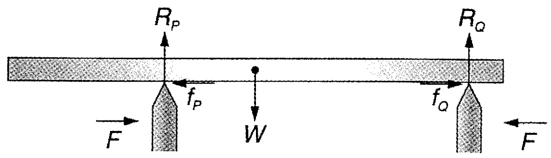
1 Statics

Self Evaluation Exercise 1.2 (p.46)

1. A



The centre of mass is at the mid-point of the ruler. Because support P is closer to the centre of mass than support Q (take moments about the centre of mass), the normal reaction at P is greater than that at Q.

$$R_P > R_Q$$

Since the friction at each support is proportional to the respective normal reaction, thus the frictional force f_P is greater than f_Q.

$$f_P > f_Q$$

Less force is needed to overcome the frictional force at Q to make the support slide.
So, when equal forces F of increasing magnitude are applied at P and Q, sliding first occurs between the ruler and support Q. The magnitude of the applied force F just exceeds frictional force f_Q but is still smaller than the frictional force f_P.

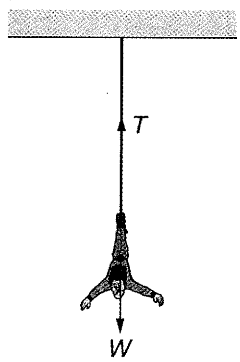
2. D

3. B

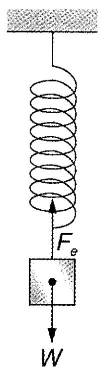
4. E

5. Let T = tension
   W = weight
   F_e = elastic force
   R = normal reaction
   F = friction

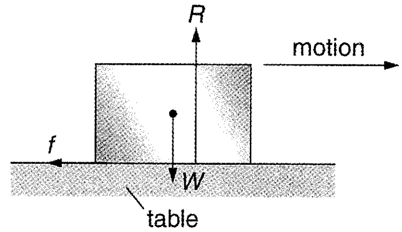
(a)



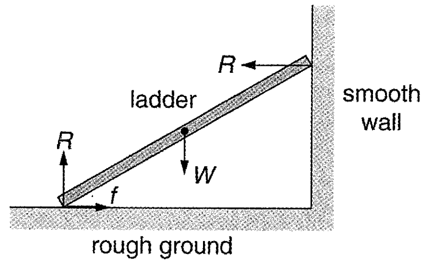
(b)



(c)

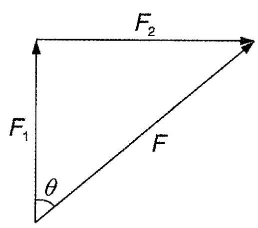


(d)



Self Evaluation Exercise 1.3A (p.51)

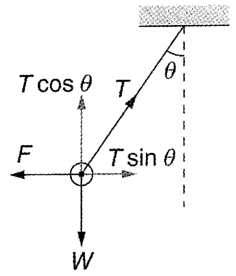
1. B



$$F_1^2 + F_2^2 = F^2$$

$$F = \sqrt{F_1^2 + F_2^2}$$

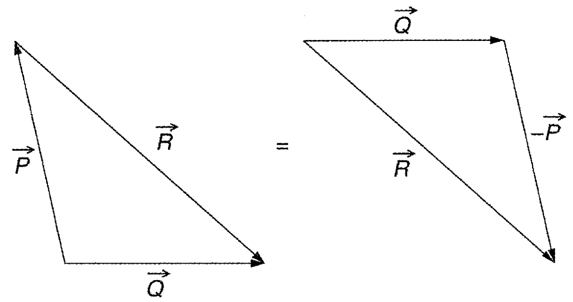
2. B



The thread experiences tension T.
And

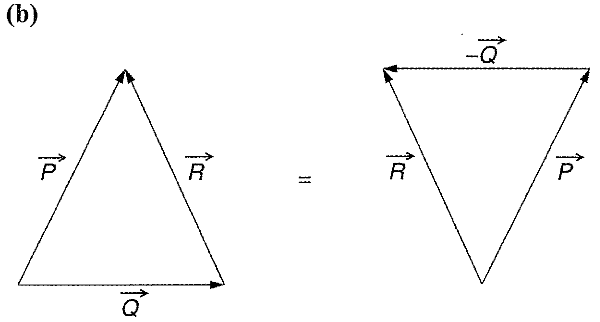
$$T\cos\theta = W$$
$$T\sin\theta = F$$
$$\frac{T\sin\theta}{T\cos\theta} = \frac{F}{W}$$
$$\tan\theta = \frac{F}{W}$$

3. **(a)**



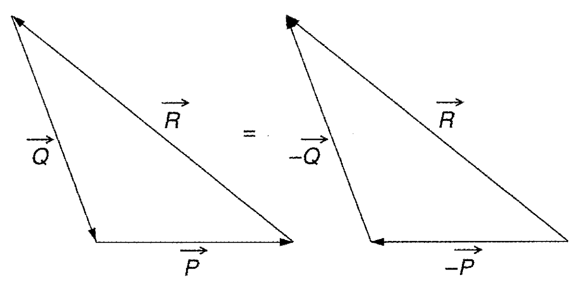
$$\vec{R} = \vec{Q} + (-\vec{P})$$
$$= \vec{Q} - \vec{P}$$

(b)



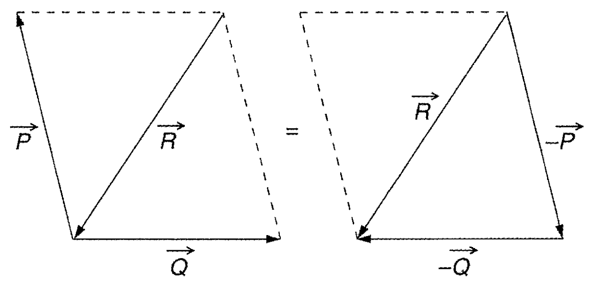
$$\vec{R} = \vec{P} + (-\vec{Q})$$
$$= \vec{P} - \vec{Q}$$

(c)



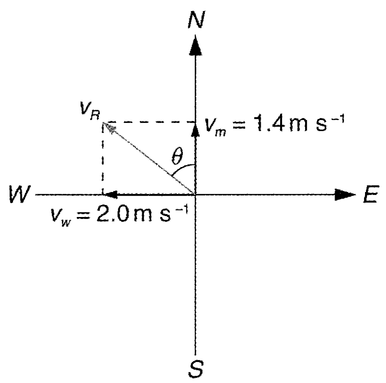
$$\vec{R} = (-\vec{P}) + (-\vec{Q})$$
$$= -(\vec{P} + \vec{Q})$$

(d)



$$\vec{R} = (-\vec{P}) + (-\vec{Q})$$
$$= -(\vec{P} + \vec{Q})$$

4. **(a)**



The magnitude of the resultant velocity is:
$\sqrt{1.4^2 + 2.0^2}$
$= 2.44$ m s^{-1}
The direction of the resultant velocity is:
$\theta = \tan^{-1}(\frac{2.0}{1.4})$
= N55°W

5. The value of $\vec{R}$ is maximum when $\theta = 0$, i.e., $\vec{P} + \vec{Q}$; the value of $\vec{R}$ is minimum when $\theta = 180°$, i.e., $\vec{P} - \vec{Q}$.

$\therefore \quad |\vec{P}| + |\vec{Q}| = 60$ and $|\vec{P}| - |\vec{Q}| = 10$

i.e. $\quad |\vec{P}| = 35$ and $|\vec{Q}| = 25$

By cos law,

$|\vec{R}| = \sqrt{35^2 + 25^2 + 2\times 35\times 25\times \cos 40°}$
$= 56.49$ N

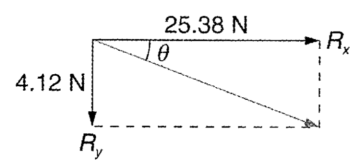
The direction of $\vec{R}$:
$\tan^{-1}(\frac{35\sin 40°}{25 + 35\cos 40°})$
$= 23.47°$ with $\vec{Q}$

6. **(a)** $\vec{R}_x = 20\cos 35° + 18\cos 60°$
$= 25.38$ N

$\vec{R}_y = 20\sin 35° - 18\sin 60°$
$= -4.12$ N

(b) $|\vec{R}| = \sqrt{|\vec{R}_x|^2 + |\vec{R}_y|^2}$
$= \sqrt{25.38^2 + 4.12^2}$
$= 25.71$ N

The direction of $\vec{R}$:

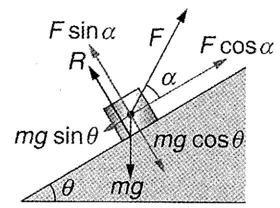


$\theta = \tan^{-1}(\frac{4.12}{25.38})$
$= 9.22°$ (clockwise from x-axis)

Self Evaluation Exercise 1.3B (p.60)

(p.60)

1. D



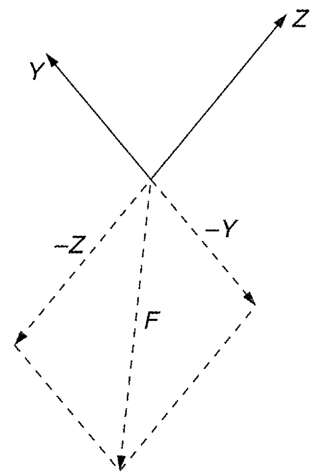
In the direction perpendicular to the plane, there is no net force.

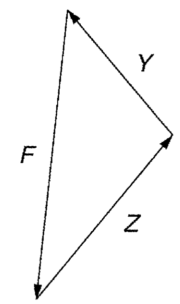
$$R + F\sin\alpha = mg\cos\theta$$

The net force is along the plane.

The magnitude of the resultant force on the object is
$F\cos\alpha - mg\sin\theta$

2. A





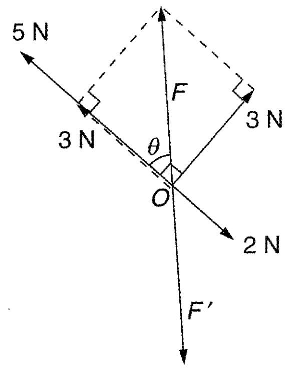
The balancing force F contains components which are opposite in direction and equal in magnitude to forces Y and Z.

$$F = -Y + (-Z)$$
$$-F = Y + Z$$
$$= Z + Y$$

3. A

4. D

5.



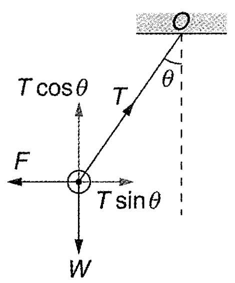
The resultant force F is shown in the figure.
The balancing force F' is opposite in direction but equal in magnitude to the resultant force F.
The magnitude of the balancing force F' is

$$F'^2 = (5-2)^2 + 3^2$$
$$F' = \sqrt{3^2 + 3^2}$$
$$= 4.24 \text{ N}$$

The angle θ is 45°. $\quad (\tan\theta = \frac{3}{3})$

Thus, the required force F' is at an angle 45° to the 2 N force (clockwise).

6.



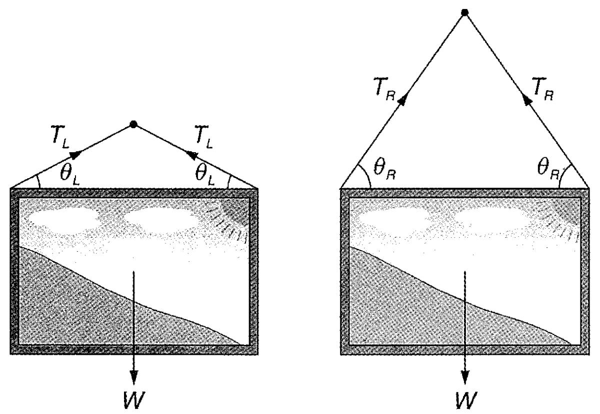
The tension T in the string in terms of W and θ is

$$T\cos\theta = W$$

The force F in terms of W and θ is

$$F = T\sin\theta$$
$$F = \frac{W\sin\theta}{\cos\theta}$$
$$F = W\tan\theta$$

7.



The wires of the left picture are more likely to break because the tensions of those wires are bigger.

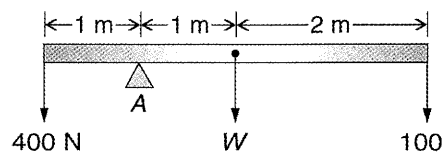
$$T_L = \frac{W}{2\sin\theta_L} \qquad T_R = \frac{W}{2\sin\theta_R}$$

As $\theta_L < \theta_R$, $\sin\theta_L < \sin\theta_R$

$\therefore \quad T_L > T_R$

Self Evaluation Exercise 1.5 (p.66)

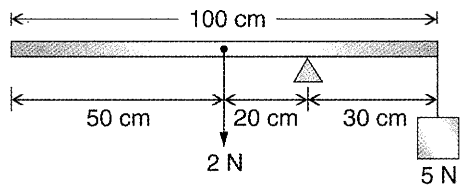
1. D



Take moment about A, clockwise moment is equal to anticlockwise moment.

$$400 \times 1 = W \times 1 + 100 \times 3$$
$$400 = W + 300$$
$$W = 100 \text{ N}$$

2. B

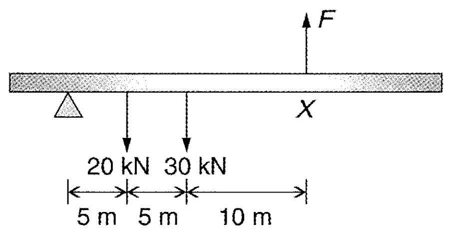


Take moment about pivot, the resultant moment is

$$5 \times 0.3 - 2 \times 0.2$$
$$= 1.5 - 0.4$$
$$= 1.1 \text{ N m}$$

The moment is clockwise.

3. B



Take the rear wheel as pivot, the clockwise moment is equal to the anticlockwise moment.
Let the upward force exerted by the cab be F.

$$20\,000 \times 5 + 30\,000 \times 10 = F \times 20$$
$$400\,000 = 20F$$
$$F = 20\,000$$
$$= 20 \text{ kN}$$

4. C

5. D

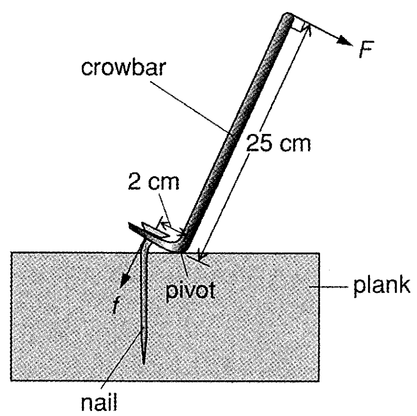
6. **(a)** The friction between the nail and the plank is f.
Take moment about the pivot.

$$\text{Clockwise moment} = 300 \times d$$
$$= 300 \times (0.25 \times \sin 20°)$$
$$= 25.65 \text{ N m}$$

Anticlockwise moment $= f \times 0.02$
By the principle of moments,
Clockwise moment = Anticlockwise moment

$$25.65 = f \times 0.02$$
$$\therefore \quad f = 1\,282.5 \text{ N}$$

(b) In order to minimize the force required, the force (F) should be applied in a direction perpendicular to the crowbar as shown in the figure.



Take moment about the pivot.
Clockwise moment $= F \times 0.25$
Anticlockwise moment $= f \times 0.02$

$$= 1\,282.5 \times 0.02$$

By the principle of moments,
Clockwise moment = Anticlockwise moment

$$F \times 0.25 = 1\,282.5 \times 0.02$$
$$\therefore \quad F = 102.6 \text{ N}$$

7. **(a)** The moment of the weight W about the pivot is:

$$\text{Moment} = Wl$$
$$= 2 \times 0.35$$
$$= 0.7 \text{ N m}$$

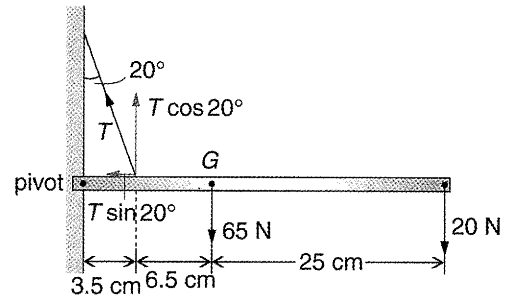
(b) To balance the system, the moment of the ball bearings should be equal to the moment of the weight W about the pivot.

$$Wl = W_{\text{ball}}\, l'$$
$$0.7 = W_{\text{ball}} \times 0.3$$
$$W_{\text{ball}} = 2.33 \text{ N}$$

(c) If the ball bearings have identical size but lower density, the total weight of the ball bearings is less. Then, the moment of the ball bearings is not enough to balance that of the weight W. The end A of the beam would tip up.

Self Evaluation Exercise 1.6 (p.74)

1.



(a) Take moment about the pivot, the clockwise moment is equal to the anticlockwise moment. The tension T in the supporting muscle is

$$T\cos 20° \times 0.035 = 65 \times 0.1 + 20 \times 0.35$$
$$0.032\,9\,T = 13.5$$
$$T = 410.33 \text{ N}$$

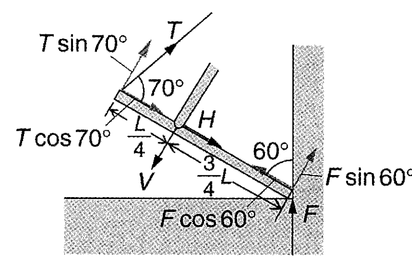
(b) The horizontal component of the force at the elbow is equal to the horizontal component of tension.

$$T\sin 20°$$
$$= 410.33 \sin 20°$$
$$= 140.34 \text{ N}$$

The vertical component of the force at the elbow is equal to the vertical component of tension minus the weight.

$$T\cos 20° - 65 - 20$$
$$= 410.33\cos 20° - 85$$
$$= 300.58 \text{ N}$$

2.



(a) Take moment about the ankle,

$$T\sin 70° \times \frac{L}{4} = F\sin 60° \times \frac{3}{4}L$$
$$0.235TL = 0.650FL$$
$$T = 2.76F$$

(b) The force V at the ankle is

$V = T \sin 70° + F \sin 60°$
$= 2.76F(0.940) + F(0.866)$
$= 3.46F$

The force H at the ankle is

$H = T \cos 70° - F \cos 60°$
$= 2.76F(0.342) - 0.5F$
$= 0.44F$

3. The torque I acting on the pulley is

$I = Fr$
$= 10 \times 0.2$
$= 2$ N m

4. Let x be the CG of the system.

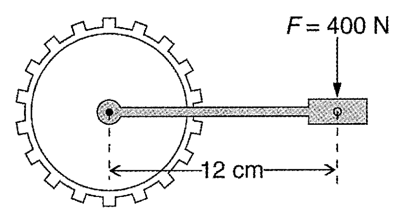
$$x = \frac{\sum_{i=1}^{3} m_i x_i}{M}$$

$$x = \frac{1\times0+2\times2+3\times(5+2)}{1+2+3}$$

$= 4.17$

The position of CG is 4.17 m from the 1 kg mass.

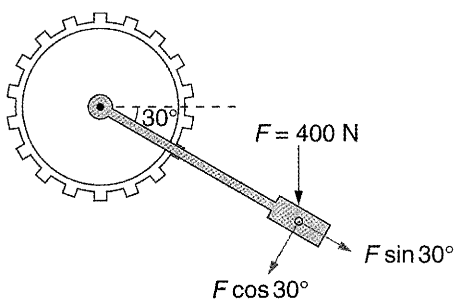
5. (a)



The moment of the force when the crank is horizontal is

$Fd = 400 \times 0.12$
$= 48$ N m

(b)

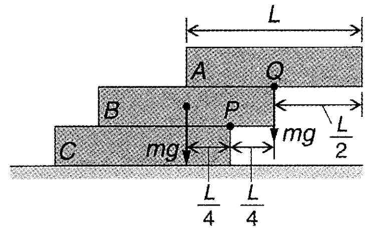


The moment of the force when the crank is turned 30° is

$F \cos 30°\ d$
$= 400 \cos 30° \times 0.12$
$= 41.57$ N m

Self Evaluation Exercise 1.7 (p.77)

1.



To have maximum horizontal overhang, there are two requirements.

(1) Brick A has maximum overhang above brick B and does not topple over.

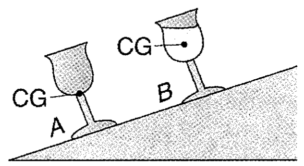
(2) System of bricks A and B has maximum overhang above brick C and does not topple over.

For requirement **(1)**, the maximum overhang can be obtained by locating the CG of brick A about the pivot Q. The overhang of A is $\frac{L}{2}$.

For requirement **(2)**, take moment about P, the overhang is maximized if the moment by brick A and moment by brick B are equal in magnitude. So the maximum overhang of brick B over brick C is $\frac{L}{4}$.

By the above arrangement, the top brick A has maximum horizontal overhang above the bottom brick C.

2.



The glass B is unstable because the CG is higher. The water in the glass B is in the upper part of the glass, and this makes the CG higher.

Review Exercise 1 (p.81)

A. *Multiple Choice Questions*

1. A

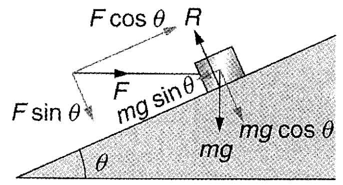
The work done (heat energy form) produced by the friction is given by:

Work done = Fx

2. A

The minimum resultant force is $(6 - 4)$ N = 2 N. Other magnitudes of resultant force can be obtained by varying the angle between the two forces.

3. A



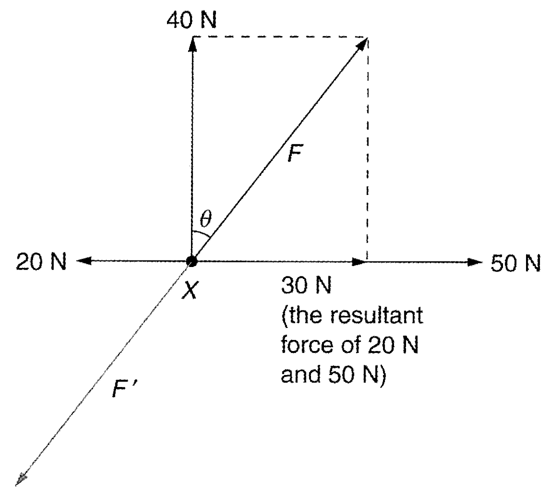
There is no net force in the direction perpendicular to the plane.

$R = mg \cos \theta + F \sin \theta$

The net force is along the plane. The magnitude of the resultant force acting on the body is

$F \cos \theta - mg \sin \theta$

4. D



The balancing force F' should be opposite to the resultant force F.

The angle θ is

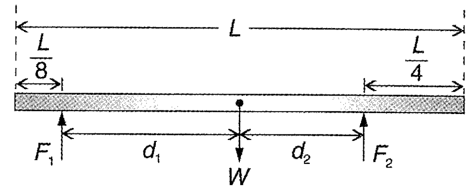
$$\tan \theta = \frac{30}{40}$$

$\theta = 36.9°$

The approximate bearing of the balancing force required is

$180° + 36.9°$
$= 216.9°$
$\approx 217°$

5. C



The centre of mass is at the mid-point of the plank. Take moment about the position of F_1, the clockwise moment is equal to the anticlockwise moment.

$W \times d_1 = F_2 \times (d_1 + d_2)$

$$W \times \left(\frac{L}{2} - \frac{L}{8}\right) = F_2 \times \left(L - \frac{L}{8} - \frac{L}{4}\right)$$

$$\frac{3}{8}WL = \frac{5}{8}F_2 L$$

$$\frac{3}{8}W = \frac{5}{8}F_2$$

$$F_2 = \frac{3}{5}W$$

Similarly, take moment about the position of F_2,

$W \times d_2 = F_1 \times (d_1 + d_2)$

$$W \times \left(\frac{L}{2} - \frac{L}{4}\right) = F_1 \times \left(L - \frac{L}{8} - \frac{L}{4}\right)$$

$$\frac{1}{4}WL = \frac{5}{8}F_1 L$$

$$\frac{1}{4}W = \frac{5}{8}F_1$$

$$F_1 = \frac{2}{5}W$$

Therefore, the ratio of F_1 to F_2 is

$$\frac{F_1}{F_2} = \frac{\frac{2}{5}W}{\frac{3}{5}W}$$

$$= \frac{2}{3}$$

6. C

Take moment about pivot, the clockwise moment is equal to the anticlockwise moment.

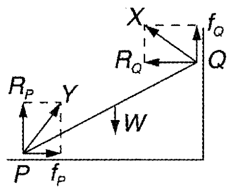
The anticlockwise moment is

$100 \sin 60° \times 4 \approx 346.4$ N m

Hence, the value of F is

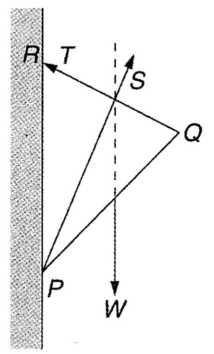
$F \times 4 = 346.4$
$F = 86.6$ N
≈ 87 N

7. C



At P, there are upward normal reaction R_P on the ladder and friction f_P pointing to the right. The resultant force is Y. At Q, there are upward friction f_Q and normal reaction R_Q pointing to the left. The resultant force is X.

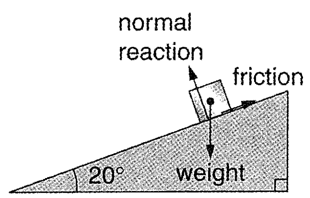
8. B



The whole system is in equilibrium.
The force acting on the flagpole by the wall must meet the tension T and weight W at the same point.
Therefore, the direction of the force is $\vec{PS}$.

B. Structured Questions

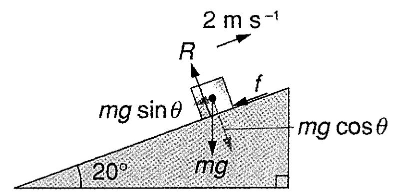
9. (a)



(b) The kinetic frictional force f between the block and the plane is equal to the magnitude of sine component of the weight,

$$f = mg \sin\theta$$
$$= 4 \times 9.81 \times \sin 20°$$
$$= 13.42 \text{ N}$$

(c)



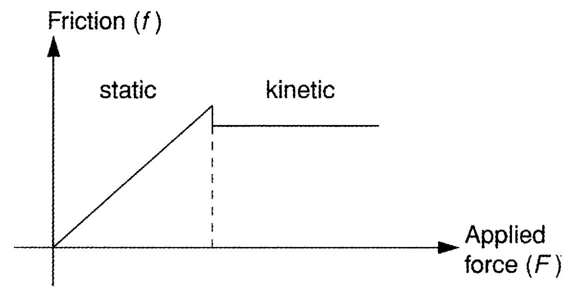
The block moves up the plane with an initial velocity of 2.0 m s^{-1}. When the block is moving up, it will exert a kinetic frictional force of 13.42 N. Hence, the deceleration of the block is

$$mg \sin\theta + f = m(-a)$$
$$13.42 + 13.42 = 4(-a)$$
$$a = -6.71 \text{ m s}^{-2}$$

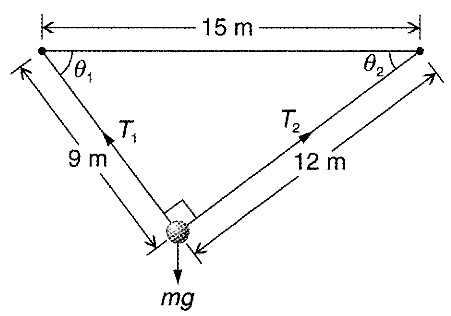
The distance s moved by the block before it stops is

$$v^2 = u^2 + 2as$$
$$0 = (-2)^2 + 2(-6.71)s$$
$$s = 0.30 \text{ m}$$

The block will not slide down again. Because when it stops, it is exerted by static friction. The static friction is greater than the kinetic force which is equal to $mg \sin\theta$.



10.

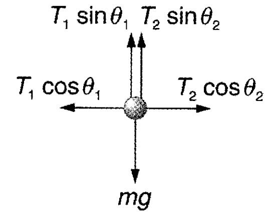


The two strings make a right angle

$$9^2 + 12^2 = 15^2$$

The angles θ_1 and θ_2 are given by:

$$\tan\theta_1 = \frac{12}{9}$$
$$\theta_1 = 53.13°$$
$$\tan\theta_2 = \frac{9}{12}$$
$$\theta_2 = 36.87°$$



Thus, the tension T_1 and T_2 can be calculated by

$T_1 \sin\theta_1 + T_2 \sin\theta_2 = mg$ .... (1)

$T_1 \cos\theta_1 = T_2 \cos\theta_2$ .... (2)

By substitution, we obtain:

From (1), $T_1 \sin 53.13° + T_2 \sin 36.87° = 10 \times 9.81$

From (2), $T_1 \cos 53.13° = T_2 \cos 36.87°$

Thus,

From (1), $0.8\, T_1 + 0.6\, T_2 = 98.1$

From (2), $0.6\, T_1 = 0.8\, T_2$

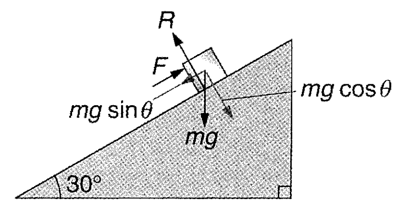
Sub (2) into (1),

$$0.8\left(\frac{0.8}{0.6}T_2\right) + 0.6\,T_2 = 98.1$$
$$1.67\,T_2 = 98.1$$

Hence, $T_2 = 58.74$ N

$T_1 = 78.32$ N

11.



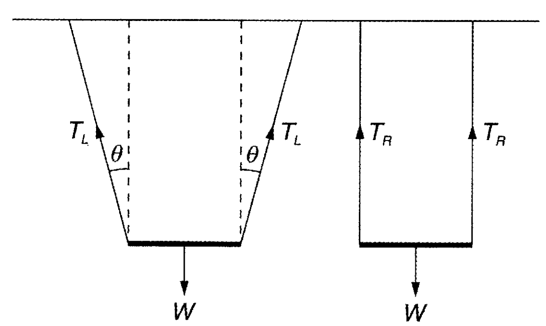
(a) The minimum force F is along and up the inclined plane

$$F = mg \sin\theta$$
$$= 2 \times 9.81 \times \sin 30°$$
$$= 9.81 \text{ N}$$

(b) The normal reaction R on the mass by the plane is

$$R = mg \cos\theta$$
$$= 2 \times 9.81 \times \cos 30°$$
$$= 16.99 \text{ N}$$

12.

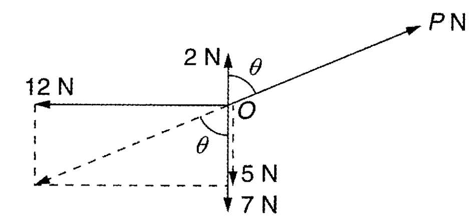


The left swing is more likely to break. Because the strings make an angle θ with the vertical, the tensions of the strings are bigger.

$$T_L = \frac{W}{2\cos\theta},\ T_R = \frac{W}{2}$$

As $\cos\theta < 1$, $T_L > T_R$.

13.



P N force can be treated as the balancing force of 2 N, 12 N and 7 N forces.

The value of P is

$$P^2 = 12^2 + 5^2$$
$$P = \sqrt{12^2 + 5^2}$$
$$= 13$$

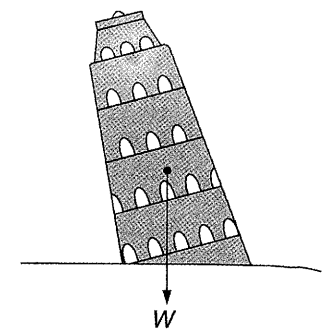
The value of θ is

$$\tan\theta = \frac{12}{5}$$
$$\theta = 67.38°$$

14. Although the forces do not point at the same point, the moments of these forces about the centre of mass cancel each other. The net moment is zero. So the body is in equilibrium.

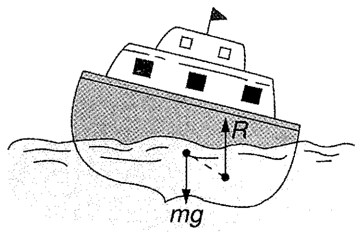
15. The CG of the balanced wheel should be located at the centre of wheel.

16.



This is because the line of action of weight falls within the base of tower.

17.



The boat is able to right itself because the centre of gravity G and centre of upthrust R are at different positions. There is a restoring couple provided by the weight mg and upthrust R.

$$R = mg$$

The restoring couple is anticlockwise, so it can right the boat.

C. Overseas & HKALE Questions

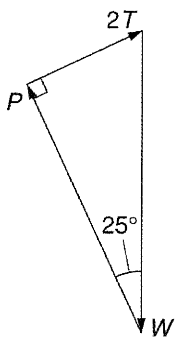
18. (a) A body is in equilibrium if there is
1. no resultant force,
2. no resultant torque

acting on it.

(b) (i) Weight of cylinder,

$W = mg$
$= 160 \times 9.81$
$= 1\ 569.6$ N

(ii) Draw vector triangle
scale: 1 cm represents 400 N



From the vector diagram,
$2T = W \sin 25° = (1\ 569.6) \sin 25° = 663.34$
$\therefore$ Tension $T = 331.67$ N

(c) Work done
$= Fs$
$= 2T \times 3.0$
$= 663.34 \times 3.0$
$= 1990.02$ J