13 Electric Fields

Self Evaluation Exercise 13.2 (p.11)

1. C

The Coulomb's force experienced by a point charge q due to another point charge Q is:

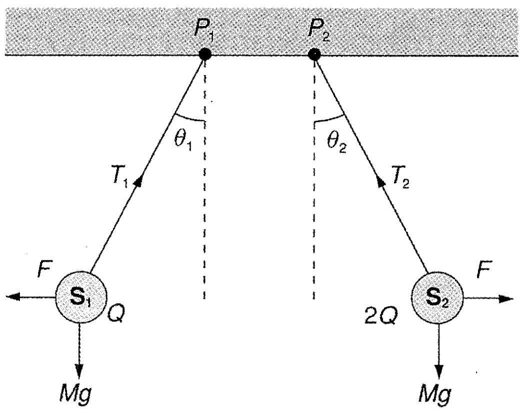
$$F = \frac{Qq}{4\pi\varepsilon_0 x^2}$$

$$\therefore \quad F \propto \frac{1}{x^2}$$

When $\frac{1}{x^2}$ increases, the force F increases.

2. C

The force diagram of the system is:



According to Newton's Law, the Coulomb's force experienced by each sphere is the same.
To balance the forces,

$$T_1 \cos\theta_1 = Mg$$
$$T_2 \cos\theta_2 = Mg$$
$$T_1 \sin\theta_1 = F$$
$$T_2 \sin\theta_2 = F$$
$$\therefore \quad \frac{T_1 \sin\theta_1}{T_1 \cos\theta_1} = \frac{T_2 \sin\theta_2}{T_2 \cos\theta_2} = \frac{F}{Mg}$$
$$\tan\theta_1 = \tan\theta_2$$
$$\therefore \quad \theta_1 = \theta_2$$

3. B

Similar to Question 2.
No matter how much the quantity of charge on X and Y are, the Coulomb's force experienced by each sphere is the same.

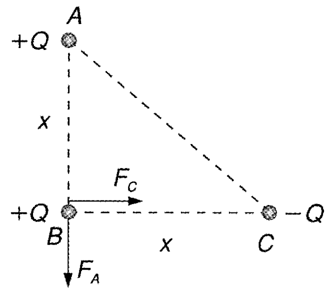
$$\therefore \quad T_X \sin\alpha = T_Y \sin\beta = F$$
$$\frac{T_X}{T_Y} = \frac{\sin\beta}{\sin\alpha}$$

And
$$T_X \cos\alpha = M_X g$$
$$T_Y \cos\beta = M_Y g$$
$$\therefore \quad \frac{M_X}{M_Y} = \frac{T_X}{T_Y} \cdot \frac{\cos\alpha}{\cos\beta}$$
$$= \frac{\sin\beta}{\sin\alpha} \cdot \frac{\cos\alpha}{\cos\beta}$$
$$= \frac{\tan\beta}{\tan\alpha}$$

$\because \alpha > \beta \ \therefore \tan\alpha > \tan\beta$ (Both $\alpha, \beta < 90°$)
$\therefore m_X < m_Y$

4. C

5.

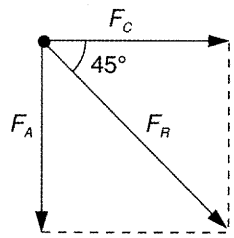


The magnitudes of the Coulomb's force acted on B due to the charge at A and C are the same. But they differ in direction. F_C points to the right (attraction) and F_A points downwards (repulsion).

(a) The magnitude of resultant force is:

$$(F_A)^2 + (F_C)^2 = (F_R)^2$$
$$\sqrt{2}\left(\frac{Q^2}{4\pi\varepsilon_0 x^2}\right) = F_R$$
$$F_R = \frac{\sqrt{2}Q^2}{4\pi\varepsilon_0 x^2}$$
$$= \frac{\sqrt{2}(2\times10^{-7})^2}{4\pi(8.85\times10^{-12})(0.1)^2}$$
$$= 0.0509 \text{ N}$$

(b)



$\because \quad F_C = F_A$

$\tan\theta = \frac{F_C}{F_A} = 1 \quad \therefore \theta = 45°$

6. The electrostatic force between 2 protons is:

$$F_e = \frac{Q_1Q_2}{4\pi\varepsilon_0 x^2} = \frac{(Q_p)^2}{4\pi\varepsilon_0 x^2} \quad (Q_p \text{ – charge of a proton})$$

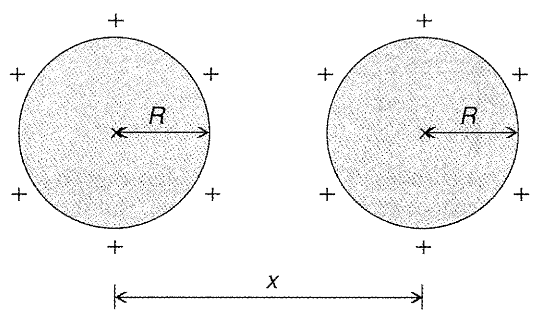
The gravitational force between them is:

$$F_g = \frac{Gm_1m_2}{x^2} = \frac{G(m_p)^2}{x^2} \quad (m_p \text{ – mass of a proton})$$

The ratio is:

$$\frac{F_e}{F_g} = \frac{(Q_p)^2}{G(m_p)^2 4\pi\varepsilon_0}$$
$$= \frac{(1.6\times10^{-19})^2}{6.67\times10^{-11}\times(1.67\times10^{-27})^2\times4\pi(8.85\times10^{-12})}$$
$$= 1.23\times10^{36}$$

7. Because the system is not two point charges at a distance x apart. The charges are evenly distributed on the spheres' surfaces.



For example, the closest distance between two spheres is $x - 2R$ and the farest distance is $x + 2R$. After squaring of distance, the force due to the charges at points on the spheres surface does not equal to $F = \frac{Q^2}{4\pi\varepsilon_0 x^2}$.

8. **(a)** Charging by friction refers to the seeds loosing electrons due to friction between the seeds and their surrounding, and so acquiring a positive charge.

(b) Electric force = weight

i.e. $\frac{ee}{4\pi\varepsilon_0 x^2} = mg$

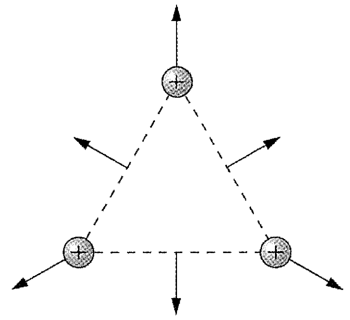
$$\Rightarrow x^2 = \frac{ee}{4\pi\varepsilon_0 mg}$$
$$= \frac{(1.60\times10^{-19})(1.60\times10^{-19})}{4\pi(8.85\times10^{-12})(9.0\times10^{-14})(9.81)}$$
$$= 2.607\times10^{-16}$$
$$\therefore x = \sqrt{2.607\times10^{-16}}$$
$$= 1.614\times10^{-8}\text{ m} = 16\text{ nm}$$

Self Evaluation Exercise 13.3 (p.14)

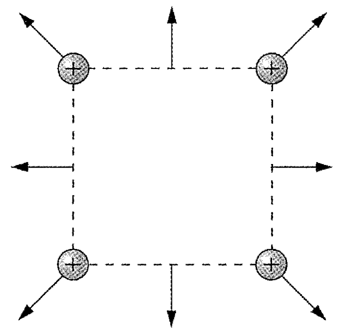
1. D

The electric force on a negative charge is opposite in direction to the electric field.

2. **(a)**

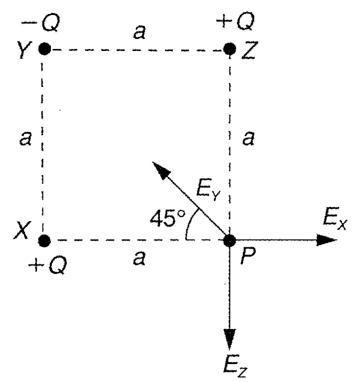


(b)



Self Evaluation Exercise 13.4 (p.19)

1. To study the electric field at point P, a positive test charge is placed at point P to decide the direction of electric field. And the resultant field can be calculated by vector addition.



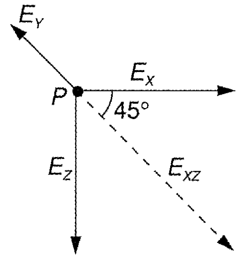
Since the distance between P and X is the same as that of P and Z, the magnitude of electric field is the same:

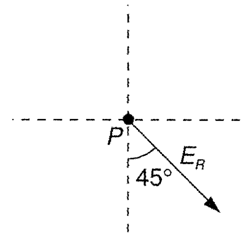
$$\frac{Q}{4\pi\varepsilon_0 a^2}$$

And the electric field due to charge at point Y is:

$$\frac{Q}{4\pi\varepsilon_0(\sqrt{2}a)^2} = \frac{Q}{4\pi\varepsilon_0 a^2(2)}$$

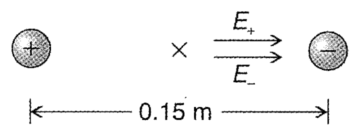
The resultant field (E_R) is:





$$E_R = E_X + E_Z + E_Y$$
$$= E_{XZ} + E_Y$$
$$= \frac{Q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2} - \frac{1}{2}\right)$$

2. **(a)**



The magnitude of electric field due to positive charge is the same as that of negative charge.

∴ The magnitude of total electric field is:

$$2\left(\frac{q}{4\pi\varepsilon_0 x^2}\right) = \frac{2\times10^{-7}}{2\pi(8.85\times10^{-12})\times(0.15\div2)^2}$$
$$= 6.39\times10^5\text{ N C}^{-1}$$

The direction of electric field is along the negative charge.

(b) The magnitude of force is:

$$F = Eq$$
$$= (6.39\times10^5)(1.6\times10^{-19})$$
$$= 1.02\times10^{-13}\text{ N}$$

∵ The electron is negatively charged.

∴ The direction of force is opposite to the direction of electric field. It is towards the positive charge.

3. A

Electric field will not be zero between two opposite charges. Point of zero field will be further away from the point charge of greater magnitude ($-2Q$) than from the point charge of smaller magnitude (Q).

Self Evaluation Exercise 13.5 (p.28)

1. C

For a point charge:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ and } V = \frac{Q}{4\pi\varepsilon_0 r}$$

2. D

Electric potential at a point is the work done to move a unit positive charge from infinity to that point.

$$V = \frac{W}{q}$$
$$= \frac{15}{3} = 5\text{ J C}^{-1} = 5\text{ V}$$

3. B

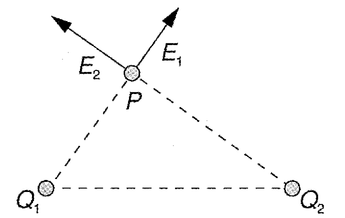
Electric potential is a scalar quantity.

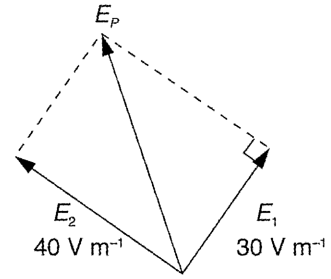
∴ The total electric potential due to Q_1 and Q_2 is simply scalar addition:

$$V_P = V_1 + V_2$$
$$= 60 + 120$$
$$= 180\text{ V}$$

Electric field is a vector quantity.

∴ The total electric field due to Q_1 and Q_2 is calculated by vector addition.





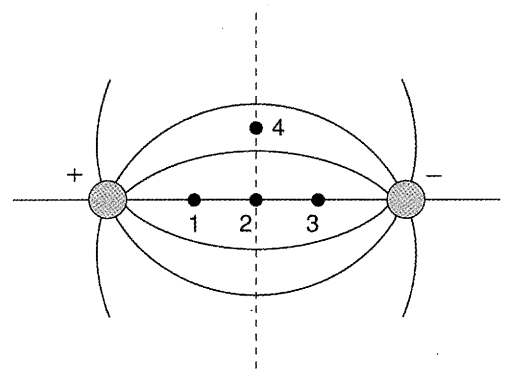
$$E_P^2 = E_1^2 + E_2^2$$
$$E_P = \sqrt{30^2 + 40^2}$$
$$= 50\text{ V m}^{-1}$$

4. D

$$\because E = -\frac{dV}{dx}$$

The negative sign of the equation means that along the direction of electric field, the slope (the rate of change of electric potential with distance) is negative. In other words, along the direction of electric field line, the electric potential decreases.

5. B



The resultant electric potential of each point is the addition of electric potential due to positive charge and also due to negative charge at that point.

∴ At the mid-point between the two charges, the electric potential is zero.

∴ $V_2 = V_4 = 0$

On the right side from the central axis, the electric potential is negative. On the left side, electric potential is positive.

∴ The greatest potential difference is between points 1 and 3.

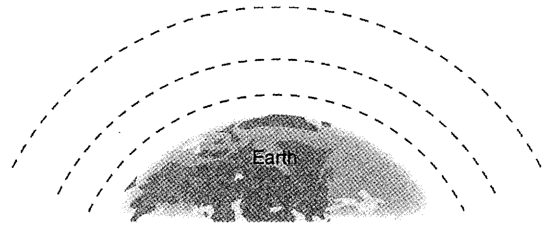
6. A

7. C

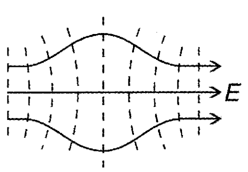
8. C

9. B

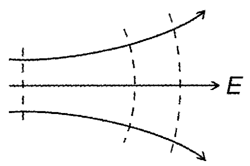
10. **(a)**



(i)

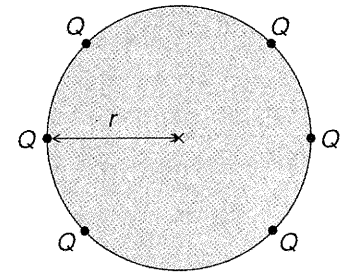


(ii)



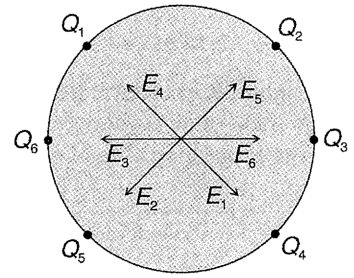
11. Yes. These are two examples.

(1) The system of six identical charges Q placed symmetrically around a circle.



At the centre, the electric potential is $\frac{6Q}{4\pi\varepsilon_0 r}$. It is not zero. However, the electric field at the centre is zero.

∵ The charges are equally apart from the centre. By symmetry, the resultant electric field is zero.

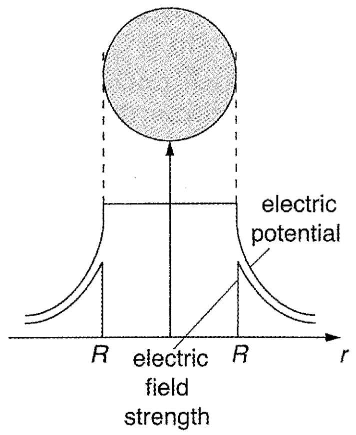


Magnitude: $E_1 = E_2 = E_3 = E_4 = E_5 = E_6$

∴ E_1 cancels E_4, E_2 cancels E_5 and E_3 cancels E_6.

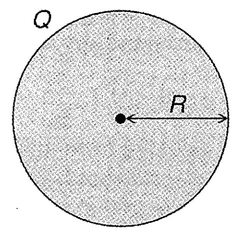
(2) Inside a charged conducting sphere,

$$V = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{R} \text{ and } E = 0$$



Electric potential is constant inside the conducting sphere. By symmetry, the electric field strength is zero.

12. **(a)**



The definition of electric potential is:

$$V = \frac{Q}{4\pi\varepsilon_0 x}$$

$$500 = \frac{3\times10^{-11}}{4\pi(8.85\times10^{-12})R}$$

$$R = 5.39 \times 10^{-4} \text{ m}$$

(b) The charge of the new drop is double that of the original one. And the volume of the larger spherical drop is also double that of the original drop.

The volume of a sphere is:

$$\text{Vol} = \frac{4}{3}\pi R^3 \propto R^3$$

And the electric potential is:

$$V = \frac{Q}{4\pi\varepsilon_0 R} \Rightarrow V \propto \frac{Q}{R} \Rightarrow R \propto \frac{Q}{V}$$

$$\therefore \left(\frac{\text{Vol}_{\text{large}}}{\text{Vol}_{\text{original}}}\right)^{\frac{1}{3}} = \frac{R_{\text{large}}}{R_{\text{original}}} = \frac{V_{\text{original}}}{V_{\text{large}}} \cdot \frac{Q_{\text{large}}}{Q_{\text{original}}}$$

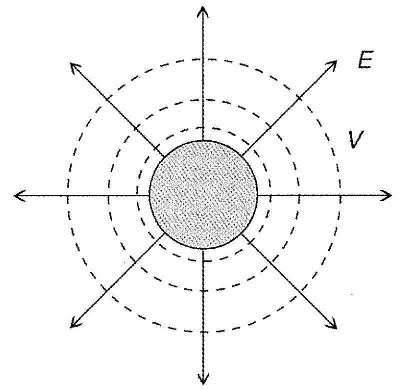
$$\frac{V_{\text{original}}}{V_{\text{large}}} \cdot \frac{Q_{\text{large}}}{Q_{\text{original}}} = \left(\frac{\text{Vol}_{\text{large}}}{\text{Vol}_{\text{original}}}\right)^{\frac{1}{3}}$$

$$\frac{V_{\text{original}}}{V_{\text{large}}} \cdot (2) = 2^{\frac{1}{3}}$$

$$V_{\text{large}} = \frac{2(500)}{2^{\frac{1}{3}}} = 794 \text{ V}$$

13. **(a)** **(i)** Sketch as solid arrows radiating out from sphere, labelled E.

(ii) Sketch as concentric circles in dashed lines with increasing spacing, labelled V.



(b) **(i)** Calculate Vx in 3rd column:

x / cm	V / V	Vx / V cm
19	1.50×10^5	28.5×10^5
25	1.14×10^5	28.5×10^5
32	0.89×10^5	28.5×10^5
39	0.73×10^5	28.5×10^5

$$Vx = 28.5 \times 10^5 = \text{constant}$$

$$\Rightarrow \quad V \propto \frac{1}{x}$$

(ii) $V_{\text{surface}}\, r$ = constant (28.5×10^5)

∴ Radius of sphere,

$r = (28.5 \times 10^5) \div (1.9 \times 10^5) = 15$ cm

(c) $V = \frac{Q}{4\pi\varepsilon_0 r}$

∴ $Q = 4V\pi\varepsilon_0 r$

$= 4\,(1.9 \times 10^5)\pi(8.85 \times 10^{-12})(15 \times 10^{-2})$

$= 3.17 \times 10^{-6}$ C

Self Evaluation Exercise 13.7 (p.35)

1. C

The electric potential due to 1 charge Q at the centre is:

$$\frac{Q}{4\pi\varepsilon_0 r}$$

∵ Electric potential is a scalar quantity. The total electric potential at the centre due to the 6 charges Q is $\frac{6Q}{4\pi\varepsilon_0 r}$.

And the work done to remove a point charge q is:

$$W = Vq = \frac{6Qq}{4\pi\varepsilon_0 r}$$

2. E

3. D

4. The electric force between two protons is:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 x^2} = \frac{(1.6\times10^{-19})^2}{4\pi(8.85\times10^{-12})(1.5\times10^{-15})^2} = 102.3\text{ N}$$

The electric potential due to a proton at a distance x is:

$$V = \frac{Q_1}{4\pi\varepsilon_0 x}$$

And the work done against the force to bring the other proton at distance 1.5×10^{-15} m is:

$$W = Vq = \frac{Q_1}{4\pi\varepsilon_0 x}\cdot Q_2$$

$$= \frac{(1.6\times10^{-19})^2}{4\pi(8.85\times10^{-12})(1.5\times10^{-15})}$$

$$= 1.53 \times 10^{-13}\text{ J}$$

Electric potential at a point is the energy required to bring a unit positive charge from infinity to that point.

$$V = \frac{U}{q} \quad \therefore U = Vq$$

The work done of moving a charge from infinity is the force acting on it times the displacement. Because the direction of the force is opposite to that of the displacement, there is a negative sign in the equation. And suppose the displacement is small, then

$$dU = -Fdx \quad \therefore F = -\frac{dU}{dx}$$

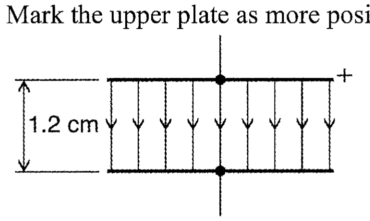
And $F = -\frac{dU}{dx}$

$$qE = -\frac{dU}{dx}$$

$$E = -\frac{\frac{dU}{q}}{dx} = -\frac{dV}{dx}$$

Self Evaluation Exercise 13.8 (p.38)

1. (a) (i) Mark the upper plate as more positive.



(ii) $E = \frac{V}{d}$

$\therefore$ p.d. between plates,

$V = Ed = (3.0 \times 10^4)(0.012) = 360$ V

(b) Let a_e = acceleration of an electron

Force = mass × acceleration

i.e. $eE = m_e a_e$

$$a_e = \frac{eE}{m_e} = \frac{(1.60\times10^{-19})(3.0\times10^4)}{9.11\times10^{-31}} = 5.3 \times 10^{15}\text{ m s}^{-2}$$

2. C

When the electric potential is zero, the electric field may not be equal to zero. An example is the case inside a charged conducting sphere.

3. D

Choice A (incorrect)

Joule is not the unit of electric potential. The correct units are Joule per Coulomb and Volt.

Choice B (incorrect)

The correct statement is that the electric field is given by the rate of change of electric potential with distance.

Choice C (incorrect)

By the definition of electric potential at a point, it should be the work done to move a unit positive charge from infinity to the point.

4. A

$$E = -\frac{dV}{dx} \Rightarrow \int_P^Q Edx = -\int_P^Q dV = \int_Q^P dV = V_P - V_Q$$

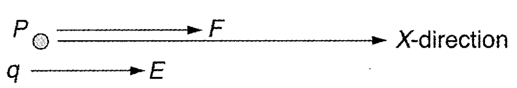
The shaded area between P and Q

$\therefore$ It represents the potential difference between P and Q.

5. D

Self Evaluation Exercise 13.9 (p.46)

1. D



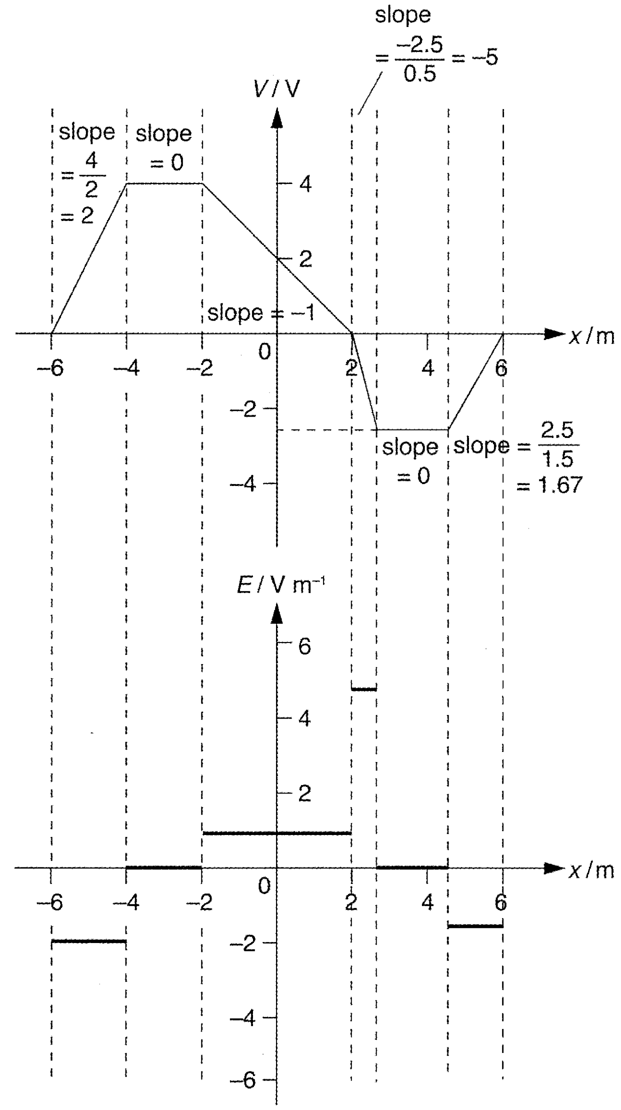
By definition, electric field is the force acting on a unit positive charge.

$$E = \frac{F}{q} \quad \therefore F = Eq$$

6. $E = -\frac{dV}{dx}$

where $-\frac{dV}{dx}$ is the slope of curve.

The negative sign means that when the slope is positive, the electric field strength is negative.



7. The electric potential at any point inside a conductor is the same.

And $E = -\frac{dV}{dx}$

$\because$ V is constant, $\frac{dV}{dx} = 0$

$\therefore$ $E = 0$

From another point of view, the electric field strength must be zero inside a conductor. Otherwise, there will be a force to move charge inside the conductor. Then, the case is not applicable to static charge (electrostatic).

8. $E = -\frac{dV}{dx}$

If the electric potential on the surface of a charged conductor is not the same everywhere, then $\frac{dV}{dx} \neq 0$.

And there is resultant electric field that would drive the charges to move from a point to another. The charges are not static and it is not the case for electrostatic.

Self Evaluation Exercise 13.10 (p.49)

1. (a) Coulomb force is the force between charged particles:

$$F_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1Q_2}{r^2} \propto \frac{1}{r^2}$$

Gravitational force is the force between masses:

$$F_G = \frac{GM_1M_2}{r^2} \propto \frac{1}{r^2}$$

For similarity, both forces are directly proportional to $\frac{1}{r^2}$ (obeys inverse square law). That means if the distance between 2 charges (masses) doubles, the force between them is decreased by 4 times.

For difference, Coulomb force can be attractive or repulsive. The force between like charges (+, +), (–, –) is repulsive. The force between unlike charges (+, –) is attractive. However, gravitational force between masses must be attractive. This means masses has the tendency to move towards each other.

(b) In Hydrogen atom, there is one proton and one electron. The magnitude of charge of a proton is equal to that of an electron.

The magnitude of the Coulomb force is:

$$F_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_pQ_e}{r^2}$$

$$= \frac{(1.6\times10^{-19})^2}{4\pi(8.85\times10^{-12})(5.3\times10^{-11})^2}$$

$$= 8.19 \times 10^{-8}\text{ N}$$

The magnitude of the gravitational force is:

$$F_G = \frac{GM_pM_e}{r^2}$$

$$= \frac{(6.67\times10^{-11})(1.673\times10^{-27})(9.11\times10^{-31})}{(5.3\times10^{-11})^2}$$

$$= 3.62 \times 10^{-47}\text{ N}$$

The ratio between F_C and F_G is:

$$\frac{F_C}{F_G} = \frac{8.19\times10^{-8}}{3.62\times10^{-47}} = 2.26 \times 10^{39} \sim 10^{39}$$

(c) The electric force experienced by a charge in electric field is:

$F_E = Eq$

The weight of electron is mg.

$\therefore\ F_E = 1.6\,mg$

$Eq = 1.6\,mg$

$$E = \frac{1.6 \times (9.11 \times 10^{-31}) \times 9.8}{1.6 \times 10^{-19}}$$

$= 8.9 \times 10^{-11}$ V m^{-1}

2. (a) Electric field at a point is the force acting on a unit of positive charge at that point.

$\therefore$ Unit for electric field is N C^{-1}.

Gravitational field at a point is the force acting on a unit of mass at that point.

$\therefore$ Unit for gravitational field is N kg^{-1}.

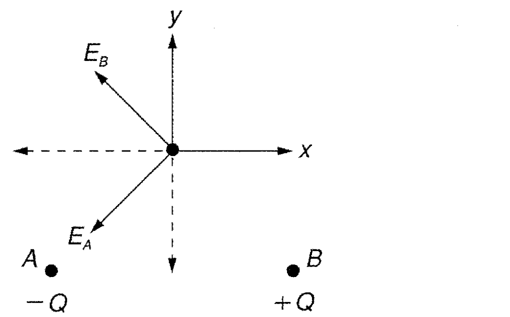
Electric potential at a point in an electric field is the work done (energy) required to move a unit positive charge from infinity to that point.

$\therefore$ Unit for electric potential is J C^{-1}.

Gravitation potential at a point in a gravitational field is the work done by the gravitational force to bring a unit mass from infinity to that point.

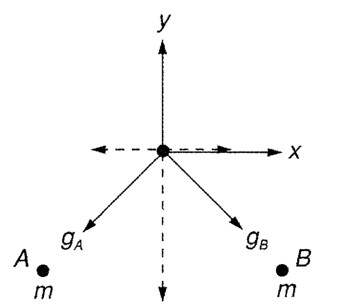
$\therefore$ Unit for gravitational potential is J kg^{-1}.

(b) (i)



A and B are equidistant from C. After breaking the fields into x and y components, the y components of two field cancel each other. The resultant electric field points to negative x-direction.

(ii)



Since gravitational force is always attractive, the fields due to two masses point to the masses. After breaking the field into components, the resultant field points to negative y-direction.

(iii) The electric potentials due to the charges at A and B are:

$$V_A = \frac{-Q}{4\pi\varepsilon_0 r} \qquad V_B = \frac{Q}{4\pi\varepsilon_0 r}$$

The resultant potential is:

$$V_A + V_B = \frac{-Q}{4\pi\varepsilon_0 r} + \frac{Q}{4\pi\varepsilon_0 r} = 0$$

The gravitational potentials due to the masses at A and B are:

$$V_A = \frac{-Gm}{r} \qquad V_B = \frac{-Gm}{r}$$

The resultant potential is:

$$V_A + V_B = \frac{-Gm}{r} - \frac{Gm}{r} = \frac{-2Gm}{r}$$

Review Exercise 13 (p.53)

A. Multiple Choice Questions

1. D

A conductor has the same potential at any point on its surface. $\therefore\ V_P = V_Q$.

And at deformed or sharp region, the surface charge density is higher. And thus the electric field there is also higher.

$\therefore$ charge density at $Q < \sigma$.
electric intensity at $Q < E$.

2. A

Because of the repulsion between like charges, the charges arrange themselves as far as possible from others. Therefore, charges appear only on the surface of the sphere.

3. C

The rate of charge leakage is 64 μA. That means 64 μC charge is removed per second. And the area of belt running through it is:

$0.04 \times 0.8 = 0.032$ m^2 s^{-1}

Therefore, the charge density is:

$$\text{Charge density} = \frac{\text{Charge}}{\text{Area}} = \frac{64 \times 10^{-6}}{0.032} = 2.0 \times 10^{-3}\ \text{C m}^{-2}$$

4. D

Choice A (incorrect)

By definition, electric potential at a point is the work done required to move a unit positive charge from infinity to that point.

Choice B (incorrect)

By definition, potential gradient at a point is:

$$\frac{dV}{dx} = -E$$

It is the negative of electric field at that point.

Choice C (incorrect)

By definition, electric field strength at a point is the force acting on a unit positive charge placed at that point.

Choice D (correct)

See explanation in choice B.

5. D

The maximum potential is the potential at which the electric field intensity at the sphere's surface just equal to air breakdown field. Therefore, the maximum potential is determined by maximum electric field at the surface. And only choice D is a factor affecting electric field at surface.

$$E_{max} = \frac{Q}{4\pi\varepsilon_0 R^2}$$

R is the radius of the sphere.

Choice C is a factor affecting the time needed to reach maximum potential. Other choices have no relation with the maximum potential.

6. A

The potential at any point of a conductor is always the same. Therefore, the potentials of the two spheres are equal.

And by the conservation of charge, charges on the two spheres only transfer and arrange themselves to make the two spheres having the same potential. No charge is being destroyed or created as the system is closed. Therefore, the charge is conserved.

And because there is a transfer of charges, current passes through the conducting wire. As wire prossesses resistance, there is a loss of electrical energy against resistance.

$$W = I^2R$$

B. Structured Questions

7. The relation between charge and electric potential is:

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

(a) For the sphere of 1.0 m radius,

$$1.0 \times 10^6 = \frac{Q}{4\pi(8.85 \times 10^{-12})1}$$

$Q = 1.11 \times 10^{-4}$ C

(b) For the sphere of 1.0 cm radius,

$$1.0 \times 10^6 = \frac{Q}{4\pi(8.85 \times 10^{-12})0.01}$$

$Q = 1.11 \times 10^{-6}$ C

8. (a) The total charge on the earth's surface is:

Charge = Surface charge density × Surface area
$= (2.0 \times 10^{-19}) \times 4\pi(6.4 \times 10^6)^2$
$= 1.03 \times 10^{-4}$ C

The electric potential of earth is:

$$V = \frac{Q}{4\pi\varepsilon_0 R} = \frac{1.03 \times 10^{-4}}{4\pi\varepsilon_0(6.4 \times 10^6)} = 0.145\ \text{V}$$

(b) The electric field strength at a point close to the earth's surface is:

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{0.145}{6.4 \times 10^6} = 2.26 \times 10^{-8}\ \text{V m}^{-1}$$

9. (a) Electric potential at point A is:

$$V_A = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r_{QA}} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{4} = \frac{Q}{16\pi\varepsilon_0}$$

Electric potential at point B is:

$$V_B = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r_{QB}} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{3} = \frac{Q}{12\pi\varepsilon_0}$$

The potential difference between two points is:

$$|V_A - V_B| = \frac{Q}{\pi\varepsilon_0}\left|\left(\frac{1}{16} - \frac{1}{12}\right)\right| = \frac{1.0 \times 10^{-6}}{\pi(8.85 \times 10^{-12})}(0.0208) = 7.49 \times 10^2\ \text{V}$$

(b) $\because$ $V_B > V_A$
$\therefore$ When a positive charge moves from B to A, work is done by the charge.
The work done is:
$W = Vq$
$= (7.49 \times 10^2) \times (2.0 \times 10^{-4})$
$= 0.1498$ J

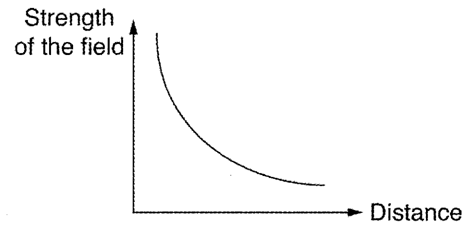
10. (a) Coulomb's Law in electrostatics states that the electric field due to a charge Q is:

$$E = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2} \propto \frac{1}{r^2}$$

The magnitude of the electric field is inversely proportional to the square of distance from the point.
Similarly, Newton's Law of Gravitation states that the gravitational field due to a mass M is:

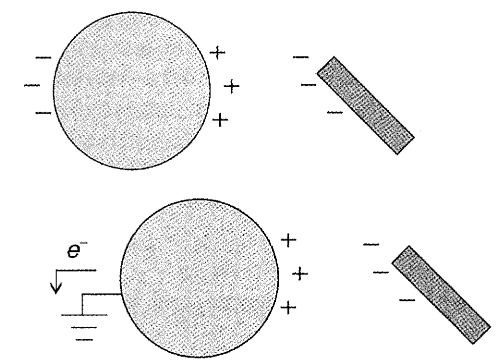
$$g = \frac{GM}{r^2} \propto \frac{1}{r^2}$$

The magnitude of gravitational field is also inversely proportional to the square of distance from the point.
These two are examples of the inverse square law field. The strength of this kind of field is inversely proportional to the square of distance.
The relation between strength and distance is shown below.

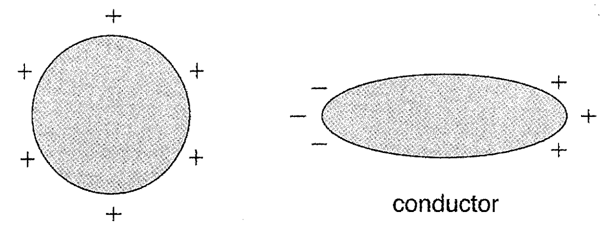


(b) Electric potential at a point is the energy required to bring a unit positive charge from infinity to that point. It is a property of electric field regardless of whether a charged object has been placed in that field. However, electric potential energy is the energy of a system consisting of the charged object and the external electric field.
Moreover, electric potential is measured in Joules per Coulomb or in Volt, but electric potential energy is measured in Joules.

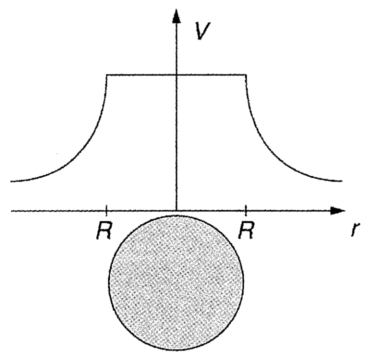
(c) (i) When a conductor is connected to the earth, it is always at the earth's potential. Conductors contain 'free electrons'. When a negatively-charged rod is placed nearby, the electrons of the conductor repel and move to the earth through the wire. As a result, the conductor carries a net positive charge.



(ii)



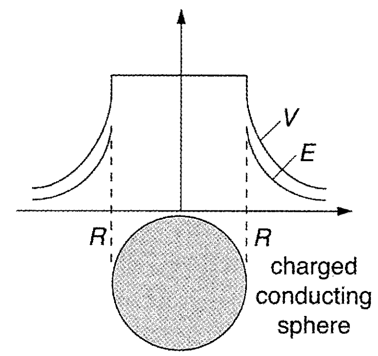
Around the positively-charged sphere, positive electric potential is generated as the graph shown below:

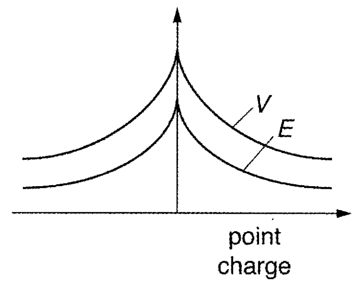


So, at the position of the conductor, the potential is positive.
And the conductor is isolated, although positive charges and negative charges are separated, it is always neutral (no net charge).

11. (a) (i) $\frac{Q}{4\pi\varepsilon_0 r^2}$ (ε_0 is the permittivity of free space)

(ii) Only for points outside the conducting sphere. Inside a sphere, the electric field is zero by symmetry and potential is a constant within the sphere. This case is different from the case of point charge, whose electric field and potential decrease with distance when $r < R$.





(b) (i) Both He nucleus and Au nucleus are positively charged, the force between them is repulsive. Therefore, energy is needed to move the He nucleus towards the Au nucleus.
By conservation of energy, the closest point the He nucleus can reach should be the point where the electric potential energy of the system (He & Au) is equal to the kinetic energy of the He nucleus.

$$\frac{1}{2} m_{He}v^2 = \frac{Q_{He}Q_{Au}}{4\pi\varepsilon_0 x}$$

(Q_{He}-charge of He, 2 positive charge,
Q_{Au}-charge of Au, 79 positive charge
M_{He}-mass of He, 2 neutrons and 2 protons)
The kinetic energy is:

$\frac{1}{2} m_{He}v^2$
$= \frac{1}{2}(2 \times 1.673 \times 10^{-27} + 2 \times 1.675 \times 10^{-27}) \times (7.0 \times 10^6)^2$
$= 1.64 \times 10^{-13}$ J

And electric potential energy = kinetic energy

$$\frac{Q_{He}Q_{Au}}{4\pi\varepsilon_0 x} = 1.64 \times 10^{-13}$$

$$\frac{2 \times 79 \times (1.6 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})x} = 1.64 \times 10^{-13}$$

$$x = 2.2 \times 10^{-13} \text{ m}$$

(ii) The gravitational attraction is negligible since the magnitude of electric force is much greater than gravitational force
($\because$ The masses of He and Au are small.)

C. Overseas & HKALE Questions

12. (a) Drawn on Fig. (a), the direction of the electric field is from the positive plate to the negative plate.

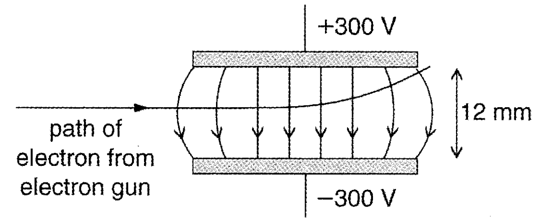


Fig. (a)

(b) (i) Electric field strength

$E = \frac{\Delta V}{d}$
$= \frac{300 - (-300)}{12 \times 10^{-3}}$
$= 5.0 \times 10^4$ N C^{-1}

(ii) Force on an electron
$= eE = (1.60 \times 10^{-19})(5.0 \times 10^4)$
$= 8.0 \times 10^{-15}$ N

(c) Referring to Fig. (a), the electron will be deflected towards the positive plate, describing a parabolic path.

13. (a) (i) The gravitational force is a force of attraction between masses with a magnitude that is proportional to the product of the masses and is inversely proportional to the square of their distance.
Whereas the gravitational field strength at a point in a gravitational field is the gravitational force per unit mass acting on any object at that point.

(ii) The electric potential V at a point in an electric field E is defined as the work done in bringing a unit positive charge from infinity to the point.
Whereas the electric potential energy U of a charge q in an electric field is defined as the work done in bringing the charge q from infinity to the point, i.e. $U = qV$.

(b) (i) Charge on the gold nucleus
$= n_p \times e = 79 \times 1.60 \times 10^{-19}$ C
$= 1.264 \times 10^{-17} = 1.26 \times 10^{-17}$ C

(ii) 1. Electric potential due to the gold nucleus

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$= \frac{1.264 \times 10^{-17}}{4\pi(8.85\times10^{-12})(2.5\times10^{-12})}$$

$= 4.546 \times 10^4 = 45$ kV

2. α-particle, $^4_2$He, has 2 protons.

Electric potential energy of the α-particle

$U = QV = (2 \times 1.60 \times 10^{-19})(4.546 \times 10^4)$

$= 1.5 \times 10^{-14}$ J

(c) (i) Relationship between the energies:

$E_T = E_G + E_E$

(ii) 1. In an α-particle scattering experiment, the ratio of the gravitational forces to the electrical forces is in the order of 10^{-13}. Therefore gravitational effects are ignored in the experiment.

2. The electric potential of the α-particle at a distance r from the nucleus is independent of the path taken to reach that position. Therefore the direction of approach need not be considered.

14. (a) (i) 1. The electric field strength at a point in an electric field is the electrostatic force acting per unit positive charge placed at that point.

2. The electric potential at a point in an electric field is the work done per unit charge in bringing the charge from infinity to the point, irrespective of the path taken.

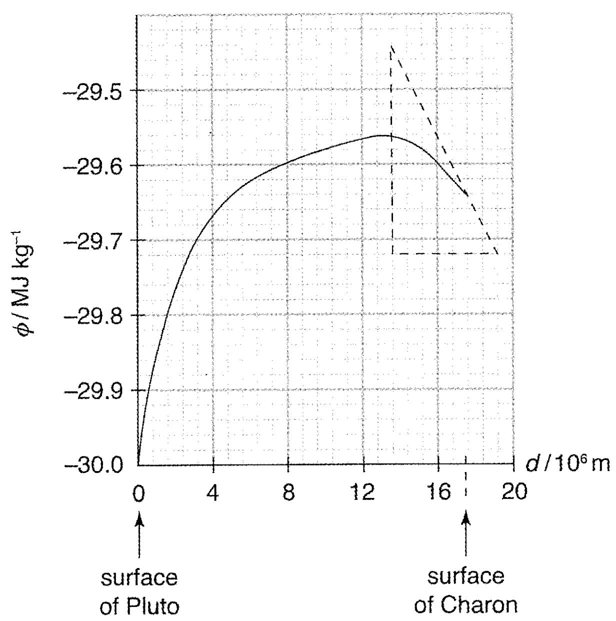
(ii) $$E = \frac{\Delta V}{\Delta x}$$

Hence, the electric field strength at a point may be computed from the gradient of the tangent at the point on the graph of electric potential versus distance from the point.

(b) (i) All values of gravitational potential are negative because masses always attract one another, i.e. the gravitational force between two masses is always attractive.

(ii) The gradient at a point on the graph of Fig. (a) equals the change in gravitational potential per unit distance from the surface. This equals the magnitude of the gravitational field strength at the point, according to the answer to **(a)(ii)** and is also the acceleration of free fall at that point.

(iii)



1. The acceleration of free fall is zero where the gradient of the graph in Fig. (a) is zero. From the graph, the distance from the surface of Pluto at which the acceleration of free fall is zero

$= 13.6 \times 10^6$ m.

2. The acceleration of free fall on the surface of Charon

= gradient at d is equal to 15.4×10^6 m

$$= \frac{[-29.52-(-29.76)]\times10^6}{(19.2-14.8)\times10^6}$$

(taking only the magnitude)

$= 0.055$ m s^{-2}

(c) (i) In order to reach Pluto, the rock must make it past the point where the acceleration of free fall is zero, i.e. where $d = 13.6 \times 10^6$ m. From that point to the surface of Pluto, change in potential,

$\Delta\phi = [-29.565 - (-30)] \times 10^6$ J kg^{-1}

$= 0.435 \times 10^6$ J kg^{-1}

The rock reaches Pluto with the minimum velocity when its velocity at $d = 13.6 \times 10^6$ m is zero.

$$\therefore \quad \frac{1}{2}mv^2 = m\Delta\phi$$

$$v = \sqrt{2\Delta\phi} = \sqrt{2\times0.435\times10^6}$$

$= 933$ m s^{-1}

(ii) In order for the rock to travel from Pluto to Charon, it must make it past the point $d = 13.6 \times 10^6$ m. The change in potential between this point and the surface of Charon is smaller. Hence the minimum speed on reaching Charon is less than that calculated in **(i)**.

15. – 16. HKALE Questions