6 Gravitation

Self Evaluation Exercise 6.1 (p.202)

1. A

2. Let M_S be the mass of the sun,
and d be the distance of the sun from the earth.

$$\frac{GM_E m}{d^2} = \frac{GM_S m}{(1.50\times10^{11}-d)^2}$$

$$\frac{1}{d^2} = \frac{3.24\times10^5}{(1.50\times10^{11}-d)^2}$$

$$1.50\times10^{11} - d = 569\,d$$

$$d = 2.63\times10^8 \text{ m}$$

Self Evaluation Exercise 6.2A (p.207)

1. C

$$F = \frac{GMm}{R^2}$$

$$G = \frac{FR^2}{Mm}$$

$$[G] = \frac{[F][R^2]}{[M][m]} = \frac{(\text{MLT}^{-2})(\text{L}^2)}{\text{M}^2} = \text{L}^3\,\text{M}^{-1}\,\text{T}^{-2}$$

Thus, SI unit for G is $\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$.

2. C

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{MG}{R^2}$$

3. D

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2} \propto \frac{M}{R^2} = \frac{\text{mass}}{(\text{radius})^2}$$

4. D

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

$$\because \quad mg = \frac{GMm}{r^2}$$

$$\frac{g}{G} = \frac{M}{r^2}$$

$$\therefore \quad \text{Density} = \frac{3g}{4\pi rG}$$

5. B

$$mg = \frac{GMm}{R^2}$$

$$G = \frac{gR^2}{M}$$

6. Eclipse of the sun:

$$a_1 = \frac{GM_E}{R^2} + \frac{GM_M}{r^2}$$

Eclipse of the moon:

$$a_2 = +\frac{GM_E}{R^2} - \frac{GM_M}{r^2}$$

$$|a_1 - a_2| = \frac{2GM_M}{r^2}$$

7.

$$\frac{GMm}{r^2} = mr\omega^2$$

$$M = \frac{r^3\omega^2}{G} = \frac{(20\times10^3)^3(2\pi)^2}{6.67\times10^{-11}} = 4.73\times10^{24} \text{ kg}$$

8.

$$mg = \frac{GMm}{r^2}$$

$$M = \frac{gr^2}{G} = \frac{9.81\times(6\times10^6)^2}{6.67\times10^{-11}} = 5.29\times10^{24} \text{ kg}$$

9.

$$mg = \frac{GMm}{r^2}$$

$$M = \frac{gr^2}{G}$$

$$\because \quad \rho = \frac{M}{\frac{4}{3}\pi r^3}$$

$$M = \frac{4}{3}\pi\rho r^3$$

$$\therefore \quad g = \frac{4}{3}\pi\rho Gr$$

Self Evaluation Exercise 6.2B (p.213)

1. A

$[g] = [a]$
$= \text{LT}^{-2}$

$\therefore$ SI unit for $g = \text{m s}^{-2}$

2. B

For $x < R_E$,
the outer shell exerts no gravitational force on the mass inside it.

$$F = \frac{GMm}{x^2} = (Gm)\frac{\rho\left(\frac{4}{3}\pi x^3\right)}{x^2} = \frac{4}{3}\pi\rho Gmx \propto x$$

For $x > R_E$,
the gravitational force follows the inverse square law.

3. A

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\left(\rho\frac{4}{3}\pi R^3\right) \quad (R = \text{radius of the sphere})$$

$$= \frac{4}{3}\pi\rho GR \propto R$$

When R is doubled, g is doubled.

4. D

At a point outside the earth,

$$g = \frac{GM}{X^2}$$

$$\frac{g_1}{g_2} = \frac{X^2}{R^2}$$

$$2 = \frac{X^2}{R^2}$$

$$R = \frac{X}{\sqrt{2}}$$

5. D

At the pole,
$W = mg$
At the equator,
$W = mg - mr\omega^2$
Hence, it would be smaller at the equator. This is because the gravitational force of the earth must provide the weight and the centripetal force due to the circular motion of the body.

6. D

$$F = \frac{GMm}{r^2}$$

$$\frac{W'}{W} = \frac{R_E^2}{\left(R_E + \frac{R_E}{50}\right)^2}$$

$$W' = \left(\frac{50}{51}\right)^2 W = 0.96W$$

7. A

8. For a pendulum,

$$a = -\frac{g}{\ell}x$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\because \quad g = \frac{GM}{R^2}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\ell R^2}{GM}}$$

On the earth, $T = 2$ s

$$\therefore \quad \ell = g\left(\frac{T}{2\pi}\right)^2 = 10\times\left(\frac{2}{2\pi}\right)^2 = 1.01 \text{ m}$$

On the moon,

$$T = 2\pi\sqrt{\frac{1.01\times(1\,740\times10^3)^2}{(6.67\times10^{-11})\times(7.35\times10^{22})}} = 4.97 \text{ s}$$

9. $mg' = mg - mr\omega^2$

$$9.81 = g - (6.38 \times 10^6)\left(\frac{2\pi}{8.6\times10^4}\right)^2$$

$$g = 9.84 \text{ m s}^{-2}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}}$$

$$T = \sqrt{\frac{9.81}{9.84}} = 0.998 \text{ s}$$

Self Evaluation Exercise 6.3 (p.222)

1. C

$$F = \frac{GMm}{r^2}$$

$$F = -\frac{dU}{dr}$$

$$U = \frac{GMm}{r}$$

When r increases, the magnitude of F decreases much faster than that of U.

2. C

By the relationship between a gravitational force and the potential energy,

$$F = -\frac{dU}{dr}$$

$= -$(Gradient of the curve)

As the gradient of the curve > 0 for all values of r, $F < 0$ which is an attractive force.

∴ The gradient at any point on the curve represents the force pulling the body towards the planet.

3. B

If we take the gravitational potential energy of the body as zero at infinite distance, then

$$U = -\frac{GMm}{r}$$

Therefore, for any values of r, the gravitational potential energy is negative. For a larger value of r, U would be greater.

4. D

$$V = -\frac{GM}{r}$$

At X, $-800 = -\dfrac{GM}{R}$

At Y, $V = -\dfrac{GM}{2R}$

$$= \frac{1}{2}\left(-\frac{GM}{R}\right)$$

$$= \frac{1}{2} \times (-800)$$

$$= -400 \text{ kJ kg}^{-1}$$

∴ Work done = [(–400 – (–800)) × 1
= +400 kJ

5. A

$$V = \frac{-GMm}{(R+x)}$$

where R is the radius of the earth.

$$V = -\frac{GMm}{R}\left(1+\frac{x}{R}\right)^{-1}$$

$$= -\frac{GMm}{R}\left[1-\frac{x}{R}+\left(\frac{x}{R}\right)^2-\left(\frac{x}{R}\right)^2+...\right]$$

for $x << R$, the terms with power of $\left(\frac{x}{R}\right)$ higher than one can be neglected.

$$\therefore \quad V = -\frac{GMm}{R} + \left(\frac{GMm}{R^2}\right)x$$

At height = 0, $V = -\dfrac{GMm}{R}$

$$\therefore \quad \Delta V = \left(-\frac{GMm}{R}+\frac{GMmx}{R^2}\right) - \left(-\frac{GMm}{R}\right)$$

$$= \left(\frac{GMm}{R^2}\right)x$$

Since $\dfrac{GMm}{R^2}$ is a constant, $\Delta V - x$ graph is a straight line passing through the origin.

6. **(a)** Gravitational potential $V = -\dfrac{GM}{R}$

Earth:

$$V = -\frac{(6.67\times10^{-11})\times(6.0\times10^{24})}{(6.4\times10^6)}$$

$$= -6.26 \times 10^7 \text{ J kg}^{-1}$$

(b) Sun:

$$V = -\frac{(6.67\times10^{-11})\times(2.0\times10^{30})}{(1.5\times10^{11})}$$

$$= -8.90 \times 10^8 \text{ J kg}^{-1}$$

7. Let r be the distance from the earth,
M_E be the mass of the earth,
M_M be the mass of the moon.

In the mid-way,

$$r = \frac{3.8\times10^8}{2} = 1.9 \times 10^8 \text{ m}$$

$$V = \frac{-GM_E}{r} + \left(\frac{-GM_M}{3.8\times10^8 - r}\right)$$

$$= \frac{-GM_E}{1.9\times10^8} - \frac{GM_M}{1.9\times10^8}$$

$$= -\frac{(6.67\times10^{-11})}{(1.9\times10^8)}\left[6.0\times10^{24}+7.4\times10^{22}\right]$$

$$= -2.13 \times 10^6 \text{ J}$$

8. **(a)** Acceleration $= -g$
$= -68.5 \text{ m s}^{-2}$

(b) Rate of change of $V = \dfrac{dV}{dr}$

$= -g$

$= -68.5 \text{ J kg}^{-1} \text{ m}^{-1}$

(c) $g = \dfrac{GM}{r^2}$

$$M = \frac{gr^2}{G} = \frac{68.5\times(1.392\times10^9)^2}{(6.67\times10^{-11})}$$

$$= 2.00 \times 10^{30} \text{ kg}$$

(d) $V = -\dfrac{GM}{r}$

$= -gr$

$= -(68.5) \times (1.392 \times 10^9)$

$= -9.54 \times 10^{10} \text{ J kg}^{-1}$

Self Evaluation Exercise 6.4 (p.226)

1. B

To escape from the gravitational field,

$$\frac{1}{2}mv^2 \geq \frac{GMm}{r}$$

$$v \geq \sqrt{\frac{2GM}{r}}$$

Escape speed $(v) = \sqrt{\dfrac{2GM}{r}}$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{0.2R_E + R_E}{R_E}} = \sqrt{1.2}$$

$$v_2 = 0.91v_1$$

$$= 0.91 \times (1.1 \times 10^4)$$

$$= 1.0 \times 10^4 \text{ m s}^{-1}$$

2. D

3. D

4. To escape from the gravitational field,

$$\frac{1}{2}mv^2 \geq \frac{GMm}{r}$$

$$v \geq \sqrt{\frac{2GM}{r}}$$

$$\frac{v_2}{v_1} = \sqrt{\left(\frac{M_2}{r_2}\right)\left(\frac{r_1}{M_1}\right)}$$

$$= \sqrt{\left(\frac{1}{81}\right)(3.7)}$$

$$v_2 = 0.21v_1$$

$$= 0.21 \times (1.1 \times 10^4)$$

$$= 2\,350.99 \text{ m s}^{-1}$$

Self Evaluation Exercise 6.5A (p.232)

1. D

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GMm}{r} = mv^2$$

∴ Numerical value of kinetic energy

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2} \cdot \frac{GMm}{r}$$

$= \dfrac{1}{2}$ (numerical value of gravitational potential energy)

Hence, its gravitational potential energy is not equal to its kinetic energy.

2. C

$$\frac{GPS}{R^2} = PR\omega^2$$

$$\frac{GS}{R^3} = \left(\frac{2\pi}{T}\right)^2$$

$$T = \frac{2\pi^2}{\sqrt{GS}}(R)^{\frac{3}{2}}$$

$$\propto R^{\frac{3}{2}}$$

3. A
Let m be the mass of the satellite,
M be the mass of the earth.

Then $\frac{GMm}{R^2} = mR\omega^2$

$\frac{GM}{R^3} = (\frac{2\pi}{T})^2$

$R^3 = \frac{GM}{4\pi^2}T^2$

which is independent of m.
Provided that their periods are the same, the radii of their orbits are the same.

4. C
Since the gravitational forces acting on each other are an action and reaction pair, by Newton's third law, their magnitude must be equal.

5. A
$\frac{GM_E m}{r^2} = mr\omega^2$

$r^3 = \frac{GM_E}{\omega^2}$

$r = (\frac{GM_E}{\omega^2})^{\frac{1}{3}}$

6. C
Originally, the gravitational force acting on the satellite provided the necessary centripetal force for it to move in a circular orbit. But when the engine was fired, the resultant force acting on it becomes zero. According to Newton's first law, it moves with uniform velocity without changing its direction of motion and hence along a tangent to its orbit.

7. A
To move in a circular orbit,

$\frac{GMm}{r^2} = \frac{mv^2}{r}$

$v^2 = \frac{GM}{r}$

when r increases, v decreases.
Since the radius of the earth is R_E, there is no solution for $r \le R_E$.

8. A
By Kepler's third law,

$r^3 \propto T^2$

$\frac{r_1^3}{r_2^3} = \frac{T_1^2}{T_2^2}$

When $r_1 = 4r_2$

$T_1 = (4)^{\frac{3}{2}} T_2$

$= 8T_2$

$\therefore$ In terms of the year on the planet X, his age becomes 10 years old.

9. C

$F = -\frac{GMm}{r^2}$

$\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$

$\frac{F_1}{F_2} = \frac{(5\,760+144)^2}{(5\,760)^2}$

$F_2 = 0.95F_1$

The force in 144 km above the earth is less than the force on the earth's surface by 5%.

10. **(a)** The orbital plane will then be perpendicular to the axis of the self-rotating earth and thus the angular velocity of the satellite does not vary with time. The satellite must travel from west to east because the direction of spinning of the earth is from west to east. The satellite will stay over the same place when viewed from the surface of the earth.

(b) **(i)** Speed of the satellite (v)

$= \omega_0 r$

$= (\frac{2\pi}{24\times60\times60}) \times (2.32 \times 10^7)$

$= 1.69 \times 10^3$ m s^{-1}

(ii) Acceleration of the satellite (a)

$= \omega_0^2 r$

$= (\frac{2\pi}{24\times60\times60})^2 \times (2.32 \times 10^7)$

$= 0.12$ m s^{-2}

(iii) It provides the centripetal force between them

$F = ma$

$= 1\,500 \times 0.12$

$= 184.04$ N

(iv) $F = G\frac{Mm}{r^2}$

$184.04 = (6.67 \times 10^{-11}) \times \frac{M \times 1\,500}{(2.32\times10^7)}$

$M = 4.27 \times 10^{16}$ kg

Self Evaluation Exercise 6.5B (p.238)

1. B
In order to escape from the gravitational field of the earth,

$\frac{1}{2}mv^2 \ge \frac{GMm}{R_E}$

$\frac{1}{2}mv^2 \ge gR_E$

$v \ge \sqrt{2gR_E}$

2. D
It is in fact its gravitational potential energy,

$W = \frac{GMm}{r}$

$= \frac{(6.7\times10^{-11})\times(5.0\times10^{24})\times(2.0)}{(6.1\times10^6)}$

$= 1.1 \times 10^8$ J

3. A
In a higher orbit, the gravitational force acting on it is:

$F = \frac{GMm}{r^2}$

$\therefore F \propto \frac{1}{r^2}$

$a = \frac{GM}{r^2} \propto \frac{1}{r^2}$

$v^2 = ar = \frac{GM}{r} \propto \frac{1}{r}$

$\omega^2 = \left(\frac{v}{r}\right)^2 = \frac{GM}{r^3} \propto \frac{1}{r^3}$

$\therefore$ When r increases, F, a, v and ω will decrease.
But the gravitational potential energy which is $-\frac{GMm}{r}$ will be less negative.

4. B
In order to move in a circular orbit,

$\frac{GM_E m}{r^2} = \frac{mv^2}{r}$

$\therefore \frac{1}{2}mv^2 = \frac{GM_E m}{2r}$

Energy supply
= Change in potential energy and kinetic energy

$= \left[\left(\frac{-GM_E m}{r_2}\right) - \left(\frac{-GM_E m}{r_1}\right)\right] + \left[\frac{GM_E m}{2r_2} - \frac{GM_E m}{2r_1}\right]$

$= GM_E m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{GM_E m}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$= \frac{GM_E m}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

5. A
In a circular orbit,

$\frac{GM_E m}{r^2} = \frac{mv^2}{r}$

$\therefore \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{GM_E m}{r}\right)$

Total energy = Potential energy + Kinetic energy

$= -\frac{GM_E m}{r} + \frac{1}{2}\left(\frac{GM_E m}{r}\right)$

$= -\frac{GM_E m}{2r}$

6. B

7.
$\frac{GMm}{r^2} = \frac{mv^2}{r}$

$\frac{1}{2}mv^2 = \frac{GMm}{2r}$

KE per unit mass $= \frac{1}{2}v^2$

$\therefore \frac{\frac{1}{2}mv^2}{m} = \frac{GM}{2r}$

$= \frac{(6.67\times10^{-11})\times(7.35\times10^{22})}{2\times(1.74\times10^6)}$

$= 1.41 \times 10^6$ J kg^{-1}

8. **(a)** **(i)** $U = -\frac{GM_E m}{(R_E + h)}$

(ii) $V = \frac{GM_E}{(R_E + h)}$

(b) Work done = Change in potential energy

$= \frac{GM_E m}{R_E} - \left(\frac{GM_E m}{(R_E + h)}\right)$

$= GM_E m\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right)$

$= GM_E m\left[\frac{h}{R_E(R_E + h)}\right]$

For $h << R_E$, $R_E(R_E + h) \approx R_E^2$

$\therefore$ Work done $= \dfrac{GM_E mh}{R_E^2}$

(c) $\dfrac{1}{2}mv^2 = -\dfrac{GM_E m}{2R_E} - \left(-\dfrac{GM_E m}{R_E}\right)$

$= \dfrac{1}{2}\left(\dfrac{GM_E m}{R_E}\right)$

$v = \sqrt{\dfrac{GM_E}{R_E}}$

Self Evaluation Exercise 6.6 (p.241)

1. D

Review Exercise 6 (p.245)

A. Structured Questions

1. **(a)** $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$

$v = \sqrt{\dfrac{GM}{r}}$

$= \sqrt{\dfrac{(6.67\times10^{-11})\times(5.98\times10^{24})}{(6.38\times10^6+160\,000)}}$

$= 7\,810 \text{ m s}^{-1}$

(b) $v = \dfrac{2\pi r}{T}$

$T = \dfrac{2\pi\times(6.38\times10^6+160\,000)}{7\,810}$

$= 5\,261 \text{ s}$

2. $\dfrac{GM_E M_M}{r^2} = M_M r\omega^2$

$\dfrac{GM_E}{r^3} = \left(\dfrac{2\pi}{T}\right)^2$

$M_E = \left(\dfrac{r^3}{T^2}\right)\left(\dfrac{4\pi^2}{G}\right)$

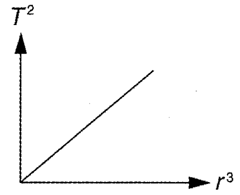
$= \left[\dfrac{(3.85\times10^8)^3}{(2.36\times10^6)^2}\right]\times\dfrac{4\pi^2}{6.67\times10^{-11}}$

$= 6.07\times10^{24}$ kg

3. $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r} = mr\omega^2$

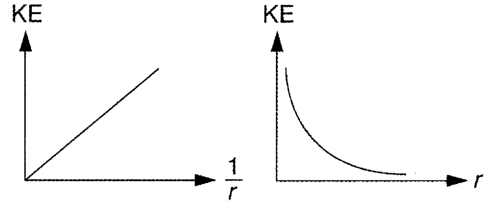
(a) $\dfrac{GM}{r^2} = r\left(\dfrac{2\pi}{T}\right)^2$

$T^2 = \dfrac{4\pi^2}{GM}r^3$



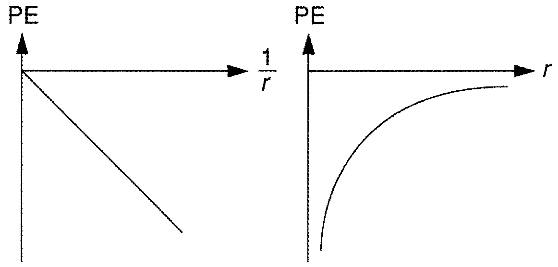
(b) $\dfrac{GMm}{r} = mv^2$

$\text{KE} = \dfrac{1}{2}mv^2 = \left(\dfrac{GMm}{2}\right)\left(\dfrac{1}{r}\right)$



(c) Gravitational potential energy $= \int_\infty^r \dfrac{GMm}{r^2}dr$

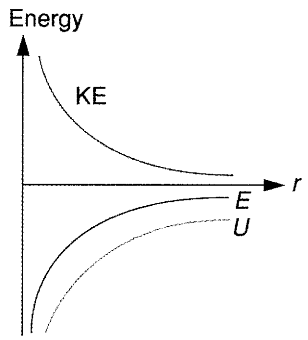
$= -\dfrac{GMm}{r}$



(d) Total energy = Kinetic energy + Gravitational potential energy

$= \dfrac{1}{2}\left(\dfrac{GMm}{r}\right) - \dfrac{GMm}{r}$

$= -\dfrac{1}{2}\left(\dfrac{GMm}{r}\right)$



(e) $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$

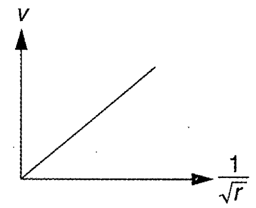
$GMm = mv^2r$

$(mvr)^2 = GMm^2r$

Angular momentum $= \sqrt{GMm^2r}$

(f) $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$

$v = \sqrt{\dfrac{GM}{r}}$



4. **(a)** Density of Mars, $\rho_M = \dfrac{0.11M_E}{\frac{4}{3}\pi\left(\frac{6900}{2}\right)^3}$

Density of earth, $\rho_E = \dfrac{M_E}{\frac{4}{3}\pi\left(\frac{1.3\times10^4}{2}\right)^3}$

$\dfrac{\rho_M}{\rho_E} = 0.11\times\left(\dfrac{1.3\times10^4}{6900}\right)^3$

$= 0.736$

(b) $g_M = \dfrac{GM_M}{r_M^2}$

$= 0.11\times\left[\dfrac{GM_E}{r_E^2}\right]\left[\dfrac{r_E^2}{r_M^2}\right]$

$= (0.11\times10)\times\left(\dfrac{1.3\times10^4}{6900}\right)^2$

$= 3.9 \text{ m s}^{-2}$

(c) For $\dfrac{1}{2}mv^2 \geq \dfrac{GMm}{r}$

$v \geq \sqrt{\dfrac{2GM}{r}}$

Escape speed $= \sqrt{\dfrac{2GM}{r}} = \sqrt{2gr}$

$\therefore$ Escape speed on Mars

$= \left(2\times3.9\times\dfrac{6900\times10^3}{2}\right)^{\frac{1}{2}}$

$= 5.2\times10^3 \text{ m s}^{-1}$

5. Let v_e be the escape speed, v_o be the orbital speed.

$\dfrac{1}{2}mv_e^2 = \dfrac{GMm}{r}$

$v_e = \sqrt{\dfrac{2GM}{r}}$

$\dfrac{GMm}{r^2} = \dfrac{mv_o^2}{r}$

$v_o = \sqrt{\dfrac{GM}{r}}$

$\therefore$ $v_e = \sqrt{2}\,v_o$

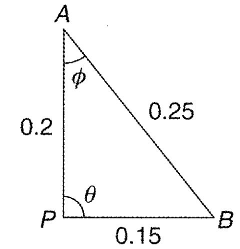
6. **(a)** By cosine rules in triangle,

$a^2 = b^2 + c^2 - 2bc\cos A$

$(0.25)^2 = (0.20)^2 + (0.15)^2 - 2(0.20)(0.15)\cos\theta$

$\cos\theta = 0$

$\theta = 90°$



$\therefore$ The ABP in fact forms a right–angled triangle,

$g = \dfrac{GM_A}{r_A^2}\sin\phi + \left(\dfrac{GM_B}{r_B^2}\right)\sin(180° - 90° - \phi)$

$= G\left[\dfrac{M_A}{r_A^2}\sin\phi + \dfrac{M_B}{r_B^2}\cos\phi\right]$

$\sin\phi = \dfrac{BP}{AB}$, $\cos\phi = \dfrac{AP}{AB}$

$\therefore$ $g = (6.67\times10^{-11})\times\left[\dfrac{8\,000}{0.2^2}\left(\dfrac{0.15}{0.25}\right) + \dfrac{6\,000}{0.15^2}\left(\dfrac{0.2}{0.25}\right)\right]$

$= 2.22\times10^{-5} \text{ N kg}^{-1}$

(b) $V = -\dfrac{GM_A}{r_A} - \dfrac{GM_B}{r_B}$

$= -(6.67\times10^{-11})\times\left(\dfrac{8\,000}{0.2} + \dfrac{6\,000}{0.15}\right)$

$= -5.34\times10^{-6} \text{ J kg}^{-1}$

B. Overseas & HKALE Questions

7. (a) (i) According to Newton's Law of Gravitation, gravitational force acted on the 1.00 kg mass,

$$F_G = \frac{GMm}{r^2}$$
$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.00)}{(6\ 370\times10^3)^2}$$
$$= 9.83\ \text{N}$$

(ii) Force to maintain circular path of the mass,

$$F_C = mr\omega^2$$
$$= mr\left(\frac{2\pi}{T}\right)^2$$
$$= (1.00)(6\ 370\times10^3)\left(\frac{2\pi}{24\times60\times60}\right)^2$$
$$= 0.033\ 7\ \text{N}$$

(iii) Accurate reading of resultant force,

$$F_R = F_G - F_C$$
$$= 9.829\ 9 - 0.033\ 7$$
$$= 9.80\ \text{N}$$

(b) (i) Acceleration due to F_G alone

$$= \frac{F_G}{m} = \frac{9.83}{1.00} = 9.83\ \text{m s}^{-2}$$

(ii) Acceleration due to F_R,

$$= \frac{F_R}{m} = \frac{9.80}{1.00} = 9.80\ \text{m s}^{-2}$$

(c) The statement is not correct.
As illustrated in **(b)**, the acceleration due to the resultant force is slightly less than the acceleration due to gravity (alone).

8. (a) According to Newton's Law of Gravitation, gravitational force acted on 1.00 kg mass on surface of the earth,

$$F = \frac{GMm}{r^2}$$
$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.00)}{(6.37\times10^6)^2}$$
$$= 9.83\ \text{N}$$

(b) Earth's gravitational field strength at its surface,
$g = 9.83\ \text{N kg}^{-1}$

(c) Gravitational potential at a point in a gravitational field is the work done in bringing a unit mass from infinity to the point.

(d) Since 800 m is very small compared to the earth's radius, g is approximately constant.
∴ Difference in gravitational potential

$$= \text{Work done on unit mass}$$
$$= mgh$$
$$= 1\times9.83\times800$$
$$= 7\ 860\ \text{J kg}^{-1}$$

9. (a) Assuming that the earth is a perfect sphere, distance from the equator to the North pole

$$= \frac{1}{4}(2\pi r)$$
$$= \frac{1}{2}\pi(6.378\times10^6)$$
$$= 10\ 018.6\ \text{km}$$

Percentage error

$$= \frac{10\ 018.6 - 10\ 000}{10\ 000}\times100\%$$
$$= 0.186\%$$

(b) Gravitational force by the earth on the moon

$$= \frac{GMm}{r^2}$$
$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(7.35\times10^{22})}{(3.84\times10^8)^2}$$
$$= 1.99\times10^{20}\ \text{N}$$

(c)

$$F = ma$$
$$1.988\times10^{20} = (7.35\times10^{22})a$$

∴ Acceleration of the moon,

$$a = 2.70\times10^{-3}\ \text{m s}^{-2}$$

- The direction of the acceleration is towards the centre of the earth.
- The acceleration is always perpendicular to the velocity and only changes its direction.

(d)

$$a = r\omega^2$$
$$2.704\ 7\times10^{-3} = (3.84\times10^8)\omega^2$$
$$\omega^2 = 7.043\times10^{-12}$$

∴ Angular velocity of the moon,

$$\omega = 2.65\times10^{-6}\ \text{rad s}^{-1}$$

Period of the orbit,

$$T = \frac{2\pi}{\omega}$$
$$= \frac{2\pi}{(2.654\times10^{-6})}$$
$$= 2.37\times10^6\ \text{s}$$

(e) Gravitational force on satellite
= Mass × Centripetal acceleration of satellite

i.e. $$\frac{GMm}{r^2} = m\times r\omega^2$$
$$\omega = \sqrt{\frac{GM}{r^3}}$$

Period of the satellite

$$T = \frac{2\pi}{\omega}$$
$$= 2\pi\sqrt{\frac{r^3}{GM}}$$
$$= \sqrt{\frac{4\pi^2r^3}{GM}}$$

(f) For geostationary orbit, T = 24 hrs

$$T = \sqrt{\frac{4\pi^2r^3}{GM}}$$
$$24\times60\times60 = \sqrt{\frac{4\pi^2r^3}{(6.67\times10^{-11})(5.98\times10^{24})}}$$
$$r^3 = 7.542\times10^{22}$$

∴ Radius of geostationary orbit,

$$r = 4.23\times10^7\ \text{m}$$

10. (a) (i) The gravitational field strength at a point in free space is the gravitational force per unit mass acting on any object placed there.

(ii) A unit for gravitational field strength is N kg^{-1}.

(iii) Base units of gravitational field strength
= (Base units of force)(Base units of mass)$^{-1}$
= (kg m s^{-2})(kg^{-1})
= m s^{-2} = base units of acceleration

(b) (i) Newton's Law of Gravitation:

$F = G\dfrac{Mm}{r^2}$ where

F = gravitational force of attraction
G = universal gravitational constant
M, m = masses of the 2 bodies involved
r = distance between centres of the masses

(ii) Gravitational field strength,

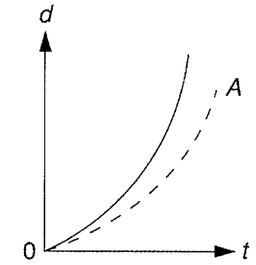
$$g = \frac{\text{Gravitational force}}{\text{Mass}}$$
$$= \frac{F}{m}$$
$$= \left(\frac{GMm}{r^2}\right)\left(\frac{1}{m}\right)$$
$$= \frac{GM}{r^2}$$

(iii) The radius of the earth, $R = 6.38\times10^6$ m, is many times larger than the distance 1 000 m.
i.e. 1 000 m is a small percentage of R.

$$\therefore\ g = \frac{GM}{(R+1\ 000)^2}$$
$$\approx \frac{GM}{R^2}$$

(c) (i) The gradient of a d–t curve of an object is a measure of the speed of the object.
The changing gradient of the d–t curve indicates that the speed of the object is changing, i.e. the object is accelerating.

(ii) Through air, due to air resistance, the object will fall a shorter distance over the same time.



11. (a) (i) The angular velocity of a body is the rate of change of the angular displacement of the body with respect to the centre of the circular path that it describes.

(ii) Period of the earth, T = 362.25 days
Angular velocity of the earth,

$$\omega_E = \frac{2\pi}{T}$$
$$= \frac{2\pi}{(365.25\times24\times60\times60)}$$
$$= 0.199\ \mu\text{rad s}^{-1}$$

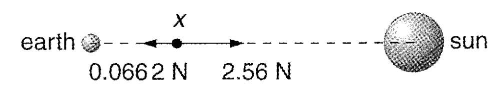
(b) (i) 1. Pull of the earth on the satellite,

$$F_g = \frac{GMm}{r^2}$$
$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(425)}{(1.60\times10^9)^2}$$
$$= 0.066\ 2\ \text{N}$$

2. Pull of the sun on the satellite,

$$F_S = \frac{GMm}{r^2}$$
$$= \frac{(6.67\times10^{-11})(1.99\times10^{30})(425)}{(1.50\times10^{11}-1.60\times10^9)^2}$$
$$= 2.56\ \text{N}$$

(ii)



(iii) 1. Resultant force

$F_R = F_S - F_g$
$= 2.562 - 0.066\ 2$
$= 2.50$ N (towards the sun)

2. Acceleration of the satellite

$a = \frac{F_R}{m}$
$= \frac{2.496}{425}$
$= 5.87 \times 10^{-3}$ m s^{-2}

(iv) $a = r\omega^2$

$5.872 \times 10^{-3} = (1.50 \times 10^{11} - 1.60 \times 10^9)\omega^2$
$\omega^2 = 3.956\ 9 \times 10^{-14}$

$\therefore$ Angular velocity of the satellite,
$\omega_S = 0.199\ \mu$ rad s^{-1}

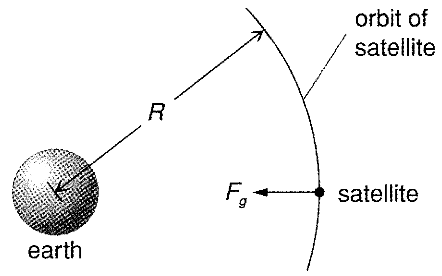
(v) With $\omega_S = \omega_E$, the satellite will remain on the same relative position between the earth and the sun.
This orbit around the sun is preferred because with a satellite orbiting round the earth, there will be times where the sun is obscured by the earth.

(vi) Two disadvantages are:

- As the satellite is nearer to the sun, it will have to withstand greater heat and so will cost more to build.
- Since the satellite is always between the sun and the earth, it will always cast a shadow over the earth's surface.

12. (a) Gravitational potential ϕ at a point in a gravitational field is defined as the work done in bringing unit mass from infinity to that point.

(b) (i)



(ii) For a satellite in orbit, the gravitational force provides the centripetal force. It implies that the acceleration due to the gravitational force is equal to the centripetal acceleration, which will continuously change the satellite's velocity, but will not move the satellite in the direction of the force.

(iii) Gravitational force = Centripetal force

$\frac{GMm}{R^2} = \frac{mv^2}{R}$
$\frac{GM}{R} = v^2$
$v = \sqrt{\frac{GM}{R}}$

(c) Speed of satellite in new orbit,

$v_f = \sqrt{\frac{GM}{R}}$
$= \sqrt{\frac{(6.67\times10^{-11})(6.0\times10^{24})}{6\ 890\times10^3}}$
$= 7\ 620$ m s^{-1}

(d) (i) 1. Change in kinetic energy,

$\Delta E_k = E_{k,\ final} - E_{k,\ initial}$
$= \frac{1}{2}m\left[v_f^2 - v_i^2\right]$
$= \frac{1}{2}(120)[7\ 620^2 - 7\ 780^2]$
$= -1.48 \times 10^8$ J

2. Change in potential energy,

$\Delta E_p = m\Delta\phi = m(\phi_{final} - \phi_{initial})$
$= m\left[-\frac{GM}{R_f} - \left(-\frac{GM}{R_i}\right)\right]$
$= mGM\left[-\frac{1}{R_f} + \frac{1}{R_i}\right]$
$= m(6.67 \times 10^{-11})(6.0 \times 10^{24})\left[-\frac{1}{6\ 890\times10^3} + \frac{1}{6\ 610\times10^3}\right]$
$= 2.95 \times 10^8$ J

3. Change in total energy,

$\Delta E = \Delta E_p + \Delta E_k$
$= (2.953 \times 10^8) + (-1.478 \times 10^8)$
$= 1.48 \times 10^8$ J

(ii) The change in total energy is an increase.

13. (a) (i) All values of gravitational potential are negative because gravity is a force of attraction and work is done by a mass moving from infinity towards an attracting body, i.e. the work done in moving the mass is negative.

(ii) The gradient at a point on the graph of the figure gives the gravitational field strength which in turn is numerically equal to the acceleration of free fall at that point.

(iii) 1. From the figure, the distance at which the gradient is zero, and so acceleration of free fall is zero,
$d_o = 13.6 \times 10^6$ m

2. Acceleration of free fall on surface of Claron,
a_C = Potential gradient at surface of Claron
$= \frac{0.135\times10^6}{2.6\times10^6}$
$= 0.052$ m s^{-2}

(b) (i) Speed (and kinetic energy E_k) is minimum if it is due only to loss in potential energy, ΔE_p, from the point with zero potential gradient.

i.e. $E_k = \Delta E_p$

$\frac{1}{2}mv_{min}^2 = \Delta\phi m$
$v_{min}^2 = 2\Delta\phi$
$= 2[(-29.56) - (-30.0)] \times 10^6$
$= 880\ 000$

$\therefore$ Minimum speed with which rock hits surface,

$v_{min} = \sqrt{880\ 000}$
$= 938.1$ m s^{-1}

(ii) From Pluto to Claron, minimum speed on reaching surface of Claron is different because the loss in potential, $\Delta\phi$, is different.

14. (a) $g = \frac{GMm}{r^2}$

Mass of the earth,

$M = \frac{gr^2}{G}$
$= \frac{(9.81)(6.38\times10^6)^2}{6.67\times10^{-11}}$
$= 5.99 \times 10^{24}$ kg

(b) (i) For geostationary orbit,
period $T = 24 \times 60 \times 60$
$= 86\ 400$ s
Angular speed,

$\omega = \frac{2\pi}{T}$
$= \frac{2\pi}{86\ 400}$
$= 7.27 \times 10^{-5}$ rad s^{-1}

(ii) For a mass m in the orbit,

$m\frac{GM}{r^2} = mr\omega^2$
$r^3 = \frac{GM}{\omega^2}$
$= \frac{(6.67\times10^{-11})(5.987\times10^{24})}{(7.272\times10^{-5})^2}$
$= 7.551 \times 10^{22}$

Radius of the orbit,

$r = \sqrt[3]{7.55\times10^{22}}$
$= 4.23 \times 10^7$ m

15. (a) The gravitational potential at a point is defined as the work done in taking a unit mass from infinity to that point.

(b) Work has to be done against the gravitational field of an isolated mass when moving an object positioned close to the mass to a position further away from the mass. Since the potential at infinity is the maximum value and taken to be zero by convention, the values of gravitational potential near the mass are hence negative.

(c) (i) Change in gravitational potential

$= -\frac{GM}{r_f} - \left(-\frac{GM}{r_i}\right)$
$= GM\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$
$= 6.67 \times 10^{-11} \times 6.0 \times 10^{24}\left(\frac{1}{6.4\times10^3\times10^3} - \frac{1}{(6.4\times10^3+1.3\times10^4)\times10^3}\right)$
$= 4.19 \times 10^7$ J kg^{-1}

(ii) By conservation of total energy,
Loss in kinetic energy = Gain in potential energy

$\frac{1}{2}mv^2 = m\Delta\phi$

where $\Delta\phi$ = change in gravitational potential

$v = \sqrt{2\Delta\phi}$
$= \sqrt{2\times4.19\times10^7}$
$= 9.15 \times 10^3$ m s^{-1}

(d) The gravitational acceleration of the object varies during its flight from the surface to the earth to the altitude of 13×10^4 km. Hence, the equation is not appropriate for the calculation in **(c)(ii)**, since the equation can only be used in cases where the acceleration, a is uniform during travel.

16. – 17. HKALE Questions