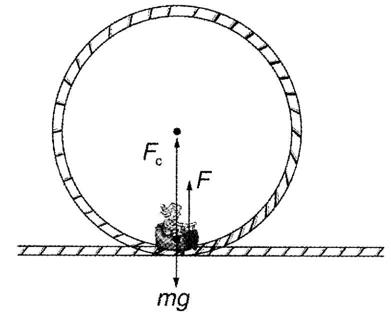
If the bucket moves with minimum speed and the speed is constant throughout the cycle, R is

$$R = mg + \frac{mv^2}{r}$$

$$= 0.5 \times 10 + \frac{0.5\times3.46^2}{1.2}$$

$$= 9.99 \text{ N}$$

2. (a) The angular speed of the rider is $\omega = \frac{2\pi}{3.2}$ = 1.96 rad s^{-1} at the bottom of the circle, the force by the structure F balances the weight mg, and provides the centripetal force F_c.



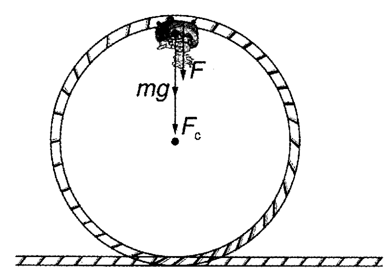
$$F = mg + F_c$$

$$= mg + mr\omega^2$$

$$= 63 \times 10 + 63 \times 6.6 \times 1.96^2$$

$$= 2\,227.34 \text{ N}$$

(b)



At the top of the circle, the force by the structure F and the weight mg provide the centripetal force F_c.

$$F + mg = F_c$$

Thus, the force by structure, F is

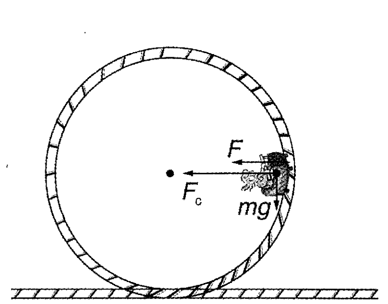
$$F = F_c - mg$$

$$= mr\omega^2 - mg$$

$$= 63 \times 6.6 \times 1.96^2 - 63 \times 10$$

$$= 967.34 \text{ N}$$

(c)



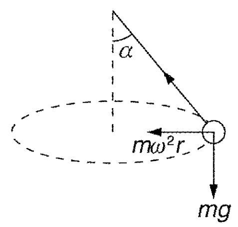
When the rider is half way up, the force by structure F points radially inwards and the weight is vertically downwards. Thus, only the force by structure F contributes the centripetal acceleration. Thus, the force by structure F

$$F = F_c$$
$$= mr\omega^2$$
$$= 63 \times 6.6 \times 1.96^2$$
$$= 1\ 597.34\ \text{N}$$

Review Exercise 5 (p.195)

A. Multiple Choice Questions

1. D

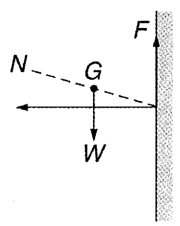


From the diagram, we obtain

$$\tan\alpha = \frac{mr\omega^2}{mg}$$
$$= \frac{\omega^2 r}{g}$$

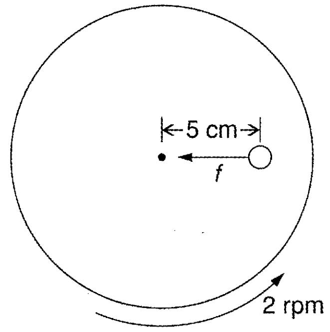
2. A

The outward force P is not a real force. It is a frictional force. Only normal reaction, friction and weight act on the motorcycle.
The motorcyclist must make a small angle to the horizontal to balance the moment by the friction and prevent toppling.



B. Structured Questions

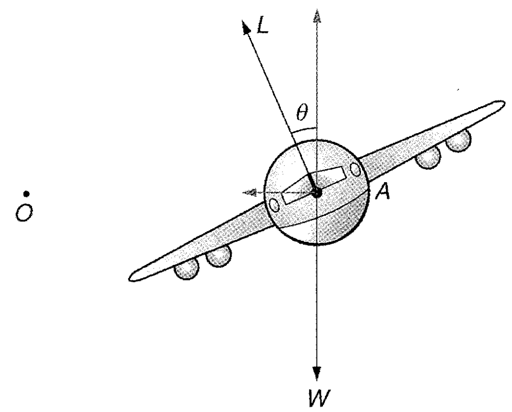
3. The mass can stay on the top of the turntable because the friction between the mass and the turntable is sufficient for the centripetal acceleration of the mass. But when the distance from the axis is more than 5 cm, the maximum friction is not enough for the change of velocity of the mass.



The frictional force f is

$$f = mr\omega^2$$
$$= 0.05 \times 0.05 \times \left(\frac{2 \times 2\pi}{60}\right)^2$$
$$= 1.10 \times 10^{-4}\ \text{N}$$

4.



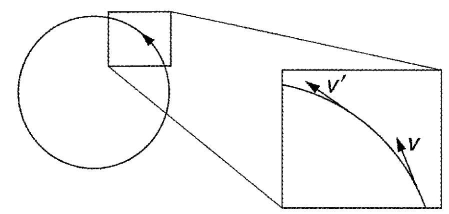
From the diagram, we obtain

$$L\sin\theta = mr\omega^2$$
$$L\cos\theta = W$$
$$\therefore\quad \theta = \cos^{-1}\left(\frac{W}{L}\right)$$

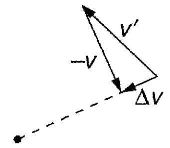
then

$$L\sin\theta = ma$$
$$L\sin\left(\cos^{-1}\left(\frac{W}{L}\right)\right) = \frac{W}{g}a$$
$$a = \frac{gL\sin\left(\cos^{-1}\left(\frac{W}{L}\right)\right)}{W}$$

5. By Newton's second law, the rate of change of momentum of a body is proportional to and in the direction of the force acting on it.
When an object is moving in a circle, the direction of the motion of the object changes. Thus, the velocity v and the momentum mv change. A force towards the centre of circle is the resultant force of motion.



The change of velocity is $v' - v = v' + (-v)$



The force is in the same direction as Δv, thus points to the centre.

(a) The angular velocity ω of the aeroplane is

$$\omega = \frac{v}{r}$$
$$= \frac{200}{1\,500}$$
$$= 0.133\ \text{rad s}^{-1}$$

(b) The angle θ between the lift L and the vertical is

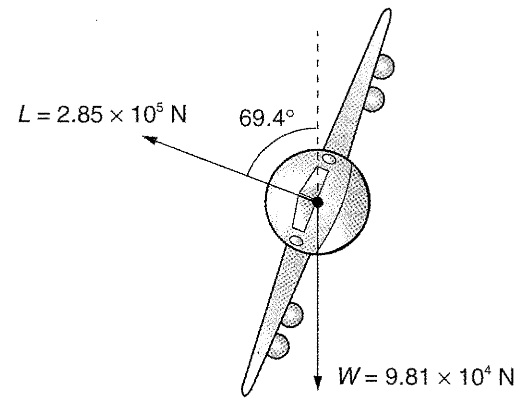
$$\tan\theta = \frac{v^2}{rg}$$
$$\tan\theta = \frac{(200)^2}{1\,500 \times 10}$$
$$\theta = 69.4°$$

The magnitude of the lift L is equal to

$$L\cos\theta = mg$$
$$L\cos 69.4° = (1.0 \times 10^4) \times 10$$
$$L \approx 2.85 \times 10^5\ \text{N}$$

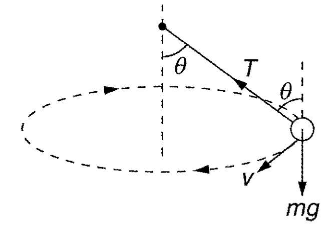
The weight of the aeroplane is

$$W = mg$$
$$= (1.0 \times 10^4) \times 10$$
$$= 1 \times 10^5\ \text{N}$$



(c) Because the passenger also moves along with the aeroplane and changes in the direction of motion. Thus a force acts on the passenger.

6. (a)



The tension of the string is equal to

$$T\cos\theta = mg$$
$$T\sin\theta = \frac{mv^2}{r}$$

When the angle θ becomes 90°, the tension becomes infinitely large to balance the weight. And it is impossible. The tension acting on the string is so large that it breaks and the mass will fly off.

(b) The angle θ the pilot should bank is

$$\tan\theta = \frac{v^2}{rg}$$
$$\tan\theta = \frac{\left(360 \times \frac{1\,000}{3\,600}\right)^2}{5\,000 \times 10}$$
$$\theta = 11.3°$$

7. (a) (i) $a = \dfrac{v^2}{r}$

The direction of the force is always towards the centre of rotation.

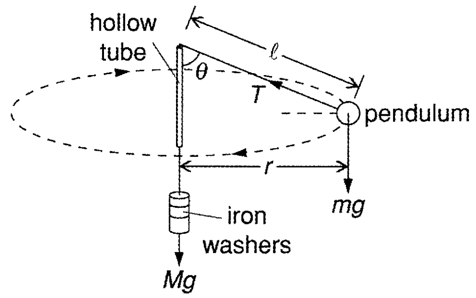
(ii) The period T of rotation is

$$T = \frac{2\pi r}{v}$$
$$v = \frac{2\pi r}{T}$$

Thus, the centripetal force F can be written as

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

(b) The experiment would be carried out as follows:



As the pendulum rotates, the tension of the string T is equal to the mass of the iron washers Mg.
The centripetal force F required for the pendulum is provided by the horizontal component of the tension T.

$$F = mr\omega^2 = T\sin\theta$$

In the experiment, the number of rotation per certain time, (e.g. 1 min) is counted. Then, the number of rotation per second, n can be obtained. The relationship between the centripetal force F and the period T is

$$F = mr\omega^2 = mr(2\pi n)^2 = 4\pi^2 n^2 mr = \frac{4\pi^2 mr}{T^2} \quad (n = \frac{1}{T})$$

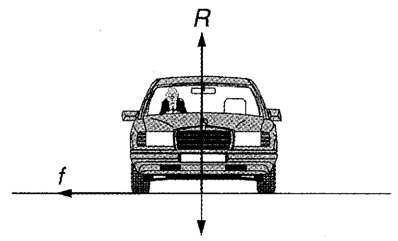
(c) **(i)** The magnitude of the centripetal force F is

$$F = \frac{mv^2}{r} = 800 \times \frac{15^2}{100} = 1\ 800\ \text{N}$$

(ii) The centripetal force is provided by the static frictional force between the wheels and the ground.

(d) For a car moving around a curve on a horizontal surface, the centripetal force F is provided by the static friction.

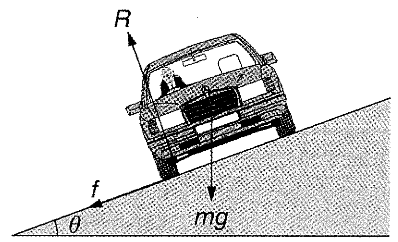
$$F = f$$



If the road is banked, the normal force R is at right angle of the surface. The horizontal component of the normal reaction R provides the centripetal force instead of the friction.

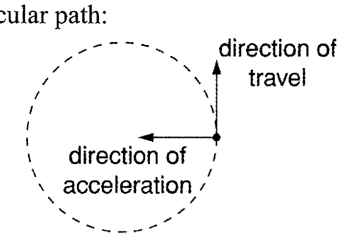
$$F = R\sin\theta$$

If the speed of the car is too high, friction points down along the road surface and prevents the car from sliding. This allows the car to move around a curve with higher speed than the horizontal one. Therefore, the banked road design is safer.



C. Overseas & HKALE Questions

8. **(a)** Sketch a body travelling at a constant speed in a circular path:



(i) The direction of its velocity is changing continuously.
The body must therefore have an acceleration.

(ii) The direction of the acceleration is perpendicular to the direction of travel, and is pointing towards the centre of the circular path.

(b) **(i)** At maximum speed $v = 25$ m s^{-1}, centripetal force = $0.8W$

$$\frac{mv^2}{r} = 0.8mg$$

∴ Minimum radius for circular link,

$$r = \frac{v^2}{0.8g} = \frac{25^2}{0.8\times 9.81} = 79.64\ \text{m}$$

(ii) When a vehicle moves in a circular path, the sideways force at the wheels produce a sideways torque about its centre of gravity. Lorries' centre of gravity are generally higher than that of cars. The same sideways force, required by the same speed, will produce a higher torque that could topple a lorry.

9. **(d)** **(i)** Speed of the moon in its orbit,

$$v = r\omega = r\frac{2\pi}{T} = (3.84 \times 10^8) \times \frac{2\pi}{2.36\times 10^6} = 1\ 022\ \text{m s}^{-1}$$

(ii) Acceleraton of the moon,

$$a = \frac{v^2}{r} = \frac{1\ 022^2}{3.84\times 10^8} = 2.72 \times 10^{-3}\ \text{m s}^{-2}$$

(iii) Force on the moon,

$$F = ma = (7.35 \times 10^{22})(2.72 \times 10^{-3}) = 2.00 \times 10^{20}\ \text{N}$$

10. **(a)** Angular velocity refers to the rate of change of angular displacement. It is a vector quantity with magnitude and direction.

(b) **(i)** The relationship is $v = r\omega$,

(ii) When ω is constant, v can be varied by varying r.

(iii) The tension in the cord is needed to provided the centripetal force which produces the centripetal acceleration that is directed towards the centre of the circle and continuously changing the direction of the velocity.

(iv) Acceleration of stone is $\frac{v^2}{r}$ towards the centre of the circle.

Force = Mass × Acceleration

$$T = \frac{mv^2}{r} = mv\left(\frac{v}{r}\right) = mv\omega$$

(c) **(i)** **(1)** The centripetal acceleration,

$$a = \frac{v^2}{r} = \frac{12^2}{7.0} = 20.6\ \text{m s}^{-2}$$

(2) Let F_s = force exerted by seat on passenger

Force = Mass × Acceleration

$F_s + mg = ma$

$F_s + 60 \times 9.81 = 60 \times 20.6$

∴ $F_s = 647.4$ N

(ii) **(1)** Change in potential energy

$\Delta E_p = mg\Delta h = 60 \times 9.81 \times 14 = 8\ 240$ J

(2) Let v_0 = speed at top of loop = 12 m s^{-1}
v_e = speed at bottom of loop
Assuming negligible loss of energy,

$$\Delta E_p = \Delta E_k = \frac{1}{2}mv_e^2 - \frac{1}{2}mv_0^2$$

$$8\ 240 = \frac{1}{2}(60)v_e^2 - \frac{1}{2}(60)(12)^2$$

∴ $v_e = 20.5$ m s^{-1}

(iii) The entry speed must be sufficient for the cart and passenger to reach the top of the loop at the speed where the centripetal acceleration that is greater or equal to their weight. Otherwise, the cart and/or passenger may fall off the track or fail to reach the top.

11. – 13. HKALE Questions