Exercise (Projectile motion)

1

1 (a) (i) $F_{1x} = 120\,N$, $F_{1y} = 0\,N$

$F_{2x} = 200\cos 60^\circ = 100\,N$

$F_{2y} = 200\sin 60^\circ = 173.2\,N$

$F_{3x} = -150\cos 45^\circ = -106.1\,N$

$F_{3y} = -150\sin 45^\circ = -106.1\,N$

(ii) $R_x = F_{1x} + F_{2x} + F_{3x}$

$= 113.9\,N$

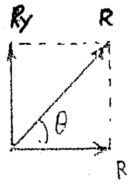
$R_y = F_{1y} + F_{2y} + F_{3y}$

$= 67.1\,N$

$R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{113.9^2 + 67.1^2}$

$= 132.2\,N$



$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{67.1}{113.9}$

$\theta = 30.5^\circ$

∴ The resultant has a magnitude 132.2 N and makes an angle 30.5° with the +ve x-axis

(b) $R_x = 200\cos 30^\circ + (-300\cos 45^\circ) + (-155\cos 53^\circ)$

$= -132.2\,N$

$R_y = 200\sin 30^\circ + 300\sin 45^\circ + (-155\sin 53^\circ)$

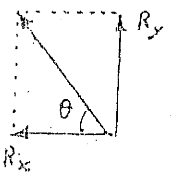
$= 188.3\,N$

$R = \sqrt{(132.2)^2 + (188.3)^2}$

$= 230.1\,N$

$\tan\theta = \dfrac{R_y}{R_x}$

$= \dfrac{188.3}{132.2}$



$\theta = 54.9^\circ$

∴ The resultant has a magnitude 230.1 N and makes an angle 54.9° with the -ve axis

2

2. (a)(i) during the 4^{th} second

$\bar{V}_x = \dfrac{\Delta x}{\Delta t} = \dfrac{8-6}{4-3} = 2\,ms^{-1}$

$\bar{V}_y = \dfrac{\Delta y}{\Delta t} = \dfrac{0.16-0.09}{4-3} = 0.07\,ms^{-1}$

(ii) during the 5^{th} second

$\bar{V}_x = \dfrac{\Delta x}{\Delta t} = \dfrac{10-8}{5-4} = 2\,ms^{-1}$

$\bar{V}_y = \dfrac{\Delta y}{\Delta t} = \dfrac{0.25-0.16}{5-4} = 0.09\,ms^{-1}$

(iii) during the 6^{th} second

$\bar{V}_x = \dfrac{\Delta x}{\Delta t} = \dfrac{12-10}{6-5} = 2\,ms^{-1}$

$\bar{V}_y = \dfrac{\Delta y}{\Delta t} = \dfrac{0.36-0.25}{6-5} = 0.11\,ms^{-1}$

(b) $a_x = \dfrac{\Delta V_x}{\Delta t} = \dfrac{2-2}{5.5-3.5} = 0\,ms^{-2}$

$a_y = \dfrac{\Delta V_y}{\Delta t} = \dfrac{0.11-0.07}{5.5-3.5} = 0.02\,ms^{-2}$

(c) During the 5^{th} second

$\bar{V}_x = 2\,ms^{-1}$

$\bar{V}_y = 0.09\,ms^{-1}$

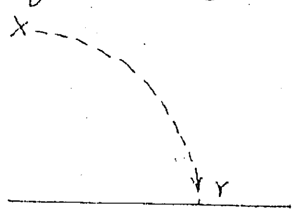
$\bar{V} = \sqrt{\bar{V}_x^2 + \bar{V}_y^2} = \sqrt{2^2 + 0.09^2}$

$= 2.0\,ms^{-1}$

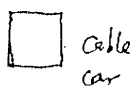
(d) An object is projected with an horizontal velocity

3

3. (a) (i)

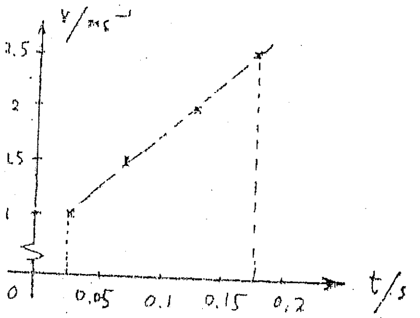


(ii) Directly above Y because the cable car and the ball have the same horizontal velocity



(b) $\bar{v}_y = \frac{\Delta y}{\Delta t}$

Downward distance/m	$\bar{v}_y$ /ms^{-1}
0.05	$\frac{0.05}{0.05} = 1$
0.075	$\frac{0.075}{0.05} = 1.5$
0.1	$\frac{0.1}{0.05} = 2$
0.125	$\frac{0.125}{0.05} = 2.5$



$$a = \frac{\Delta V}{\Delta t} = \frac{2.5-1}{0.175-0.025} = 10 \text{ ms}^{-2}$$

4

4. (a) Vertical direction

$$V_y = V_{0y} + a_y t = 0 - 10t$$

$$y = V_{0y}t + \tfrac{1}{2}a_y t^2 = 0 - 5t^2$$

at $t = 2.5$ s

$$y = -5(2.5)^2 = -31.25 \text{ m}$$

(31.25 m downward)

(b) Horizontal direction

$$V_x = V_{0x} = 25 \text{ ms}^{-1}$$

$$x = V_{0x}t = 25t$$

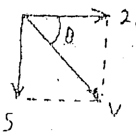
at $t = 2.5$ s

$$x = 25(2.5) = 62.5 \text{ m}$$

(c) at $t = 2.5$ s

$$V_x = 25 \text{ ms}^{-1}$$

$$V_y = -10(2.5) = -25 \text{ ms}^{-1}$$



$$v = \sqrt{25^2 + 25^2} = 35.4 \text{ ms}^{-1}$$

$$\tan\theta = \frac{25}{25} \Rightarrow \theta = 45^\circ$$

$\therefore$ 35.4 ms^{-1} at 45° downward with x-axis

5. (a) $V_{0x} = 25$ ms^{-1}, $V_{0y} = 20$ ms^{-1}

$a_x = 0$ ms^{-2}, $a_y = -10$ ms^{-2}

Horizontal direction

$$V_x = V_{0x} = 25 \text{ ms}^{-1}$$

$$x = V_{0x}t = 25t$$

Vertical direction

$$V_y = V_{0y} - gt = 20 - 10t$$

$$y = V_{0y}t - \tfrac{1}{2}gt^2 = 20t - 5t^2$$

t	V_x	V_y	x	y
2	25	0	50	20
3	25	-10	75	+15
6	25	-40	150	-60

(b) at highest point

$V_y = 0$

$\Rightarrow 20 - 10t = 0 \Rightarrow t = 2$ s

(c) at $t = 2$ s, $y = +20$ m

$\Rightarrow$ max. height = 20 m

(d) for the same level $y = 0$

$$0 = 20t - 5t^2$$

$$t(t-4) = 0$$

$t = 0$ or $t = 4$ s

$\Rightarrow$ it takes 4 s for the body to return to the original level

(e) $x = V_{0x}t = 25(4)$

$= 100$ m

6.

$$V_{0x} = V_0 \cos 30^\circ$$

$$V_{0y} = V_0 \sin 30^\circ$$

$$y = V_0 \sin 30^\circ t - \tfrac{1}{2}(10)t^2$$

When it return to the ground level

$$y = 0 \Rightarrow t = \frac{2V_0 \sin 30^\circ}{10}$$

Horizontal range

$$R = V_{0x}t = V_0 \cos 30^\circ \cdot \frac{2V_0 \sin 30^\circ}{10}$$

$$= \frac{V_0^2 \sin 60^\circ}{10}$$

Now the range should be $10(2) = 20$

$$\frac{v_0^2 \sin 60^\circ}{10} = 20$$

$$v_0 = 15.2 \text{ ms}^{-1}$$

(b) if there are n cars, the horizontal range must be greater than or equal to $n(2)$

$$\therefore \frac{v_0^2 \sin 60^\circ}{10} \geq 2n$$

$$v_0^2 \geq 23.1 n$$

$$\Rightarrow v_{0_{min}} = \sqrt{23.1 n} = 4.8\sqrt{n}$$

7 Since the cat and the rabbit have the same horizontal speed $\Rightarrow$ the cat is always directly above the rabbit

$\therefore$ the cat can intercept the rabbit

Vertical motion

$$y = 0 - \tfrac{1}{2}(10)t^2$$

when $y = -5$

$$\Rightarrow -5 = -5t^2 \Rightarrow t = 1 \text{ s}$$

$\therefore$ the cat reaches the ground level 1 s later

horizontal distance travelled during the same time

$= 10(1) = 10$ m

i.e. the cat intercepts the rabbit 10 m from the burrow

8(a), i, Vertical motion

$$v_y = 20 - (10)t$$

at maximum height, $v_y = 0$

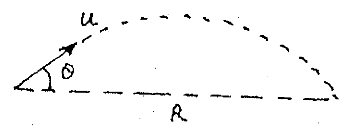
$$\Rightarrow 20 - 10t = 0 \Rightarrow t = 2 \text{ s}$$

(ii) Time needed to return to ground $= 2(2) = 4$ s

$\Rightarrow$ horizontal distance travelled

$= 15(4) = 60$ m

(b) The horizontal range for the long jumper is $R = \dfrac{u^2 \sin 2\theta}{g}$



For maximum R, $\sin 2\theta = 1$

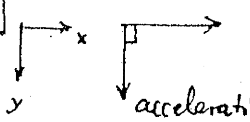
$$\Rightarrow \theta = 45^\circ$$

$\therefore$ Vertical component of velocity is $u \sin 45^\circ$

9. $\boxed{C}$ The vertical motions of X and Y are just the same because the horizontal initial velocity of X does not affect its vertical motion (The accelerations due to gravity are the same for X and Y)

$\Rightarrow$ they hit the ground at the same time

10. $\boxed{B}$



initial velocity v

acceleration $a_y = a$

$v_x = v_0$ because $a_x = 0 \Rightarrow (1)$ is ✓

$a_y = a = \text{constant}$

$\Rightarrow (2)$ is correct

$$v_y = v_{0y} + a_y t = at$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + (at)^2}$$

which is not a constant and it depends on time