2 Kinematics

Self Evaluation Exercise 2.2 (p.88)

1. B

For constant acceleration,

$$s = ut + \frac{1}{2}at^2$$

$\because$ $\frac{1}{2}a > 0$

$\therefore$ The graph s against t is a parabola and opens upwards.

2. C

When the ball is falling, it undergoes constant acceleration due to gravity. Therefore, the slope of the velocity is the same for any time except at the moment of rebounce.
Also, the ball falls with a downward velocity (negative) and after rebounce, its velocity should be upward (positive).

3. **(a)** **(i)** The velocity v of the time-interval from $t = 0$ to $t = 5$ s is

$$v = \frac{s}{t} = \frac{9}{5} = 1.8 \text{ m s}^{-1}$$

(ii) The velocity v of the time-interval from $t = 5$ s to $t = 8$ s is zero, because there is no change of displacement.

(iii) The velocity v of the time-interval from $t = 8$ s to $t = 12$ s is

$$v = \frac{s}{t} = \frac{28-9}{12-8} = 4.75 \text{ m s}^{-1}$$

(b) The total displacement of the boy is 28 m.

(c) The average velocity over the whole journey is

$$\bar{v} = \frac{s}{t} = \frac{28}{12} = 2.33 \text{ m s}^{-1}$$

4. **(a)** No. The car accelerates only in the time-intervals from $t = 0$ to $t = 4$ s and from $t = 6$ s to $t = 8$ s. In the period $t = 4$ s to $t = 6$ s, the car travelled with constant velocity.

(b) The acceleration a between $t = 0$ and $t = 4$ s is

$$a = \frac{\Delta v}{\Delta t} = \frac{12-8}{4} = 1 \text{ m s}^{-2}$$

(c) The area covered between $t = 0$ and $t = 4$ s is

$$\text{Area} = \frac{(8+12)\times 4}{2} = 40 \text{ m}$$

The area covered between $t = 4$ s and $t = 6$ s is

$$\text{Area} = (6 - 4) \times 12 = 24 \text{ m}$$

The area covered between $t = 6$ s and $t = 8$ s is

$$\text{Area} = \frac{1}{2} \times 12 \times \left[\frac{12\times(8-6)}{(12+5)}\right] + \frac{1}{2} \times (-5) \times \left[2 - \frac{12\times(8-6)}{(12+5)}\right]$$
$$= 8.47 - 1.47$$
$$= 7 \text{ m}$$

Thus, the average velocity $\bar{v}$ is

$$\bar{v} = \frac{\sum s}{\sum t} = \frac{40+24+7}{8} = 8.875 \text{ m s}^{-1}$$

Self Evaluation Exercise 2.3 (p.92)

1. A

$$v^2 = u^2 + 2as \qquad \text{......(1)}$$
$$(15)^2 = (30)^2 + 2a(75)$$
$$a = -4.5 \text{ m s}^{-2}$$

Substitute $v = 0$ m s^{-1}, $a = -4.5$ m s^{-2}, $u = 15$ m s^{-1} into (1),

$$(0)^2 = (15)^2 + 2(-4.5)s$$
$$s = 25 \text{ m}$$

2. C

$$x = ut + \frac{1}{2}at^2$$
$$\frac{x}{t} = (\frac{1}{2}a)t + u$$

When a graph of $\frac{x}{t}$ against t is plotted, a straight line with slope $\frac{1}{2}a$ and y-intercept u is obtained.

3. A

4. C

5. A

6. C

7. The acceleration a of the boat is given by

$$s = ut + \frac{1}{2}at^2$$
$$360 = 50 \times 10 + \frac{1}{2}a(10)^2$$
$$a = -2.8 \text{ m s}^{-2}$$

8. Between $t = 0$ and $t = 10$ s, the MTR travels

$$s_1 = ut + \frac{1}{2}at^2$$
$$= \frac{1}{2}(2)(10)^2 \qquad (u = 0)$$
$$= 100 \text{ m}$$

The speed at $t = 10$ s is

$$v^2 = 2as_1$$
$$v = \sqrt{2 \times 2 \times 100}$$
$$= 20 \text{ m s}^{-1}$$

Between $t = 10$ s and $t = 50$ s, the MTR moves with constant speed for 40 s. The MTR travels

$$s_2 = vt$$
$$= 20 \times 40$$
$$= 800 \text{ m}$$

Before the MTR stops, it travels

$$v^2 = u^2 + 2as_3$$
$$0 = 20^2 + 2(-4)(s_3)$$
$$s_3 = 50 \text{ m}$$

Thus, the distance d between the two stations is

$$d = s_1 + s_2 + s_3$$
$$= 100 + 800 + 50$$
$$= 950 \text{ m}$$

9. Put downward as positive.
The speed v of the stone just before hitting the water surface is

$$v^2 = u^2 + 2as$$
$$v^2 = 2 \times 10 \times 20$$
$$v = 20 \text{ m s}^{-1}$$

It penetrates the river to a depth of 2 m below the water surface. The average retardation a is

$$v^2 = u^2 + 2as$$
$$0 = 20^2 + 2(a)(2)$$
$$a = -100 \text{ m s}^{-2}$$

Self Evaluation Exercise 2.4 (p.96)

1. D

$$s = ut + \frac{1}{2}at^2$$

Let s_1 and s_2 be the displacement at t_1 and t_2 respectively.

$$s_2 - s_1 = \frac{1}{2}(g)t_2^2 - \frac{1}{2}(g)t_1^2$$
$$2h = g(t_2^2 - t_1^2)$$
$$g = \frac{2h}{t_2^2 - t_1^2}$$

2. D

3.

$$s = ut + \frac{1}{2}at^2$$
$$-0.14 = 0 + \frac{1}{2}(-10)t^2$$
$$t = 0.17 \text{ s}$$

4. The height h of the point of release above the ground is

$$h = ut + \frac{1}{2}at^2$$
$$= \frac{1}{2}(10)(2.6)^2$$
$$= 33.8 \text{ m}$$

5. Put upward as positive.
(a) The time t is

$$v = u + at$$
$$1 = 10 + (-10)t$$
$$t = 0.9 \text{ s}$$

(b) The time t is

$$v = u + at$$
$$-1 = 10 + (-10)t$$
$$t = 1.1 \text{ s}$$

Self Evaluation Exercise 2.5 (p.98)

1. D
When the man is rowing the boat across the river, the resultant velocity v_1 is

$$v_1 = \sqrt{v_w^2 - v_m^2}$$
$$= \sqrt{(v_w + v_m)(v_w - v_m)}$$

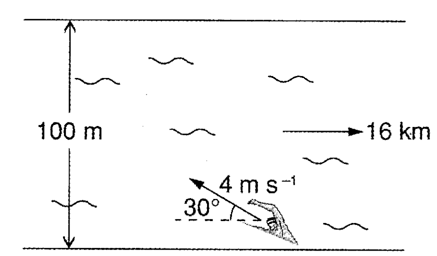
When the man is rowing the boat down the stream, the resultant velocity v_2 is

$$v_2 = v_w + v_m$$

As the distances travelled of the two cases are the same, the ratio of $t_1 : t_2$ is

$$s_1 = s_2$$
$$v_1 t_1 = v_2 t_2$$
$$\frac{t_1}{t_2} = \frac{v_2}{v_1}$$
$$= \frac{\sqrt{(v_w + v_m)^2}}{\sqrt{(v_w + v_m)(v_w - v_m)}}$$
$$= \frac{\sqrt{(v_w + v_m)}}{\sqrt{(v_w - v_m)}}$$

2.



(a) The vertical component of the velocity of the swimmer is

$$v_y = v \sin\theta$$
$$= 4 \sin 30°$$
$$= 2 \text{ m s}^{-1}$$

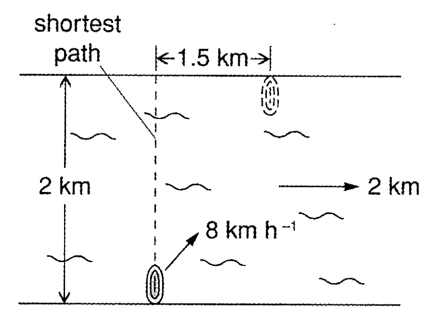
The time needed for the swimmer to cross the river is

$$t = \frac{s}{v_y}$$
$$= \frac{100}{2}$$
$$= 50 \text{ s}$$

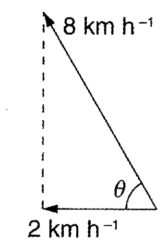
(b) If the water is still, the distance s the swimmer moves up the stream is

$$s = v \times t$$
$$= v(\cos\theta)t$$
$$= 4 \cos 30° (50)$$
$$= 173.21 \text{ m}$$

3.



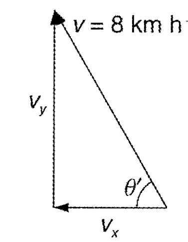
The man should control his boat at an angle θ which the horizontal component of the velocity v_x is just equal to the water velocity.



Thus, the angle θ is

$$\cos\theta = \frac{2}{8}$$
$$\theta = 75.52°$$

However, he chooses a wrong direction θ', and reaches 1.5 km downstream.



The relations between time t, v_x, v_y and v are

$$(2 - v_x)t = 1.5 \qquad \text{...... (1)}$$
$$v_y t = 2 \qquad \text{...... (2)}$$
$$v = \sqrt{v_x^2 + v_y^2} \qquad \text{...... (3)}$$

Combine (1), (2) and (3).

$$(2 - v_x)(\frac{2}{v_y}) = 1.5$$
$$(2 - v_x)(\frac{2}{\sqrt{v^2 - v_x^2}}) = 1.5$$
$$(\frac{4 - 2v_x}{\sqrt{v^2 - v_x^2}}) = 1.5$$
$$(\frac{4 - 2v_x}{\sqrt{64 - v_x^2}}) = 1.5$$
$$6.25v_x^2 - 16v_x - 128 = 0$$
$$v_x = 5.98 \text{ m s}^{-1}$$

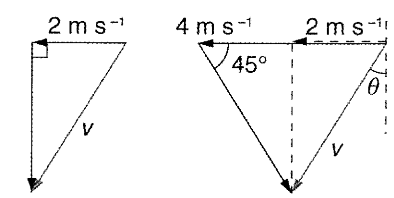
Thus, the angle θ' is

$$\cos\theta' = \frac{v_x}{v}$$
$$= \frac{5.98}{8}$$
$$= 41.63°$$

The deviation $\Delta\theta$ of the chosen direction from the required one is

$$\Delta\theta = \theta - \theta'$$
$$= 75.52° - 41.63°$$
$$= 33.89°$$

4. When the man walks at 2 m s^{-1}, the rain strikes vertically on him. When he increases velocity to 4 m s^{-1}, the rain strikes him at 45°.



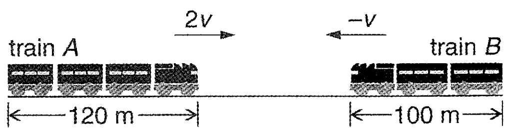
Thus, the velocity v of the rain is

$v \cos 45° = 2$

$v = 2.83\ ms^{-1}$

$\theta = 45°$

5.



The relative velocity of B relative to A is $-3v$.
After 4 s, they pass each other. Thus the velocity v is

$$-3vt = s$$
$$-3v(4) = -120 - 100$$
$$v = 18.33\ m\ s^{-1}$$

The velocity of A is $2v = 2 \times 18.33 = 36.66\ m\ s^{-1}$
The velocity of B is $-v = -18.33\ m\ s^{-1}$

Review Exercise 2 (p.101)

A. *Multiple Choice Questions*

1. A

$$s = ut + \frac{1}{2}gt^2$$

$\because \frac{1}{2}g > 0$

$\therefore$ The graph s against t is a parabola which opens upwards. Thus, h against t is a parabola opening downwards.

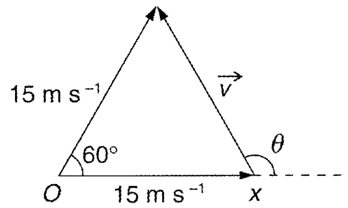
2. B
Except at the moment of rebounce, the steel ball undergoes a constant acceleration due to gravity. Each time when the ball rebounds, it experiences an upward force exerted by the table. Therefore, option **B** gives the correct description by taking the downward acceleration as positive.

The repulsive force that acts on the ball is getting smaller because the collision is inelastic and the ball hits the table with smaller momentum each time.

3. A
After the ball rebounds, it should reach a maximum height before it falls back to the horizontal surface again. At that moment, its velocity together with its momentum and kinetic energy are zero. The curve never reaches zero after the ball was released, so options **B**, **C** and **D** are wrong.

In fact, due to the constant acceleration, each time between the rebounds, the displacement-time graph should be a parabola. By taking downward displacement as positive, the graph gives the correct description of displacement.

4. A
Consider the vector $\vec{v}$ in the diagram.



$$|\vec{v}|^2 = 15^2 + 15^2 - 2 \times 15 \times 15 \times \cos 60°$$

$|\vec{v}| = 15\ m\ s^{-1}$

$\therefore$ The vectors form an equilateral triangle.
$\theta = 180° - 60° = 120°$

5. B

$$v^2 = u^2 + 2as$$
$$v^2 = 10^2 + 2(1.6)(120)$$
$$v = 22\ m\ s^{-1}$$

B. *Structured Questions*

6. **(a)** The acceleration a is

$$s = \frac{1}{2}at^2$$
$$160 = \frac{1}{2}(a)(10)^2$$
$$a = 3.2\ m\ s^{-2}$$

(b) The final velocity v is

$$v = at = 3.2 \times 10 = 32\ m\ s^{-1}$$

(c) The time required for the car to travel half the total distance is

$$s' = \frac{1}{2}at^2$$
$$\frac{160}{2} = \frac{1}{2}(3.2)t^2$$
$$t = 7.07\ s$$

(d) The distance d, the car travelled in half the total time is

$$d = \frac{1}{2}at^2 = \frac{1}{2}(3.2)(\frac{10}{2})^2 = 40\ m$$

(e) The velocity v' at half the total distance is

$$v'^2 = u^2 + 2as'$$
$$v'^2 = 2 \times 3.2 \times \frac{160}{2}$$
$$v' = 22.63\ m\ s^{-1}$$

(f) The velocity v'' at half the total time is

$$v'' = at = 3.2 \times \frac{10}{2} = 16\ m\ s^{-1}$$

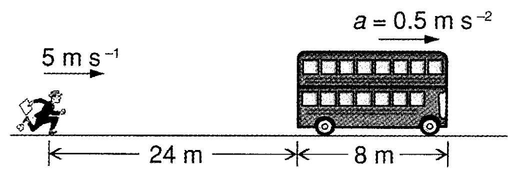
7. **(a)** Take upward as negative.
The time t required for the mass to hit the ground is

$$s = ut + \frac{1}{2}at^2$$
$$400 = (-5)t + \frac{1}{2}(10)t^2$$
$$5t^2 - 5t - 400 = 0$$
$$t = 9.46\ s \text{ or } -8.46\ s \text{ (rej.)}$$

(b) The velocity v of the mass when it hits the ground is

$$v^2 = u^2 + 2as$$
$$v^2 = (-5)^2 + 2 \times 10 \times 400$$
$$v = 89.58\ m\ s^{-1}$$

8. **(a)**



The time t the man takes to reach the rear of the bus is

Total displacement of man = Distance between the man and bus + Displacement of bus

$$5t = 24 + \frac{1}{2}(0.5)t^2$$
$$0.25t^2 - 5t + 24 = 0$$
$$t = 8\ s \text{ and } 12\ s$$

The man reaches the rear of the bus the first time at $t = 8$ s. He then runs ahead a little bit of the back of the bus. However, since the bus is accelerating, its rear catches up with the man again at $t = 12$ s.

(b) If the man can reach the front of the bus, the equation $5t = 24 + 8 + \frac{1}{2}(0.5)t^2$ has real solution.

However, for $0.25t^2 - 5t + 32 = 0$, there is no real solution. Thus, the man cannot reach the front of the bus.

9. The deceleration of the car is $4.0\ m\ s^{-2}$ and the driver's reaction time is 0.8 s.
Because of the driver's reaction time, the car would still travel at $10\ m\ s^{-1}$ for 0.8 s. Then, the distance d between the car and the stop line after 0.8 s would become:

$$d = 25 - 0.8 \times 10 = 17\ m$$

The distance s that the car still travels after the driver brakes fully and then stops is:

$$v^2 = u^2 + 2as$$
$$0 = 10^2 + 2(-4.0)s$$
$$s = 12.5\ m$$

Thus, the car stops at $d - s = 17 - 12.5 = 4.5$ m from the stop line.

10. **(a)** Take upward as negative.
The time t required for the stone to strike the ground is

$$s = ut + \frac{1}{2}at^2$$
$$15 = -6(t) + \frac{1}{2}(10)t^2$$
$$5t^2 - 6t - 15 = 0$$
$$t = 2.43\ s \text{ or } -1.23\ s \text{ (rej.)}$$

(b) The velocity v of the stone just hitting the ground is

$$v^2 = u^2 + 2as$$
$$v^2 = (-6)^2 + 2(10)(15)$$
$$v = 18.33\ m\ s^{-1}$$

11. **(a)** **(i)** Let u = velocity by which the ball leaves the hand. When the ball is caught by the hand again, displacement $s = 0$ and time $t = 4.0$ s.
Acceleration, $a = -g = -10\ m\ s^{-2}$.

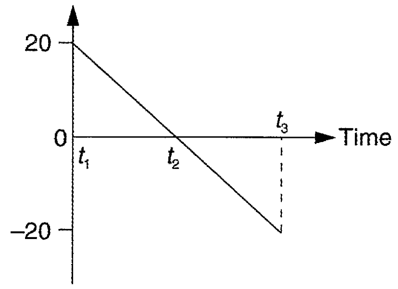
Using $s = ut + \frac{1}{2}at^2$,

$$0 = u(4) - \frac{1}{2} \times 10 \times (4)^2$$

$\therefore\ u = 20\ m\ s^{-1}$

(ii) Let H = maximum height reached.
At the maximum height, $v = 0$
Using $v^2 = u^2 + 2as$,
$0 = (20)^2 - 2 \times 10 \times H$
$\therefore \quad H = 20$ m

(b)



(c) (i) Neglecting air resistance, when the ball moves upwards, the retardation is of magnitude, g.
At the highest point, $v = 0$.
Using $v = u + at$,
$0 = u - gt$
$\therefore$ Time taken to reach the highest point is

$$t = \frac{u}{g}.$$

With air resistance, the retardation a_1 of the ball is greater than g, $a_1 > g$.
$\therefore$ Time taken to reach the highest point is

$$t' = \frac{u}{a_1} < \frac{u}{g},$$

i.e., the time is shorter.

(ii) Without air resistance, the maximum height reached (H) is given by
$v^2 = u^2 + 2as$
$0 = u^2 - 2gH$

$$H = \frac{u^2}{2g}$$

With air resistance, the new maximum height is given by

$$H' = \frac{u^2}{2a_1} < \frac{u^2}{2g},$$

i.e., the maximum height is lower.

(iii) If there is air resistance, using $s = \frac{1}{2}(u + v)\,t$,
for the upward motion, $v = 0$.

$$\therefore \quad t_u = \frac{2H'}{u}$$

For the downward motion,

$$H' = \frac{1}{2}(0 + v_1)\,t_d$$

where v_1 = velocity of ball when it reaches the hand again; t_d = time taken.

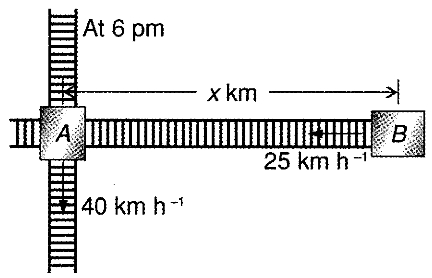
Magnitude of v_1 is less than that of u because of energy loss due to friction.

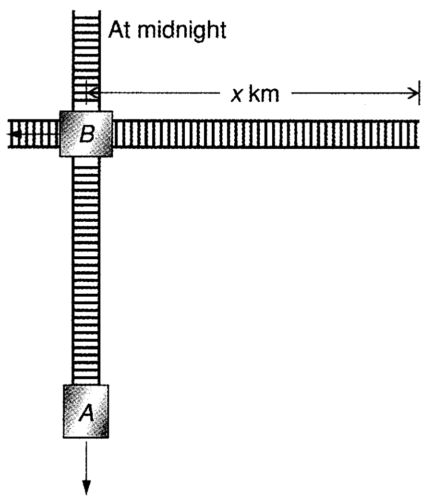
$$\therefore \quad t_d = \frac{2H'}{v_1} > \frac{2H'}{u} \qquad (\because v_1 < u)$$

$\therefore \quad t_d > t_u$

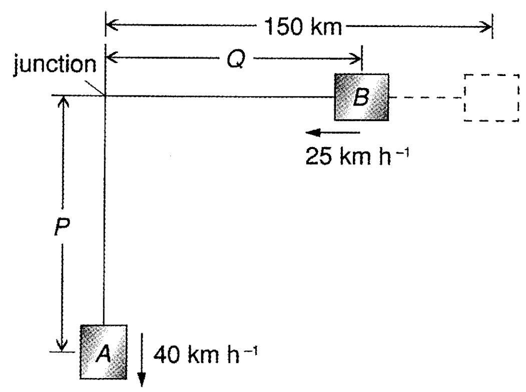
12. Using $v_{BA} = v_B - v_A$,
where v_{BA} is the relative velocity of the car relative to Nancy;
v_B is the relative velocity of the car relative to Alex;
v_A is the relative velocity of Nancy relative to Alex.
$\therefore \quad v_B = -70$ km h^{-1} and $v_A = 52$ km h^{-1}
$v_{BA} = -70 - 52$
$= -122$ km h^{-1}
Thus, the car is moving at 122 km h^{-1} due west as measured by Nancy.

13. (a)





At 6 pm, the train B is x km from the junction.
$x = v_B t$
$= 25 \times (12 - 6)$
$= 150$ km
The nearest distance to each other is where the distances between the two trains and the junction are the same.



$P = Q$
$v_A t' = v_B t'$

$$P = 40\left(\frac{150 - Q}{25}\right)$$

$P = 1.6(150 - P) \qquad (P = Q)$
$2.6P = 240$
$P = 92.31$ km

The time is $t' = \frac{P}{v_A} = \frac{92.31}{40} = 2.31$ h
The trains are the closest at 8 : 31 pm.

C. Overseas & HKALE Questions

14. (a) Trains R and V do not stop at intermediate stations.

(b) Conventional time-table for train U:

Station H	departure time	0824 hr
Station E	arrival time	0902 hr
	departure time	0908 hr
Station A	arrival time	0946 hr

(c) Average speed of train U over entire journey,

$$v = \frac{\text{Distance } HA}{\text{Time } HA} = \frac{80.0}{\frac{82}{60}} = 58.54 \text{ km h}^{-1}$$

(d) (i) Train U is moving at the maximum speed between stations E and D.

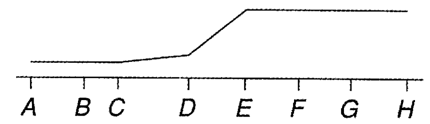
(ii) Maximum speed,

$$v_{max} = \frac{\text{Distance } ED}{\text{Time } ED} = \frac{12}{\frac{7.0}{60}} = 102.86 \text{ km h}^{-1}$$

(e) One possible reason, why all trains from D to E go slowly but trains from E to D go quickly, is that from D to E the track has an upward gradient (or downwards from E to D).

(f) (i) Assume that the trains have approximately the same power and the same mass.
Tracks are flat if travelling times in opposite directions are approximately equal.
Thus, tracks are flat from A to C and from E to H, and have an upward gradient from C to E.

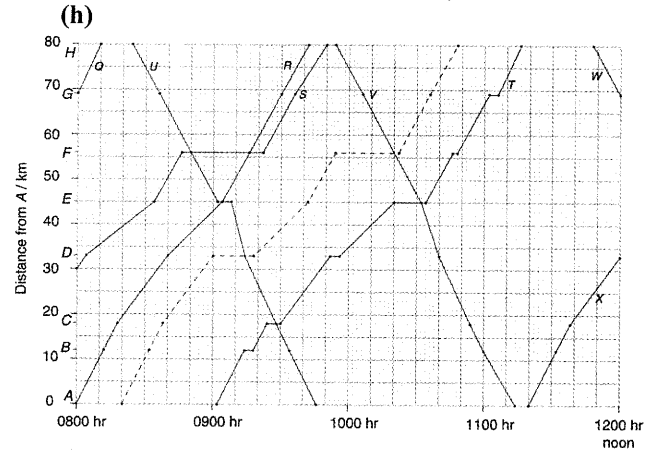
(ii) Sketch conclusion:



(g) (i) In relation to speed, one idealized aspect is that trains could maintain a constant speed between stations.

(ii) In relation to acceleration, one idealized aspect is that trains could accelerate from zero speed to the required travelling speed and vice versa immediately.

(h)



15. (a) $a = \frac{v - u}{t}$

(b) Distance travelled,
s = area under velocity-time graph

$$= \frac{1}{2}(v + u)\,t \qquad \text{.......... (1)}$$

From **(a)**,

$$a = \frac{v - u}{t}$$

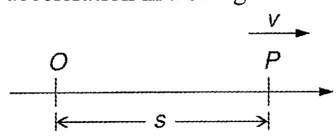
$$t = \frac{v - u}{a}$$

Substitute t into (1):

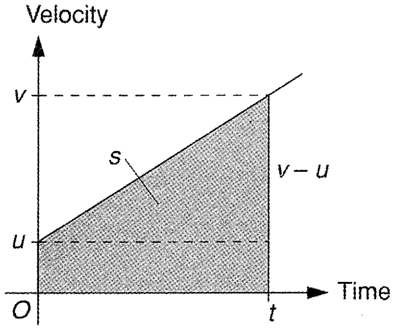
$$s = \frac{1}{2}(v + u)\frac{v - u}{a} = \frac{v^2 - u^2}{2a}$$

16. (a) Velocity is the rate of change of displacement with respect to time.
Acceleration is the rate of change of velocity with respect to time.

(b) Consider a body P moving with uniform acceleration in a straight line.



Its position at any instant of time t can be specified by its distance s measured from some reference point O fixed on the line.
If its velocity at O is u and its velocity at time t is v, the velocity-time graph is a straight-line graph as shown:



(i) By definition, acceleration of P,

$$a = \text{Rate of change of velocity with time}$$
$$= \text{Slope of } v\text{-}t \text{ graph}$$
$$= \frac{v-u}{t}$$
$$\therefore \quad v = u + at \text{ .... (1)}$$

(ii) The displacement of P,

$$s = \text{Area under } v\text{-}t \text{ graph}$$
$$= \frac{1}{2}(v+u)\,t \text{ ...... (2)}$$
$$v = u + at$$
$$t = \frac{v-u}{a} \text{ .... (3)}$$

Substitute (3) into (2):

$$s = \frac{1}{2}(v+u)\frac{v-u}{a}$$
$$= \frac{v^2-u^2}{2a}$$
$$\therefore \quad v^2 = u^2 + 2as$$

The condition necessary for these two equations to be applicable is constant acceleration a.

(c) $v^2 = u^2 + 2as$

Minimum distance of nest above ground:

$$s = \frac{v^2-u^2}{2a}$$
$$= \frac{9.0^2 - 0^2}{2 \times 9.8}$$
$$= 4.13 \text{ m}$$