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Project Editor Maggie Ngan
Book Editors Sara Wong, Maggie Ngan, Tam Sze-ying
Designer Choi Wai-hung
Illustrators Eddie Lo, Rickel Kwok
Production Team Eddie Lo, Miran Shek, Rickel Kwok

Units, Dimensions and Measurements

Self Evaluation Exercise 1 (p.12)

1. B

2. $[\text{Velocity}] = [k][\text{Density}]$
$LT^{-1} = [k]\,ML^{-3}$
$[k] = M^{-1}\,L^{4}\,T^{-1}$
$\therefore$ SI unit for $k = \text{kg}^{-1}\,\text{m}^{4}\,\text{s}^{-1}$

3. $[C] = [\alpha][T]$ and $[C] = [\beta][T^3]$
SI Unit for $C = \text{J K}^{-1}$
SI Unit for $T = \text{K}$
$\therefore$ SI unit for $\alpha = \text{J K}^{-2}$
SI unit for $\beta = \text{J K}^{-4}$

4. **(a)** $[A][g][\rho][v]$
$= (LT^{-2})(ML^{-3})(LT^{-1})$
$= ML^{-1}\,T^{-3}$
Thus, its SI unit is $\text{kg m}^{-1}\,\text{s}^{-3} = \text{N m}^{-1}\,\text{s}^{-1} \neq \text{N m}^{-2}$

(b) $[B][\rho][v]^2$
$= (ML^{-3})(LT^{-1})^2$
$= ML^{-1}\,T^{-2}$
Thus, its SI unit is $\text{kg m}^{-1}\,\text{s}^{-2} = \text{N m}^{-2}$

(c) $[C][\gamma][g][v]^{-2}$
$= (MT^{-2})(LT^{-2})(LT^{-1})^{-2}$
$= ML^{-1}\,T^{-2}$
Thus, its SI unit is $\text{kg m}^{-1}\,\text{s}^{-2} = \text{N m}^{-2}$
$\therefore$ Equations (b) and (c) are homogeneous.

5. **(a)** **(i)** Base units of $\frac{4}{3}\pi r^3$
$= \text{m}^3 \neq$ base units of surface area (m^2)

(ii) Base units of $\frac{\lambda}{T}$
$= \text{m s}^{-1} =$ base units of speed

(iii) Base units of $2\pi\sqrt{\frac{g}{\ell}}$
$= \sqrt{(\text{m s}^{-2})(\text{m}^{-1})} = \text{s}^{-1}$
$\neq$ base units of period (s)

(iv) Base units of $\frac{1}{3}\rho < c^2 >$
$= (\text{kg m}^{-3})(\text{m}^2\,\text{s}^{-2}) = \text{kg m}^{-1}\text{s}^{-2}$
= base units of pressure

surface area of a sphere $= \frac{4}{3}\pi r^3$	×
speed of a wave $= \frac{\lambda}{T}$	✓
period of an oscillating pendulum $= 2\pi\sqrt{\frac{g}{\ell}}$	×
pressure of a gas $= \frac{1}{3}\rho < c^2 >$	✓

(b)

Physical quantity	Magnitude	Unit
The weight of an adult	600	newton
The power of a hair drier	1 000	watt
The energy required to bring a kettleful of water to boil	500 000	joule
The resistance of a domestic filament lamp	1 000	ohm
The wavelength of visible light	600	nanometre

Self Evaluation Exercise 2 (p.20)

1. A
This graph represents the smallest percentage error which indicates the highest accuracy.

2. **(a)** 6.20 ± 0.05 mm
(b) 3.700 ± 0.025 mm
(c) 12.740 ± 0.005 mm

Self Evaluation Exercise 3 (p.28)

1. D
$$\left|\frac{\delta(\frac{p}{q})}{\frac{p}{q}}\right| = \left|\frac{\delta p}{p}\right| + \left|\frac{\delta q}{q}\right|$$

2. C
3. B
4. D
5. **(a)** 3.14
(b) 4.00 A
(c) 891 000 J
(d) 0.000 038 4 s

6. The radius has four significant figures and the length has three significant figures. Thus, the volume can only has three significant figures.
The volume V of the wire
$V = \pi r^2 \ell = \pi(3.167 \times 10^{-3})^2(0.135)$
$= 4.253\ 8 \times 10^{-6}$
$\approx 4.25 \times 10^{-6}\ m^3$

7. (a) The answer can only has one significant figure.
$(0.03 + 0.878\ 9) \times 0.000\ 234$
$= 0.908\ 9 \times 0.000\ 234$
$= 2.126\ 826 \times 10^{-4}$
$\approx 2 \times 10^{-4}$
(b) The answer can only has one significant figure.
$1.509 \times (2.34 - 0.435\ 7) + 1$
$= 1.509 \times 1.904\ 3 + 1$
$= 2.873\ 588\ 7 + 1$
$= 3.873\ 588\ 7$
≈ 4

8. (a) Random
(b) Systematic
(c) Random
(d) Systematic
(e) Systematic
(f) Systematic
(g) Systematic
(h) Systematic

9. $d_1 - d_2 = [(64 - 47) \pm (2 + 1)]$
$= 17 \pm 3$ mm
Max. percentage error $= \frac{3}{17} \times 100\%$
$= 17.6\%$

Self Evaluation Exercise 4 (p.31)

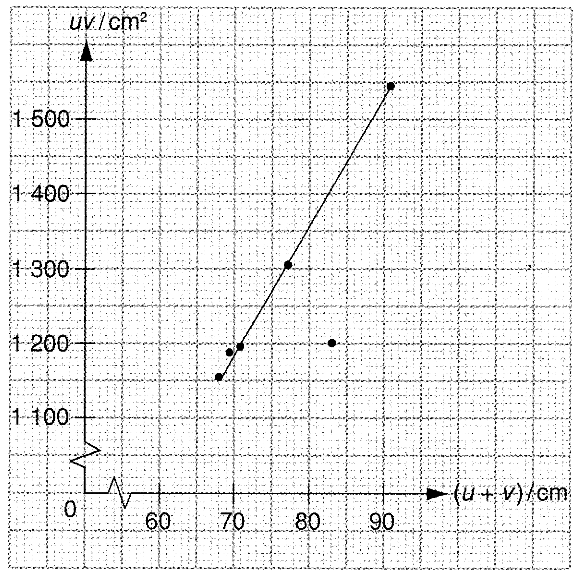
1.

Equation	$F = ma$	$F = \frac{k}{r^2}$	$E_0 = V + Ir$	$A = BC^2 + D$
x-axis	F	$\frac{1}{r^2}$	I	c^2
y-axis	a	F	E_0	A
Slope (m)	$\frac{1}{m}$	k	r	B
y-intercept (c)	0	0	V	D

2. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
$\frac{1}{f} = \frac{v+u}{uv}$
$uv = (u + v)f$

Plot a graph of uv against $u + v$

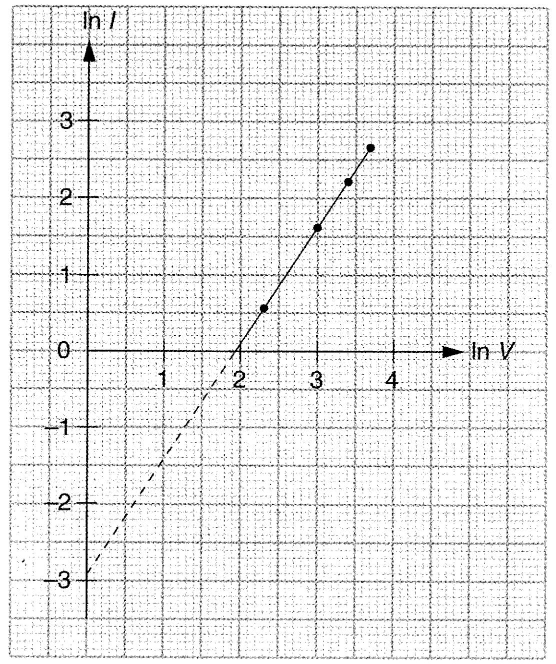
uv / cm^2	1 545.9	1 195.4	1 155.8	1 188.7	1 305.2	1 208.1
$u + v$ / cm	90.8	70.8	68	69.3	77.1	83.8



Slope $= f$
$= \frac{1\ 545.9 - 1\ 155.8}{90.8 - 68}$
$= 17.1$ cm

3. $I = kV^n$
$\ln I = \ln k V^n$
$= \ln k + \ln V^n$
$= n \ln V + \ln k$
Plot a graph of $\ln I$ against $\ln V$

$\ln I$	0.57	1.61	2.21	2.66
$\ln V$	2.30	3.00	3.40	3.69



Slope $= n$
$= \frac{2.66 - 0.57}{3.69 - 2.30}$
$= 1.50$
y-intercept $= \ln k$
$-2.9 = \ln k$
$\therefore\ k = 5.5 \times 10^{-4}$

Review Exercise (p.34)

A. Multiple Choice Questions

1. A
Since the ruler is graduated in mm, i.e. 0.1 cm, the calculated value should be corrected to 1 decimal place. Therefore, 36.3 cm^2.

2. B
Width of the metal $= (5.01 - 0.01) \pm (0.02 + 0.02)$ mm
$= 5.00 \pm 0.04$ mm

B. Structured Questions

3. (a) The number of significant figures = 2
(b) $0.050\ \mu W\ cm^{-2}$
$= 5.0 \times 10^{-8}\ W\ cm^{-2} \times 10^4\ cm^2\ m^{-2}$
$= 5.0 \times 10^{-4}\ W\ m^{-2}$
(c) The order of magnitude $= 10^{-4}$
(d) $\frac{0.05}{5.0} \times 100\%$
$= 1.0\%$
(e) The dimensions of the quantity:
$\frac{[M][L][T]^{-2}/[T]}{[L]^2}$
$= [M][T]^{-3}$

4. (a) If an equation is correct, then its dimensions must be consistent on both sides. However, if the dimensions are consistent on both sides, it does not imply that the equation is correct.
(b) Consider $[v] = LT^{-1}$, $[a] = L$, $[\eta] = ML^{-1}\ T^{-1}$ and $[\rho] = ML^{-3}$
(a) $\left[\frac{A\eta a}{\rho}\right] = [ML^{-1}\ T^{-1}][L][ML^{-3}]^{-1}$
$= L^3\ L^{-1} \neq [v]$
(b) $\left[\frac{B\eta}{a\rho}\right] = [ML^{-1}\ T^{-1}][LML^{-3}]^{-1}$
$= LT^{-1} = [v]$
(c) $\left[\frac{C\rho a}{\eta}\right] = [ML^{-3}][L][ML^{-1}\ T^{-1}]^{-1}$
$= L^{-1}\ T \neq [v]$
Equation (b) is correct.

5. (a) The dimensions of a physical quantity are the terms used to express itself in terms of the fundamental quantities (e.g. mass, time, length). For example, the dimensions of volume is (Length)^3.
(b) (i) $N\ s\ m^{-2} = (kg\ m\ s^{-2})(s\ m^{-2}) = kg\ m^{-1}\ s^{-1}$
(ii) $[\eta] = [A\rho t] = \left[\frac{B\rho}{t}\right]$
$ML^{-1}\ T^{-1} = [A]\ ML^{-3}\ T = [B]\ ML^{-3}\ T^{-1}$
$\therefore\ [A] = L^2\ T^{-2}$ $(m^2\ s^{-2})$
$[B] = L^2$ (m^2)

6. Estimated error $= \pm(\delta p + \delta q)$

7. Percentage error $= \dfrac{\frac{1}{2}\times \text{Least count}}{\text{Measured value}} \times 100\%$

$= \dfrac{\frac{1}{2}\times 0.001}{0.465} \times 100\%$

$= 0.11\%$

8. Cross-section area $(A) = \dfrac{\pi d^2}{4}$

$= \dfrac{\pi}{4}(0.56)^2$

$= 0.246\ 3\ \text{mm}^2$

Maximum fractional error in A:

$\dfrac{\delta A}{A} = \pm 2\dfrac{\delta d}{d}$

$= \pm 2 \times \dfrac{0.01}{0.56}$

∴ maximum error in A:

$\delta A = \pm 2 \times \dfrac{0.01}{0.56} \times 0.246\ 3$

$= \pm 0.009\ \text{mm}^2$

∴ cross-sectional area $(A) = 0.246 \pm 0.009\ \text{mm}^2$

Note: **1.** The error δA is calculated to only one significant figure.

2. Errors occur at the third decimal place ($\pm 0.009\ \text{mm}^2$), therefore, it is sufficient to write the value of A accurate to the third decimal place.

9. Density$(\rho) = \dfrac{\text{Mass }(M)}{\text{Volume }(V)}$

$= \dfrac{M}{\frac{4}{3}\pi(\frac{d}{2})^3}$

Maximum percentage error in ρ is:

$\dfrac{\delta\rho}{\rho} \times 100\% = \left[\dfrac{\delta M}{M} + \dfrac{3\delta d}{d}\right] \times 100\%$

$= \pm[1 + 3 \times 3]\ \%$

$= \pm 10\%$

10. $v = k(\dfrac{\gamma}{\lambda\rho})^{\frac{1}{2}}$

∴ $\dfrac{\delta v}{v} \times 100\% = \pm\left[\dfrac{\frac{1}{2}\delta\gamma}{\gamma} + \dfrac{\frac{1}{2}\delta\lambda}{\lambda} + \dfrac{\frac{1}{2}\delta\rho}{\rho}\right] \times 100\%$

$= \pm\dfrac{1}{2}\left[\dfrac{0.05}{4.30} + \dfrac{5}{100} + \dfrac{20}{1\,450}\right] \times 100\%$

$= \pm 4\%$

11. **(a)** Perimeter $= 2 \times (0.15 + 0.033)$

$= 0.37$ m

(b) Area $= 0.15 \times 0.033$

$= 0.005\ 0\ \text{m}^2$

12. $F = Cv^2$ where C is a constant

∴ $\dfrac{\delta F}{F} = 2\dfrac{\delta v}{v}$

Thus, maximum possible error in measuring the force is $2 \times 1\% = 2\%$.

13. $h = \dfrac{1}{2}gt^2$

$= \dfrac{1}{2}(10)(2.0)^2$

$= 20$ m

$\left|\dfrac{\delta h}{h}\right| = 2\left|\dfrac{\delta t}{t}\right|$

$= 2(\dfrac{0.1}{2.0})$

$= 0.1$

$\delta h = 20 \times 0.1 = 2$ m

Thus, the height of the tower is 20 ± 2 m.

14. **(a)** **(i)** Effective length $= 92.4 + \dfrac{3.12}{2}$

$\approx 92.4 + 1.6$

$= 94.0$ cm

(ii) Percentage error of effective length

$= \dfrac{0.05}{94.0} \times 100\%$

$= 0.053\%$

(iii) Percentage error of time

$= \dfrac{0.05}{36} \times 100\%$

$= 0.1\%$

(b) **(i)** The percentage error of g is:

$\dfrac{\delta g}{g} \times 100\% = \dfrac{\delta \ell}{\ell} \times 100\% + 2\dfrac{\delta T}{T} \times 100\%$

$= (\dfrac{0.05}{94.0} + 2 \times \dfrac{0.05}{36}) \times 100\%$

$= 0.3\%$

(ii) **1.** Use a longer string.

2. Measure the time for more oscillations.

C. Overseas & HKALE Questions

15. **(a)** **(i)** A systematic error is a constant deviation of the readings in one direction from the true value.

(ii) A random error is a scatter of readings about a mean value.

(b) **(i)** Due to limited sensitivity of most meters, the digital readings would probably not be constant but a scatter of readings about a mean value.
This random error could be kept to a minimum by repeating the readings using the same meters, and using the mean of the readings.

(ii) The voltmeter must have a resistance much greater than that of the wire in order to avoid drawing a significant current. A significant current through the voltmeter will make current in the circuit to be significantly higher than what its true value would be without the meters.

(c) **(i)** Resistance of the wire

$R = \dfrac{V}{I} = \dfrac{1.30}{0.76} = 1.71\ \Omega$

$\dfrac{\Delta R}{R} = \dfrac{\Delta V}{V} + \dfrac{\Delta I}{I}$

$\dfrac{\Delta R}{1.71} = \dfrac{0.01}{1.30} + \dfrac{0.01}{0.76}$

$\Delta R = 0.04$

$\therefore\ R = 1.71 \pm 0.04\ \Omega$

(ii) Resistivity of the metal of the wire

$\rho = \dfrac{RA}{\ell} = \dfrac{1.71\times\pi(0.54\times10^{-3})^2/4}{75.4\times10^{-2}}$

$= 5.2 \times 10^{-7}\ \Omega$ m

$\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta R}{R} + \dfrac{2\Delta D}{D} + \dfrac{\Delta\ell}{\ell}$

$\dfrac{\Delta\rho}{5.19\times10^{-7}} = \dfrac{0.04}{1.71} + \dfrac{2\times0.02}{0.54} + \dfrac{0.2}{75.4}$

$\Delta\rho = 0.5 \times 10^{-7}$

∴ $\rho = (5.2 \pm 0.5) \times 10^{-7}\ \Omega$ m

(d) This procedure allows a line of best fit to be drawn, which will help to reduce the effect of random errors as the resistance is no longer computed from a specific reading but from a weighted mean.
The plotted line also helps to check for "poor" readings, i.e. those are "too far" from the line.
It may also be possible to identify systematic errors, if any, e.g. from the intercept of the lines with axes, and hence to avoid it.
The resistance of the wire is calculated from the gradient of the $V - I$ line, and hence the resistivity.

16. **(a)** **1.** ampere

2. mole

(b) A derived unit is a unit expressed in terms of the product and / or quotient of base units.
The unit of energy, joule is said to be a derived unit because it is expressed in terms of the product and / or quotient of the kilogram, the metre and the second, i.e. kg $\text{m}^2\ \text{s}^{-2}$, which are base units.

(c) **(i)** **1.** Density $(\rho) = \dfrac{\text{Mass}}{\text{Volume}}$

Base units of ρ

$= \dfrac{\text{Base units of mass}}{\text{Base units of volume}}$

$= \dfrac{\text{kg}}{\text{m}^3}$

$= \text{kg m}^{-3}$

2. Pressure $(p) = \dfrac{\text{Force}}{\text{Area}}$

Base units of p

$= \dfrac{\text{Base units of force}}{\text{Base units of area}}$

$= \dfrac{\text{kg m s}^{-2}}{\text{m}^2}$

$= \text{kg m}^{-1}\ \text{s}^{-2}$

(ii) $c = \sqrt{\dfrac{\gamma p}{\rho}}$

Base units of c

$= \sqrt{\dfrac{(\text{Base units of }\gamma)(\text{Base units of }p)}{\text{Base units of }\rho}}$

$= \sqrt{\dfrac{\text{kg m}^{-1}\ \text{s}^{-2}}{\text{kg m}^{-3}}}$

$= \text{m s}^{-1}$

(iii) Since the base units [m s^{-1}] are those of speed, the symbol c may represent the speed or velocity of sound in gases.

17.

Prefix	Decimal equivalent
pico	10^{-12}
micro	10^{-6}
giga	10^{9}
tera	10^{12}

18. **(a)** The omission introduces a systematic error because it will produce a constant deviation of the readings in one direction from the true value.

(b) The readings are precise because the micrometre screw gauge can give readings that are close to their mean.
The readings are not accurate because the gauge has a zero error, and so the mean of the readings will not be closed to the true value.

19. **(a)** Mass of an apple = 0.2 kg

(b) Number of joules of energy in 1 kilowatt–hour
= 3 600 000

(c) Wavelength of red light in a vacuum
= 7×10^{-7} m

(d) Pressure due to a depth of 10 m of water
= 100 000 Pa

20.

Quantity	Unit
Speed	$\mathrm{m\ s^{-1}}$
Density	$\underline{\mathrm{kg\ m^{-3}}}$
<u>Frequency</u>	$\mathrm{s^{-1}}$
Electric field strength	$\underline{\mathrm{kg\ m\ s^{-2}\ C^{-1}}\text{ or }\mathrm{N\ C^{-1}}}$
<u>Momentum</u>	$\mathrm{kg\ m\ s^{-1}}$

21. **(a)** Mass, temperature rise

(b) **(i)** Units of thermal energy
= Units of work
= Units of force × Units of displacement
$= \mathrm{kg\ m\ s^{-2}} \times \mathrm{m}$
$= \mathrm{kg\ m^2\ s^{-2}}$

(ii) Units of the constant c

$$= \frac{\text{Units of thermal energy}}{\text{Units of mass} \times \text{Units of temperature}}$$

$$= \frac{\mathrm{kg\ m^2\ s^{-2}}}{\mathrm{kg} \times \mathrm{K}}$$

$$= \mathrm{m^2\ s^{-2}\ K^{-1}}$$