3 Dynamics

Self Evaluation Exercise 3.2 (p.112)

1. B

By Newton's second law,

$F = ma$

Given a mass m, when the acceleration is constant, the resultant force is constant.

2. D

$F = \frac{d}{dt}(mv)$

Rate of change of momentum

$= 4$ kg m s^{-2} per 2 s

$= 8$ kg m s^{-2} per s

3. D

By Newton's third law, there is a force F acting on Y.

$\because \quad F = M_y a_y$

$M_x a = M_y a_y$

$a_y = \frac{M_x}{M_y} a$

4. C

$F = \frac{d}{dt}(mv)$

$mv = \int F dt$

$\therefore$ Area under the graph represents the change of momentum.
Also, by dimensional analysis, the area under the curve has a unit of $[F][t] = \mathrm{MLT^{-1}}$. Only option **C** is correct.

5.

$$s = ut + \frac{1}{2}at^2$$

$$60 = 0 + \frac{1}{2}a(10)^2$$

$$a = 1.2 \text{ m s}^{-2}$$

$$F - mg = ma$$

$$= 5 \times (1.2 + 10)$$

$$= 56 \text{ N}$$

6. As the speed is kept constant, an increase in mass indicates that there is an increase in momentum, so an additional force is required.

$F = \frac{d}{dt}(mv)$

Since v is constant,

$F = v\frac{dm}{dt}$

$= 1.5 \times 20$

$= 30$ N

Self Evaluation Exercise 3.3 (p.118)

1. D

2. C

Self Evaluation Exercise 3.5 (p.124)

1. B

If the collision is inelastic, the particle loses all of its kinetic energy. However, the principle of conservation of momentum still applies.

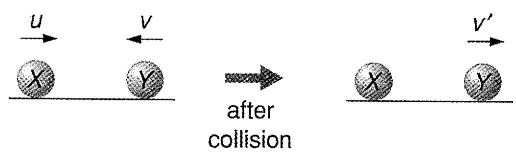
2. D

In a collision, whether it is elastic or not, momentum is always conserved provided that there is no external force acting on the system. For elastic collision, both kinetic energy and total energy are conserved.

3. A

Since the mass of the shells is negligible when comparing to the mass of the earth, the change of momentum of the earth is negligible.

4. B



By conservation of momentum, the speed of the body Y after collision is:

$$mu - mv = mv'$$

$$v' = u - v$$

The velocity v' of body Y is opposite to velocity v.

Self Evaluation Exercise 3.6 (p.136)

1. A

For the graph, $\frac{h_1}{h_0} = \frac{4}{5}$

$\therefore \quad \frac{mgh_1}{mgh_0} = 0.8$

It means that after each bounce, only 0.8 of the original energy is left.

$\therefore$ After three bounces, its energy $= (0.8)^3 \, mgh$

2. A

Let M be the mass of the piece of brass.

In case X,

$v^2 = u^2 + 2as$ (Assume that a is constant.)

$(10)^2 = 0^2 + 2as$

$s = \frac{100}{2a}$

Energy $= Fs = Mas = 50M$ J kg^{-1}

In case Y,

Energy $= Mgh$
$= M(10)(2)$
$= 20M$ J kg^{-1}

In case Z,

Energy $= Mc\,\Delta T$
$= M(380)(20 - 15)$
$= 1\,900M$ J kg^{-1}

$\therefore \quad Y < X < Z$

3. C

Assume that the acceleration is constant, then

$v^2 = u^2 + 2(-a)s$

$0 = u^2 - 2(\frac{E}{m})$

$u = \sqrt{\frac{2 \times (500 \times 10^3)}{1\,600}}$

$= 25$ m s^{-1}

4. The total momentum of the stone and the earth is always zero as there is no external force.

The attraction forces between the stone and the earth are internal forces:

$mu = M_E v$

As the downward momentum of the stone increases, the upward momentum of the earth increases. Thus, the total momentum is zero.

However, since the mass of the earth M_E is very large, the speed of the earth is extremely small.

Review Exercise 3 (p.139)

A. Multiple Choice Questions

1. B

There does not exist a type of force called "centripetal force". In fact, it is just a term used to describe the force required to maintain circular motion.

Even such a force exists, by Newton's third law, an action and reaction pair always acts on two different objects.

2. C

Area under the graph = Change in momentum

$40 = \frac{(3+5)(x)}{2}$

$x = 10$

3. D

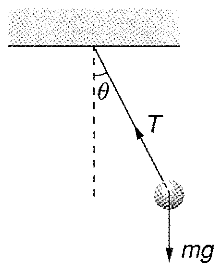
Gain in momentum = Area under the graph

$= \frac{(4+6)(10)}{2}$

$= 50$ N s

After time = 6 s, the force acts on the mass is zero, so its momentum remains at 50 N s afterwards.

4. B



$T\sin\theta = ma$

$T\cos\theta = mg$

$\therefore \quad \tan\theta = \frac{a}{g}$

$= \frac{0.5}{9.8}$

$\theta = 2.92°$

$\approx 3°$

5. D

$mu_1 = mv_1 + 14mv_2$

$v_1 = \frac{mu_1 - 14mv_2}{m}$

$= u_1 - 14v_2$

$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(14m)v_2^2$

$u_1^2 = v_1^2 + 14v_2^2$ .... (1)

$v_1^2 = u_1^2 - 14v_2^2$ .... (2)

$\therefore \quad (u_1 - 14v_2)^2 = u_1^2 - 14v_2^2$ (from(1))

$-28u_1v_2 + 196v_2^2 = -14v_2^2$

$210v_2 = 28u_1$

$v_2 = \frac{2}{15}u_1$

$v_1 = u_1 - 14v_2$

$= -\frac{13}{15}u_1$

6. C

By the principle of conservation of energy,

$0 = 4mv_1 + (238 - 4)\,mv_2$

$\frac{v_1}{v_2} = \frac{-234}{4}$

$\frac{\text{Kinetic energy of the } \alpha\text{-particle}}{\text{Kinetic energy of the recoiling daughter nucleus}}$

$= \frac{\frac{1}{2}(4m)v_1^2}{\frac{1}{2}(238-4)mv_2^2}$

$= \frac{4}{234}(\frac{v_1}{v_2})^2$

$= \frac{4}{234}(-\frac{234}{4})^2$

$= \frac{234}{4}$

7. B

(A) When it is moving upwards, its kinetic energy converts to its potential energy and becomes minimum at maximum height. But the total energy is conserved throughout the motion.

(D) During the motion, the mass undergoes acceleration, so the velocity is changing and the mass will not travel the same distance even for the same time period. Since there is no air resistance, the only force acting on the mass is the weight.

B. Structured Questions

8. (a) (i) Distance travelled = Area under the graph of interval DE

$= \frac{1}{2}(6.5 - 6.0)(1.5)$

$= 0.375$ m

(ii) Acceleration = Slope of interval DE

$= \frac{0 - 1.5}{6.5 - 6}$

$= -3$ m s^{-2}

(b) (i) Acceleration $= \frac{1.5 - 0}{1 - 0}$

$= 1.5$

$T - mg = ma$

$T = (50)(10 + 1.5)$

$= 575$ N

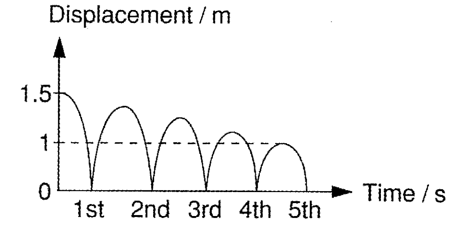
(ii) Acceleration = 0

$T = mg$

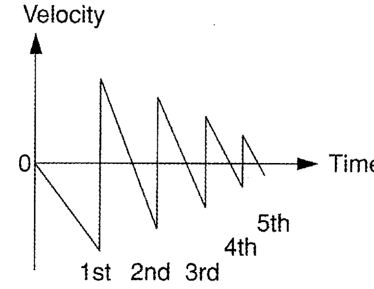
$= 50 \times 10$

$= 500$ N

9. (a) (i)



(ii)



(b) $v^2 = u^2 + 2as$

$= 0 + 2(-10)(-1.5)$

$v = -5.48$ m s^{-1}

$p = (0.025)(-5.48) = 0.137$ N s

(c) When a mass m_1 hits a stationary mass m_2, by the principle of conservation of momentum,

$m_1u_1 = m_1v_1 + m_2v_2$

then $v_2 = \frac{m_1u_1 - m_1v_1}{m_2}$

$= \frac{m_1}{m_2}(u_1 - v_1)$

In the system of ball and floor, $m_2 >> m_1$, thus, v_2 tends to zero.

However, the velocity of the ball v_1 cannot be simply determined by the principle of conservation of momentum only ($v_1 = u_1 - \frac{m_2}{m_1}v_2$). Although v_2 tends to zero, $\frac{m_2}{m_1}$ tends to infinity, so the term $\frac{m_2}{m_1}v_2$ is undefined.

When we consider the principle of conservation of energy as well, $\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + E$, where E is the energy loss.

Hence $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1u_1^2 - E$

$\therefore \quad u \geq v_1 \geq 0$

10. (a) $mu = mv = MV$ ....(1)

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$

(b) $v = V - u$

By (1), $mu = mv + MV$

$mu = mv + M(v + u)$

$v = \frac{(m-M)u}{m+M}$

Fractional loss $= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2}$

$= \frac{u^2 - v^2}{u^2}$

$= \frac{u^2 - \left[\frac{m-M}{m+M}u\right]^2}{u^2}$

$= 1 - \left(\frac{m-M}{m+M}\right)^2$

$= \frac{4Mm}{(M+m)^2}$

(c) When $M = 50m$

Fractional loss $= \frac{4(50m)m}{(50m+m)^2}$

$= \frac{200}{(51)^2}$

$= 0.076\,9$

C. Overseas & HKALE Questions

11. (a) (i) The linear momentum of a body is the product of its mass and its velocity.

(ii) It is a vector quantity.

(b) The principle of conservation of linear momentum states that when objects of a system interact, their total momentum before impact is equal to their total momentum after impact, if no net external force acts on the system.

(c) (i) Momentum of plasticine,

$p = mv$

$= 0.20 \times 8.0$

$= 1.6$ N s

The momentum of the plasticine is completely transferred to the ground (the earth).

The kinetic energy of the plasticine is dissipated as heat and sound.

(ii) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(1.00u)(6.50 \times 10^5) + (12.00u)(0)$

$= (1.00u)v_1 + (12.00u)(1.00 \times 10^5)$

∴ Velocity of the neutron after collision

$v_1 = -5.50 \times 10^5$ m s^{-1}

i.e. 5.50×10^5 m s^{-1} in the opposite direction to the initial direction.

The total kinetic energy is conserved.

(iii) Before letting go, the total momentum of the magnets is zero.

After letting go, the magnets spring apart in opposite directions, and the total momentum is also zero.

12. (a) Momentum is a vector quantity with both magnitude and direction while kinetic energy is a scalar quantity with magnitude only.

(b) (i) Momentum = mv

(ii) Kinetic energy = $\frac{1}{2}mv^2$

(c) $mv = 2.4$ .... (1)

$\frac{1}{2}mv^2 = 45$ .... (2)

$\frac{(2)}{(1)}$: $\frac{1}{2}v = 18.75$

∴ $v = 37.5$ m s^{-1}

By (1): $m\,(37.5) = 2.4$

∴ $m = 0.064$ kg = 64 g

(d) (i) Force = Rate of change of momentum

i.e. $F = \frac{mv - mu}{t}$

$-60 = \frac{0 - 2.4}{t}$

∴ Time for tennis ball to stop,

$t = 0.040$ s

(ii) Distance travelled while stopping,

$s = \frac{1}{2}(u + v)t$

$= \frac{1}{2}(37.5 + 0)(0.04) = 0.75$ m

(e) (i) $F = \frac{mv - mu}{t}$

$-60 = \frac{mv - 0}{0.06}$

∴ New momentum of the ball,

$mv = -3.6$ N s, i.e. 3.6 N s in the opposite direction to the initial momentum.

(ii) $mv = -3.6$ N s

$(0.064)v = -3.6$

∴ New velocity of the ball,

$v = -56.25$ m s^{-1}, i.e. 56.25 m s^{-1} in the opposite direction to the initial velocity.

(f) New kinetic energy of the ball

$= \frac{1}{2}mv^2 = \frac{1}{2}(0.064)(56.25)^2 = 101.25$ J

Increase in kinetic energy

$\Delta E_k = 101.25 - 45$

$= 56.25$ J

Mean power delivered to the ball

$= \frac{\Delta E_k}{\Delta t}$

$= \frac{56.25}{(0.04 + 0.06)}$

$= 562.5$ W

(g) Power = Force × Velocity.

Since the ball's velocity is changing continuously, a continuously changing power must be supplied to apply a constant force on the ball.

In practice, the power supplied by a person is approximately constant and the force on the ball builds up to a maximum and then decreases back to zero.

13. Momentum is always conserved.

Only in elastic collision will the kinetic energy be conserved.

Total energy is always conserved.

Collision	Momentum	Kinetic energy	Total energy
Elastic	✓	✓	✓
Inelastic	✓		✓

(b) (i) 1. Since the particles move off together, the collision is *inelastic*.

2. Let u = speed of neutron before capture

Conservation of momentum:

Initial Momentum = Final Momentum

$m(u) + m(0) = (2m)(3.0 \times 10^7)$

∴ $u = 6.0 \times 10^7$ m s^{-1}

(ii) Let v = speed of nitrogen atom after collision

Conservation of momentum

Initial Momentum = Final Momentum

$m(6.0 \times 10^7) + 14m(0) = 15m(v)$

∴ $v = 4.0 \times 10^6$ m s^{-1}

14. (a) (i) The linear momentum of a body is the product of its mass and its velocity.

(ii) The principle of conservation of linear momentum states that when objects of a system interact, their total momentum before impact is equal to their total momentum after impact, if no net external force acts on the system.

(b) (i) This type of interactions are called inelastic interactions.

(ii) It is observed that when no heat enters or leaves the system, comprising the gas and its containing vessel, its pressure and temperature remain constant.

The pressure of a gas depends on the mean speed of its molecules when they collide with the wall of the containing vessel. The constant pressure implies that the gas molecules have a constant mean speed when colliding with the wall of the containing vessel.

The temperature of a gas depends on its mean kinetic energy of the gas molecules, which in turn is a function of the mean speed of the molecules. Therefore, the constant temperature implies that the gas molecules have a constant mean kinetic energy and, so, a constant mean speed.

Since the mean kinetic energy and the mean speed of the gas molecules remain constant, their interactions must, on average, be elastic.

(c) (i) The α-decay could be represented by:

$^{224}_{88}\text{Ra} \rightarrow ^{220}_{86}\text{Rn} + ^{4}_{2}\text{He}$

(ii) Let m = mass of α-particle = $4u$

v = speed of emission of α-particle

Kinetic energy of α-particle,

$E_k = \frac{1}{2}mv^2$

i.e. $9.2 \times 10^{-13} = \frac{1}{2}(4 \times 1.66 \times 10^{-27})\,v^2$

$v^2 = 2.771 \times 10^{14}$

∴ $v = 1.66 \times 10^7$ m s^{-1}

(iii) Let M = mass of Rn = $220u$

V = speed of Rn on emission of α-particle

Conservation of linear momentum:

Total momentum before emission

= Total momentum after emission

i.e. $0 = MV + mv$

$0 = (220u)V + (4u)(1.66 \times 10^7)$

$V = -3.018 \times 10^5$

–ve value ⇒ momentum of Rn is opposite in direction to momentum of α-particle

∴ Speed of Rn = 3.0×10^5 m s^{-1}

(d) (i) Typical range of an α-particle in air
≈ 5 cm
Number of air molecules ionised

$$\approx \frac{\text{Initial energy of } \alpha\text{-particle}}{\text{Approximate energy per air molecule ionised}}$$

$$= \frac{9.2\times10^{-13}}{5.6\times10^{-18}} = 1.64 \times 10^{5}$$

No of air molecules ionised per mm

$$\approx \frac{1.64\times10^{5}}{50}$$

$= 3.28 \times 10^{3}$ mm^{-1}

(ii) The estimate in **(i)** is based on a constant energy per air molecules ionised. The observed increase just before the α-particle is stopped implies that the actual number of molecules ionised per mm in the earlier part of the range would be lower than the estimate. Assuming that the approximate energy lost per air molecule ionised is an average value, the estimate in **(i)** is an average value which should not be affected by the observed localised fluctuation.

15. – 16. HKALE Questions