4 Projectile Motion

Self Evaluation Exercise 4.1B (p.150)

1. B

In the x direction: $v_x = v$

In the y direction: $(v_y)^2 = (0)^2 + 2(-g)(-h)$

$$v_y = -\sqrt{2gh}$$

$$\tan\theta = \left|\frac{v_y}{v_x}\right| = \frac{\sqrt{2gh}}{v} \quad \text{where } 0° \le \theta \le 90°$$

$\therefore$ θ is maximum when $\frac{\sqrt{h}}{v}$ is maximum.

2. C

$$\text{Speed} = \sqrt{(v_x)^2 + (v_y)^2}$$

$v_x = 40 \text{ m s}^{-1}$ throughout the motion

$$\begin{aligned} v_y &= u_y + a_y t \\ &= 0 + (-10)(3) \\ &= -30 \text{ m s}^{-1} \end{aligned}$$

$\therefore$ Speed of the ball 3 s later

$$\begin{aligned} &= \sqrt{40^2 + (-30)^2} \\ &= 50 \text{ m s}^{-1} \end{aligned}$$

3. C

4. The ball is pulled downwards by gravity. The time t of flight is

$$s_y = u_y t + \frac{1}{2}at^2$$

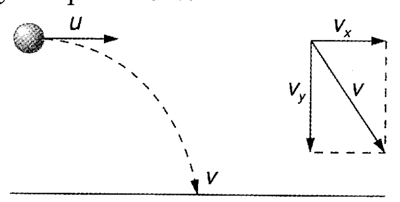
$$20 = \frac{1}{2}(10)t^2 \qquad (u_y = 0)$$

$$t = 2 \text{ s}$$

The x-component of the velocity of the ball is 10 m s^{-1}. Thus the range R is

$$\begin{aligned} R &= vt \\ &= 10 \times 2 \\ &= 20 \text{ m} \end{aligned}$$

5. v_x contributes the x-component of v. v_y contributes the y-component of v.



$$v_x^2 + v_y^2 = v^2$$

$$v_y = \sqrt{v^2 - u^2} \qquad (v_x = u)$$

At the beginning of the flight, u_y is zero. Thus, the time t of flight is

$$v_y = u_y + at$$

$$\sqrt{v^2 - u^2} = at$$

$$\begin{aligned} t &= \frac{\sqrt{v^2 - u^2}}{a} \\ &= 0.1\sqrt{v^2 - u^2} \end{aligned}$$

6. Similar to Question 5.

v is composed of the x-component v_x and the y-component v_y. When the bullet hits the wall, the vertical displacement is 0.8 m. Thus, the speed v_y is

$$v_y^2 - u_y^2 = 2as_y \qquad (u_y = 0)$$

$$v_y^2 = 2(10)(0.8)$$

$$v_y = 4 \text{ m s}^{-1}$$

The time t of flight is

$$s_y = u_y t + \frac{1}{2}at^2$$

$$0.8 = \frac{1}{2}(10)t^2$$

$$t = 0.4 \text{ s}$$

Thus, the horizontal range travelled by the object in 0.4 s is 10 m. The speed v_x is

$$v_x t = R$$

$$\begin{aligned} v_x &= \frac{R}{t} \\ &= \frac{10}{0.4} \\ &= 25 \text{ m s}^{-1} \end{aligned}$$

Finally, the speed v of the bullet when it hits the wall is

$$v^2 = v_x^2 + v_y^2$$

$$\begin{aligned} v &= \sqrt{25^2 + 4^2} \\ &= 25.32 \text{ m s}^{-1} \end{aligned}$$

7. **(a)** The time of flight, t, from B to C is

$$s_y = u_y t + \frac{1}{2}at^2$$

$$2.5 = \frac{1}{2}(10)t^2$$

$$t = 0.71 \text{ s}$$

The ball travelled a horizontal range of 4 m in 0.71 s. Then, the horizontal speed u_x at B is

$$u_x t = R$$

$$\begin{aligned} u_x &= \frac{R}{t} \\ &= \frac{4}{0.71} \\ &= 5.63 \text{ m s}^{-1} \end{aligned}$$

Assume that there is no energy loss from A to B. By conservation of energy, the potential energy of the ball at A is equal to the kinetic energy of the ball at B.

$$mgh = \frac{1}{2}mu_x^2$$

$$h = \frac{u_x^2}{2g} = \frac{5.63^2}{2\times10} = 1.59 \text{ m}$$

(b) The vertical component v_y of v is

$$v_y^2 = u_y^2 + 2as_y \qquad (u_y = 0)$$
$$v_y^2 = 2(10)(2.5) = 50 \text{ m s}^{-1}$$

Thus, the speed v of the ball at C just before hitting the ground is

$$v^2 = u_x^2 + v_y^2$$
$$v^2 = u_x^2 + v_y^2$$
$$v^2 = 5.63^2 + 50$$
$$v = 9.04 \text{ m s}^{-1}$$

8. **(a)** The height of the cliff h is equal to the vertical displacement of the diver. The height h is

$$h = u_yt + \frac{1}{2}at^2 \qquad (u_y = 0)$$
$$= \frac{1}{2}(10)(2.0)^2 = 20 \text{ m}$$

(b) The horizontal displacement s_x is

$$s_x = u_xt = 2.4 \times 2.0 = 4.8 \text{ m}$$

Self Evaluation Exercise 4.1C (p.155)

1. D

In a projectile motion, if there is no air resistance, the force acting on an object is only the gravitational force. Therefore, under a constant force, its acceleration is unchanged throughout the motion.

2. D

In the x-direction

$u_x = u\cos\theta$, $a_x = 0$

In the y-direction

$u_y = u\sin\theta$, $a_y = -g$

$\therefore$ By the equation $s = ut + \frac{1}{2}at^2$

$$\begin{cases} x = (u\cos\theta)t \\ y = (u\sin\theta)t + \frac{1}{2}(-g)t^2 \end{cases}$$

3. C

Let v_x be the horizontal component of the velocity

v_y be the vertical component of the velocity

In the projectile motion, the object is accelerating downwards only. Therefore, only v_y is changing while v_x is kept constant. It means that the rate of change of displacement is varying in the y-direction but not in the x-direction. Only option **C** is possible.

4. C

5. At the highest point, all the vertical kinetic energy is converted to the potential energy.

i.e. $F = \frac{1}{2}mu_x^2 + \frac{1}{2}mu_y^2 = mgh + \frac{1}{2}mu_y^2$

$\because$ $u_x^2 = (u\cos 60°)^2 = \frac{1}{4}u^2$

$$\frac{1}{2}mu_x^2 = \frac{1}{4}(\frac{1}{2}mu^2) = \frac{1}{4}E$$

$\therefore$ The kinetic energy at the highest point $= \frac{1}{2}mu_x^2 = \frac{1}{4}E$

6. **(a)** **(i)** The maximum height h is

$$h = \frac{u^2\sin^2\theta}{2g} = \frac{(25.0)^2\sin^2 38.0°}{2\times10} = 11.84 \text{ m}$$

(ii) The time of travel t before the football hitting the ground is

$$t = \frac{2u\sin\theta}{g} = \frac{2\times25.0\times\sin38.0°}{10} = 3.08 \text{ s}$$

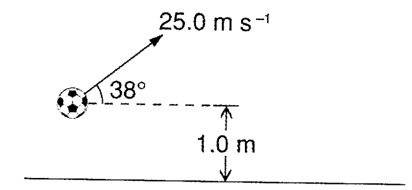
(iii) The horizontal range R of the football is

$$R = \frac{u^2\sin2\theta}{g} = \frac{(25.0)^2\sin2(38.0°)}{10} = 60.64 \text{ m}$$

(iv) The velocity of the football at the maximum height is only the horizontal component of the speed u_x. Thus, it is equal to

$$u_x = u\cos\theta = 25.0\cos38.0° = 19.70 \text{ m s}^{-1}$$

(b) **(i)**



The new maximum height h' is

$$h' = h + 1.0 = 11.84 + 1.0 = 12.84 \text{ m}$$

The time of flight t of the football from the punter's foot to the maximum height is

$$t = \frac{u\sin\theta}{g} = \frac{25.0\times\sin38.0°}{10} = 1.54 \text{ s}$$

At the maximum height, the vertical component of the speed is zero. And the time of flight from the maximum height to the ground, t_2 is

$$h' = u_yt + \frac{1}{2}at_2^2 \qquad (u_y = 0)$$
$$12.84 = \frac{1}{2}(10)t_2^2$$
$$t_2 = 1.60 \text{ s}$$

Thus, the total time of flight t' is

$$t' = t_1 + t_2 = 1.54 + 1.60 = 3.14 \text{ s}$$

(ii) The new horizontal range R' travelled by the football is

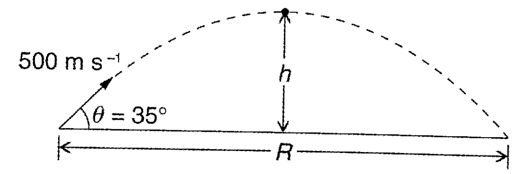
$$R' = u_xt' = 25.0 \times 3.14 = 78.5 \text{ m}$$

7. **(a)** If the horizontal range R travelled by the water is 2.0 m, the angle θ should be

$$R = \frac{u^2\sin2\theta}{g}$$
$$2.0 = \frac{13.5^2\sin2\theta}{10}$$
$$\theta = 3.15°$$

(b) By conservation of energy, the speed of the water when it lands is equal to the initial speed of water. Thus, the speed is 13.5 m s^{-1}.

8. **(a)**



The horizontal range R is

$$R = \frac{u^2\sin2\theta}{g} = \frac{500^2\times\sin2(35°)}{10} = 23\,492.32 \text{ m}$$

(b) The greatest vertical height h is

$$h = \frac{u^2\sin^2\theta}{2g} = \frac{500^2\times\sin^2 35°}{2\times10} = 4\,112.37 \text{ m}$$

(c) The time t to reach the greatest height is half of the total time of flight in air. Thus, time t is

$$t = \frac{u\sin\theta}{g} = \frac{500\sin35°}{10} = 28.68 \text{ s}$$

(d) To achieve the same range with minimum speed, the projection angle θ should be 45° in order to obtain the maximum horizontal range. Thus, the minimum speed u is

$$R = \frac{u^2\sin2\theta}{g}$$
$$R = \frac{u^2\sin2(45°)}{g}$$
$$R = \frac{u^2}{g}$$
$$u = \sqrt{Rg} = \sqrt{23\,492.32\times10} = 484.69 \text{ m s}^{-1}$$

Review Exercise 4 (p.161)

A. Structured Questions

1. (a) $u_x = 50 \cos 30° = 43.3$ m s^{-1}

$u_y = -50 \sin 30° = -25$ m s^{-1}

$v_y^2 = u_y^2 + 2(-g)(h)$

$= (-25)^2 + 2(-10)(-4\,000)$

$v_y = -284$ m s^{-1}

$v_x = u_x = 43.3$ m s^{-1}

Speed $= \sqrt{v_x^2 + v_y^2} = 287.28$ m s^{-1}

$\tan^{-1}\left|\frac{v_y}{v_x}\right| = 81.33°$

Thus, it hits the ground at 287.28 m s^{-1} with an angle 81.33° to the horizontal.

(b) $v_y = u_y + (-g)t$

$-284 = -25 - (10)t$

$t = 25.9$ s

2.

$$\begin{cases} x = u_x t & \text{..........(1)} \\ y = \frac{1}{2}(-g)t^2 & \text{..........(2)} \end{cases}$$

By (2), $-0.52 = \frac{1}{2}(-10)t^2$

$t = 0.322$ s

By (1), $1 = u_x(0.322)$

$u_x = 3.10$ m s^{-1}

3.

$$\begin{cases} x = u_x t & \text{..........(1)} \\ y = \frac{1}{2}(-g)t^2 & \text{..........(2)} \end{cases}$$

By (2), $-1\,000 = \frac{1}{2}(-10)(t^2)$

$t = 14.14$ s

By (1), $x = 50 \times 14.14$

$= 707$ m

B. Overseas & HKALE Questions

4. (a) (i) The speed of an object is the distance it travels per unit time.

(ii) Speed is a scalar quantity with only a magnitude whereas velocity is a vector quantity that has a magnitude (the speed) and a direction.

(b) (i) 1. Vertical component of initial velocity,

$u_Y = 15 \sin 60° = 12.99$ m s^{-1}

2. Horizontal component of initial velocity,

$u_X = 15 \cos 60° = 7.5$ m s^{-1}

(ii) 1. Let H = maximum height of the ball

v_Y = vertical component of velocity

v_X = horizontal component of velocity

At the maximum height, $v_Y = 0$

Vertical equation of motion:

$v_Y^2 = u_Y^2 - 2gH$

$0 = 12.99^2 - 2 \times 9.81 \times H$

$\therefore\ H = 8.6$ m

2. Let T = time of flight between the ball being thrown and returning to ground level

Deduce that at $t = T$, $v_Y = -u_Y$

Vertical equation of motion:

$v_Y = u_Y - gT$

$-12.99 = 12.99 - 9.81 \times T$

$\therefore\ T = 2.65$ s

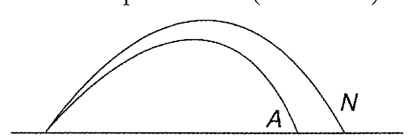
3. Horizontal range to the point where the ball strikes the ground,

$R = u_X \times T$

$= 7.5 \times 2.65$

$= 19.9$ m

(iii) Sketch the path of ball (labelled **N**):



(iv) 1. Sketch path (labelled **A**) assuming air resistance cannot be neglected.

2. Air resistance is opposite in direction to the velocity. Vertically, the ball will decelerate more when rising although it will also accelerate less when falling. The ball's horizontal speed will be reduced continuously and, if it travels long enough, would eventually be zero. Therefore, the ball will neither rise as high nor travel as far.

5. (a) (i) The directed line OA represents the velocity and not just speed because it specifies both magnitude and direction.

(ii) In Fig. (b), the length of OA = 7.0 cm

$\therefore$ The scale used is

$$1 \text{ cm} = \frac{14}{7.0} = 2.0 \text{ m s}^{-1}$$

(iii) Sketch on Fig. (a), lines OH and OV represent the horizontal and vertical components of the velocity respectively.

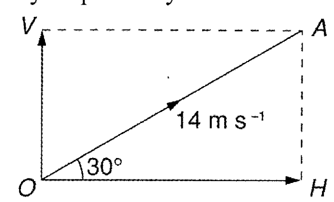


Fig. (a)

1. Horizontal component of velocity,

$u_H = (14)\cos 30° = 12.12$ m s^{-1}

2. Vertical component of velocity,

$u_V = (14)\sin 30° = 7.0$ m s^{-1}

(b) $v = u + at$

$= u_V - gt$ for the vertical direction

At maximum height, $v = 0$

$\Rightarrow\ 0 = 7.0 - (9.81)\,t$

$\therefore$ Time to reach maximum height

$t = 0.71$ s

(c) In the vertical direction, the motorcycle will return to the ramp's height in time

$t' = 2t$

$= 2 \times 0.7136$

$= 1.43$ s

Horizontal distance travelled in time t',

$s = u_H\, t'$

$= 12.12 \times 1.43$

$= 17.3$ m

Number of car lengths

$= \frac{s}{\text{width of car}}$

$= \frac{17.3}{1.6}$

$= 10.8$

$\therefore$ Maximum number of cars jumped = 10

6. – 7. HKALE Questions